Name: _____

Date: _____

Period: _____

Geometry Investigation 4

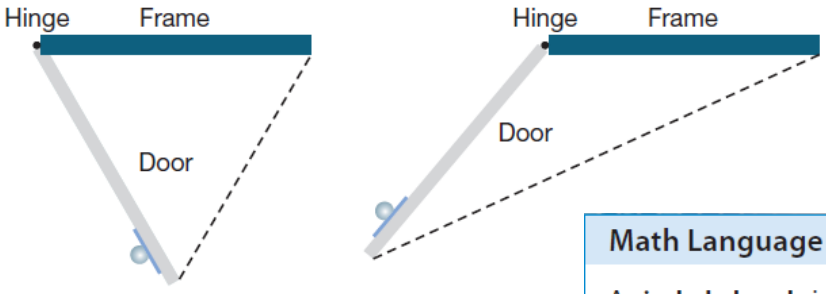
Inequalities in Two Triangles

In Lesson 39, you learned to verify whether three segments of given lengths could be used to form a triangle. This involved an inequality in one triangle. In this investigation, you will explore inequalities in two triangles.

1. Consider a door hinge. As shown in the diagram, the doorframe, door, and the distance between them make a triangle. The doorframe and the door itself are always the _____ length, so two sides of this triangle are fixed. What happens to the third side as the door is opened?

2. Model: Use a protractor to construct two triangles. Your triangles should have two pairs of congruent sides, and a third side that changes length depending on how much you open the "hinge," which is the included angle of the fixed sides. The hinge angles on your two triangles should have different

3. Write: Consider the door hinge example from step 1. If two triangles have two pairs of congruent sides, but their included angles are not congruent, what conclusion can you make about the third side?



Math Language

An **included angle** is an angle that is formed by two adjacent sides of a polygon. A hinge, like the one on a door, is an included angle of the doorframe and the door itself.

Theorem 40-1: Hinge Theorem - If two sides of one triangle are congruent to two sides of another triangle and the included angle of the first triangle is greater than the included angle of the second triangle, then the third side of the first triangle is longer than the third side of the second triangle.

The Hinge Theorem is useful when comparing two triangles with two pairs of congruent sides. Is the converse of the Hinge Theorem also true? Follow the steps below.

4. Draw an angle with side segments that are 4 and 5 inches long. Connect their endpoints to make a triangle. Label the vertex across from the 4-inch side *A*, the vertex across from the 5-inch side *B*, and the last vertex *C*. Measure $\overline{AB}$ and label its length on your diagram.

5. Draw a second angle, with a different measure from the first, that also has side segments that are 4 and 5 inches long. Connect their endpoints to make a triangle. Label the vertex across from the 4-inch side *D*, the vertex across from the 5-inch side *E*, and the last vertex *F*. Measure $\overline{DE}$ and label its length.

6. Measure the angles at vertices *C* and *F* using a protractor. What do you notice about these angles in comparison to each other and to the lengths of the sides opposite them?

7. The two triangles you drew illustrate the converse of the Hinge Theorem. State the converse of the Hinge Theorem, and determine whether it is true for your triangles.

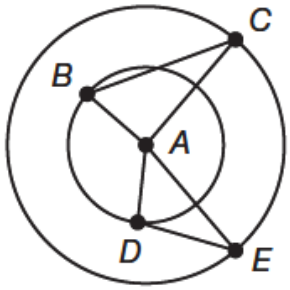
Theorem 40-2: Converse of the Hinge Theorem - If two sides of one triangle are congruent to two sides of another triangle and the third side of the first triangle is longer than the third side of the second triangle, then the measure of the angle opposite the third side of the first triangle is greater than the measure of the angle opposite the third side of the second triangle.

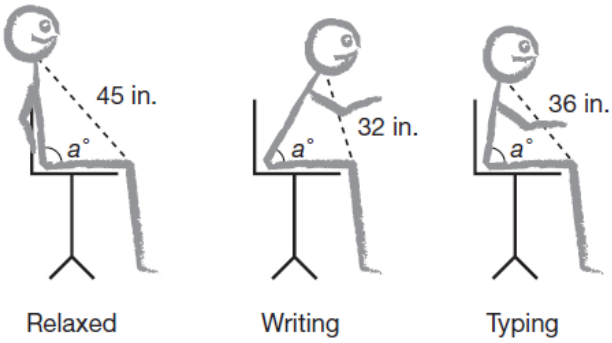
Math Reasoning

Analyze The Hinge Theorem is a more complex conditional statement than you have seen, taking the form "If (*p* and *q*), then *r*." Write out the statements *p*, *q*, and *r*.

The Hinge Theorem and the Converse of the Hinge Theorem can be used to compare two triangles.

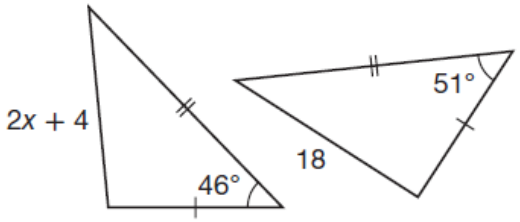
8.Multi-Step: Concentric circles are circles that have the same center but different radii measures. The circles illustrated here are concentric. The measure of $\angle BAC$ is 93°, and the measure of $\angle DAE$ is 60°. Explain why *BC* must be greater than *DE*.





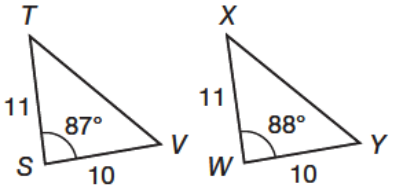
9.Ergonomics: Depending on the task a person is performing, the angle that a person's torso makes with their legs changes. The diagrams below show a student in three different sitting positions: relaxed, writing, and typing. In which position is the angle measure at the hip the greatest? … the least? Explain how you know.

10. Write an inequality that gives the possible values of *x* in the diagram.

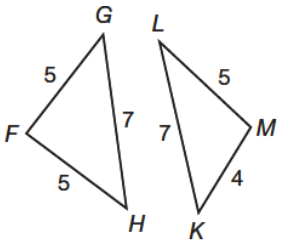


Investigation Practice

a. Use an inequality to compare the lengths of *TV* and *XY*.

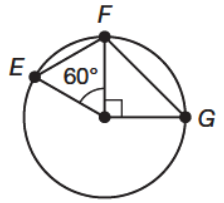


b. Use an inequality to compare the measures of $\angle G$ and $\angle L$.



c. Write: Describe how the hood of a car illustrates the Hinge Theorem.

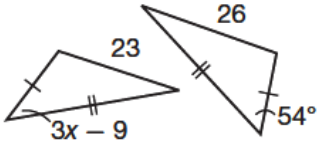
d. Multiple Choice Choose the most correct answer for the given diagram.



A $EF = FG$ B $EF < FG$

C $EF > FG$ D not enough information

e. Algebra Find the range of values for x.



f. Door Hinges: To prevent a door from opening too far and hitting a wall, a doorstop can be placed on the hinge of the door. A door in a small closet swings through a straight-line distance of 40 inches, while another door in a washroom swings through a straight-line distance of 48 inches. Which doorstop needs to be set to open to a larger angle? Explain.

g. Playground Equipment Kelvin and his friend Theo are swinging on a swing set at the local park. Both swings are the same length. Kelvin swings through an angle of 47° and Theo swings through an angle of 44°. Which of the two friends is swinging through the greatest distance? Explain.