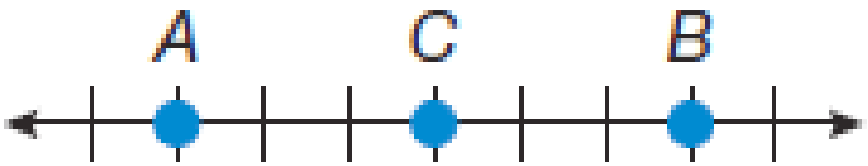
Lesson 11

Finding Midpoints

For two points on a number line A and B, the midpoint of $\overline{AB}$ is the point that is equidistant from both A and B. For point C to be equidistant from A and B means that the distance from A to C is the same as the distance from B to C.

Midpoint on a Number Line – The midpoint C of $\overline{AB}$ has a coordinate that is the average of the coordinates of A and B:

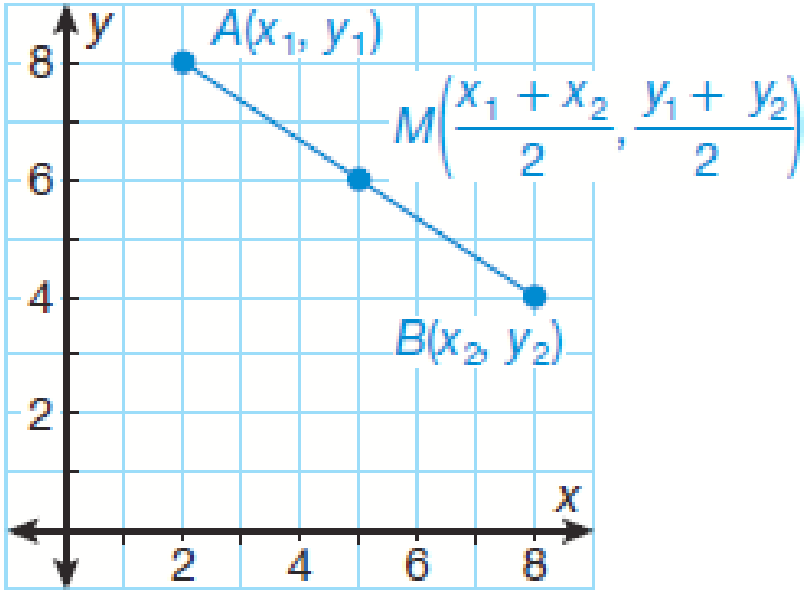
$$C = \frac{A + B}{2}$$



The midpoint of $\overline{AB}$ on a coordinate plane is the point *M* on $\overline{AB}$ that is equidistant from *A* and *B*. To find the midpoint of a segment on a coordinate plane, use the midpoint formula given below.

Midpoint on a Coordinate Plane – The midpoint M of $\overline{AB}$ with endpoints $A(x_1, y_1)$ and $B(x_2, y_2)$, has coordinates that are given by the formula:

$$M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_1}{2}\right)$$



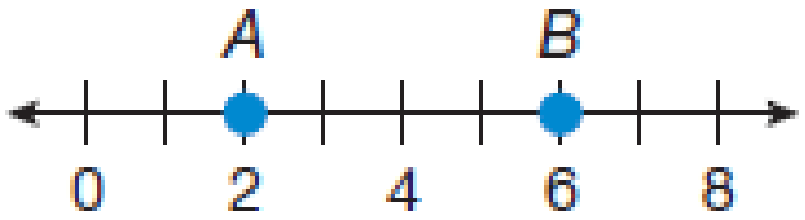
Example 1 Finding the Midpoints

a. What is the coordinate of the midpoint of $\overline{AB}$?

SOLUTION The midpoint is the coordinate on the number line that is the average of the coordinates of the points:

$$C = \frac{2 + 6}{2}$$

$$C = 4$$



b. Determine the midpoint M of $\overline{AB}$ connecting (1, 2) and (5, 6).

SOLUTION

Substitute (1, 2) for (x_1, y_1) and (5, 6) for (x_2, y_2).

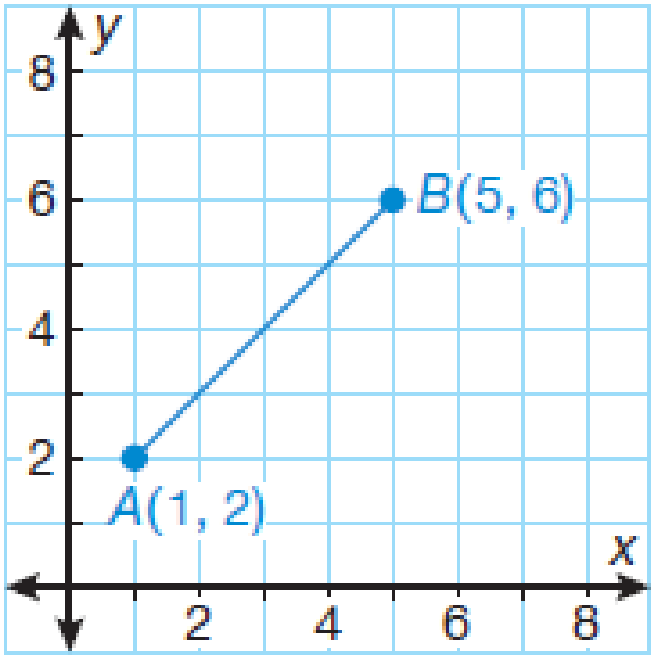
$$M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_1}{2}\right)$$

