

Lesson 10

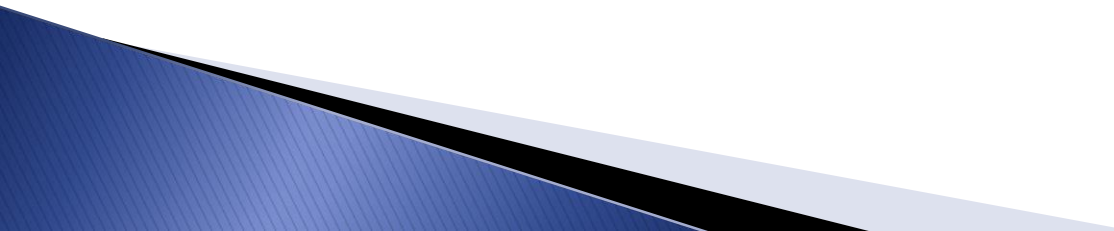
Using Conditional Statements

Conditional Statement – A statement in the form, “If p , then q ,” where p is the hypothesis and q is the conclusion. For example:

If it is morning, then the sun is in the east.

Hypothesis – The part of the statement that is between the words *if* and *then*. In the statement above, the hypothesis, p , is “it is morning.”

Conclusion – The part of the statement that follows the word *then*. In the statement above, the conclusion, q , is “the sun is in the east.”



Example 1 Identifying the Hypothesis and Conclusion

Identify the hypothesis and conclusion of each conditional statement.

a. *If $2x + 1 = 5$, then $x = 2$.*

SOLUTION

Hypothesis: $2x + 1 = 5$

Conclusion: $x = 2$

Example 1 Identifying the Hypothesis and Conclusion

Identify the hypothesis and conclusion of each conditional statement.

b. *If a plant is growing, then it needs water.*

SOLUTION

Hypothesis: *A plant is growing.*

Conclusion: *It needs water.*

Truth Value – Whether conditional statements are true or false. A statement is only false when the hypothesis is true and the conclusion is false.

For example: *If a rectangle has a width of 5 feet and a height of 4 feet, then its area is 30 square feet.*

Math Reasoning

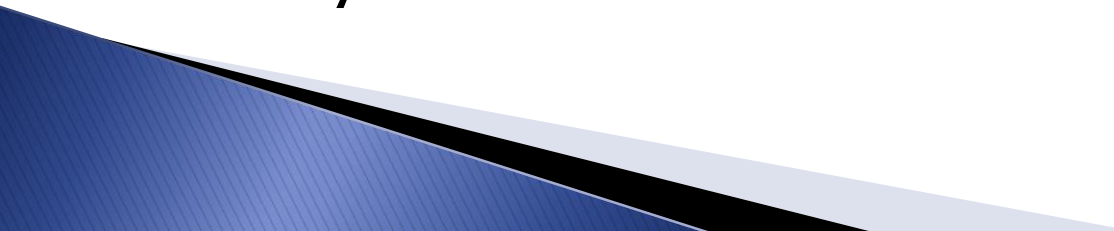
Connect Using the formula you learned in Lesson 9, write the sample statement so it is true.

The hypothesis is true, but the conclusion of this statement is false. Since the hypothesis is true but the conclusion is false, the statement's truth value is false.

If a conditional statement's hypothesis is false, then the statement could still be true.

For example, consider the statement, "If Ai wins the lottery, he will take a vacation."

The hypothesis is false if Ai does not win the lottery, but the statement is still true, because the statement only applies if Ai does win the lottery.



Example 2 Evaluating the Truth Value of a Conditional Statement

Determine whether each statement is true or false. If it is false, explain your reasoning.

a. *If an angle is obtuse, it measures 120° .*

SOLUTION

The hypothesis of this statement is true, but the conclusion is false. An obtuse angle can measure anything greater than 90° and less than 180° . Any obtuse angle that is not 120° could be used to contradict this statement. Therefore, the statement is false.

Example 2 Evaluating the Truth Value of a Conditional Statement

Determine whether each statement is true or false. If it is false, explain your reasoning.

b. *If two parallel lines intersect, then they form acute angles.*

SOLUTION

The hypothesis of this statement is false because parallel lines are defined as lines that never intersect. When the hypothesis of a conditional statement is false, the conditional statement as a whole has a truth value of “true.” The statement cannot be said to be false unless a situation exists where the hypothesis is true.

Converse of a Statement – The statement formed by exchanging the hypothesis and conclusion. The converse of a statement “if p , then q ” has the form “if q , then p .”

Consider the following conditional statement:

If it is morning, then the sun is in the east.

The converse of this statement is:

If the sun is in the east, then it is morning.

Math Reasoning

Analyze What is the result of taking the converse of a converse statement?

Even if a conditional statement is true, the converse of that statement is not necessarily true.


For example:

If an animal is a duck, then it can fly.

The converse of this statement is:

If an animal can fly, then it is a duck.

This statement is not true. There are many animals that can fly that are not ducks.



Example 3 Stating Converses

Write the converse of each statement and determine whether the converse is true.

a. *If an animal is a dog, then it has four legs.*

SOLUTION

Converse: *If an animal has four legs, then it is a dog.*

The converse is not true. We can prove it is not true by finding an untrue example. For example, a cat is also an animal with four legs, but it is not a dog.

Example 3 Stating Converses

Write the converse of each statement and determine whether the converse is true.

b. *If $x = 4$, then $3x + 7 = 19$.*

SOLUTION

Converse: *If $3x + 7 = 19$, then $x = 4$.*

The converse is also true.

Example 4 Application: Biology

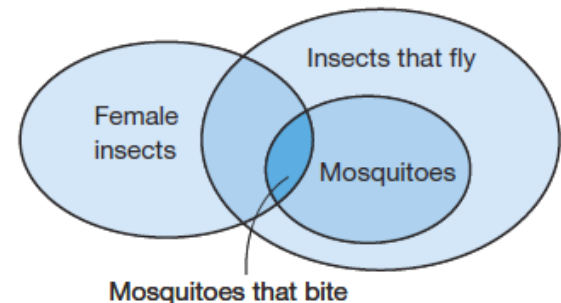
Write the converse of each conditional statement. Use the Venn diagram to determine if the converse is true.

a. *If an insect is a mosquito, then it can fly.*

SOLUTION

Converse: *If an insect can fly, then it is a mosquito.*

There are many flying insects besides mosquitoes, so the converse is false.



Example 4 Application: Biology

Write the converse of each conditional statement. Use the Venn diagram to determine if the converse is true.

b. *If a mosquito bites, then it is female.*

SOLUTION

Converse: *If a mosquito is female, then it bites.*

We see from the diagram that the entire region indicating “female mosquitoes” overlaps completely with “mosquitoes that bite,” so the converse of this statement is also true.

You Try!!!!

Find the converse of each statement and determine whether it is true.

d. *If $x^2 = 9$, then $x = 3$ or -3 .*

If $x = 3$ or -3 , then $x^2 = 9$. False

e. *If it is Thanksgiving Day, then it is Thursday.*

*If it is Thursday, then it is Thanksgiving Day.
False.*

f. *If a cardinal is a male, then it is bright red.*

*If it is bright red, then a cardinal is a male.
False*

Assignment

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Lesson Practice a–f (Ask Mr. Heintz)

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Practice 1–30 (Do the starred ones first)