Geometry Investigation 7
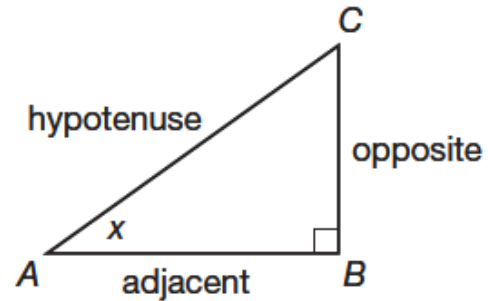
Trigonometric Ratios

The sine, cosine, and tangent ratios for a right triangle are:

$$\sin x = \frac{opposite}{hypotenuse} = \frac{}{}$$

$$\cos x = \frac{adjecent}{hypotenuse} = \frac{}{}$$

$$\tan x = \frac{opposite}{adjecent} =$$

Notice that tan x is the quotient of sin x and cos x:

$$\tan x = \frac{opposite}{adjecent}$$

$$\tan x = \left(\frac{opposite}{hypotenuse}\right) / \left(\frac{adjecent}{hypotenuse}\right)$$

$$\tan x = \underline{\quad\quad}$$

In this investigation, you will observe the values of sin x, cos x, and tan x as x varies from 0° to 90° in increments of 15°. Copy this table to record your results.

Caution

Remember that if a fraction has a radical in its denominator, you should rationalize it.

x	sin x	cos x	tan x
0°			
15°			
30°			
45°			
60°			
75°			
90°			

1. Imagine a right triangle with one angle measuring 0°. The side opposite this angle would be 0 units and the hypotenuse and adjacent side of the triangle would be congruent. Use this information to fill out the first row. Check your answers using a calculator.

2. Draw a diagram of a 30°-60°-90° triangle with the shortest side being 1 unit long. Use your diagram to fill out the 30° and 60° rows of the table.

3. Draw a diagram of a 45°-45°-90° triangle with legs that are 1 unit long. Use your diagram to fill out the 45° row of the table.

4. Use a calculator to fill out the 15° and 75° rows of the table.

5.What do you notice about the sine and cosine values of the 30° and 60° angles? Use this observation to fill out the final row of the table by comparing it to the 0° row. Find the tangent by dividing the sine of x by the cosine of x.
Review your table.

6. What do you notice about the sine of an angle, sin x, and the cosine of its complement, cos (90° – x)? Write a conjecture relating the sine and cosine of complementary angles.

7. Describe the ranges of values for cosine and sine, based on your table.

8. What is the value of sin x + cos x for x = 0° and 90°? Is this relationship true for the rest of the table?

9. Find the value of $sin^2x + cos^2x$ for several values of x. What do you notice about the value of $sin^2x + cos^2x$?

10. What is the range of the tangent function? Make a conjecture based on your table and test it by calculating the tangent of some other angles with your calculator. Explain why this is the range of the tangent function.

You Try!!!!
a. In ΔDEF, ∠E is a right angle, m∠D = 45°, and DE = 1. What is m∠F ?
Use the Converse of the Isosceles Triangle Theorem to relate DE and EF, and then use the Pythagorean Theorem to determine DF and EF. Then give exact values for sin 45°, cos 45°, and tan 45°.

b. In ΔGHI, ∠G is a right angle, m∠H = 60°, and GJ = 3. How are GH and HJ related? Determine GH and HJ. Then, give exact values for sin 60°, cos 60°, and tan 60°.

c. Use your response to a to draw a 45°-45°-90° triangle. Include all angle measures and side lengths.

d. Use your response to b to draw a 30°-60°-90° triangle. Include all angle measures and side lengths.