$$M\left(\frac{1 + 5}{2}, \frac{2 + 6}{2}\right)$$

$$M(3,4)$$

To check, plot the point (3, 4).

It should lie on $\overline{AB}$.



Also, the distance formula can be used to verify that (3, 4) is equidistant from *A* and *B*:

$MA = \sqrt{(3-1)^2 + (4-2)^2}$ $MB = \sqrt{(3-5)^2 + (4-6)^2}$

$MA = \sqrt{(2)^2 + (2)^2}$ $MB = \sqrt{(-2)^2 + (-2)^2}$

$MA = \sqrt{4+4}$ $MB = \sqrt{4+4}$

$MA = \sqrt{2(4)}$ $MB = \sqrt{2(4)}$

$MA = 2\sqrt{2}$ $MB = 2\sqrt{2}$

Example 2 Finding Midpoints of Sides

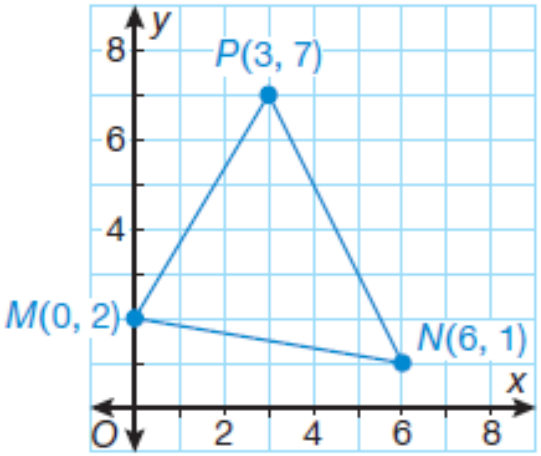
Determine the midpoint of each side of ΔMNP.

SOLUTION Use the midpoint formula to find A, the midpoint of $\overline{MN}$.

$$A\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_1}{2}\right)$$

$$A\left(\frac{0 + 6}{2}, \frac{2 + 1}{2}\right)$$

$$A(3, 1.5)$$



Similarly, the midpoints B of $\overline{NP}$ and C of $\overline{MP}$ have coordinates:

$$B\left(\frac{x_1+x_2}{2}, \frac{y_1+y_1}{2}\right)$$

$$B\left(\frac{3+6}{2}, \frac{7+1}{2}\right)$$

$$B(4.5, 4)$$

$$C\left(\frac{x_1+x_2}{2}, \frac{y_1+y_1}{2}\right)$$

$$C\left(\frac{0+3}{2}, \frac{2+7}{2}\right)$$

$$C(1.5, 4.5)$$

Example 3 Application: Navigation

A fishing boat dropped its anchor equidistant from Cape Spirit and Endeavor Rock Lighthouse, on the segment joining the two locations. Find the coordinates of the boat.

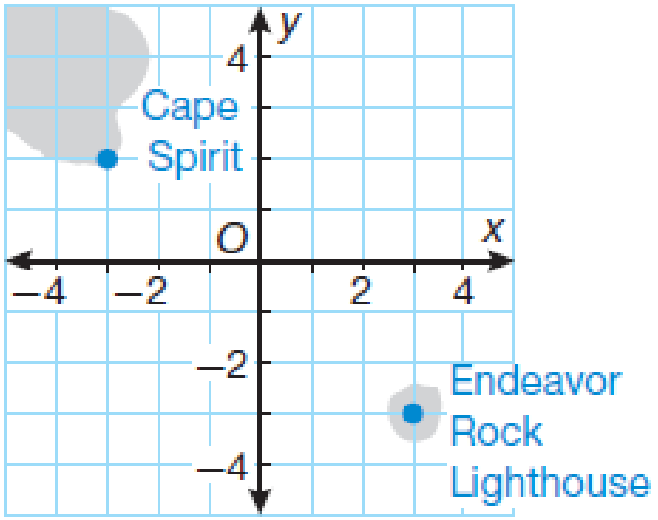
SOLUTION Let point T represent the location of the boat. Point T is the midpoint of the segment with endpoints (−3, 2) and (3, −3).

$$T\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_1}{2}\right)$$

$$T\left(\frac{-3 + 3}{2}, \frac{2 + (-3)}{2}\right)$$

$$T(0, -0.5)$$

Draw the location of the boat on the diagram.



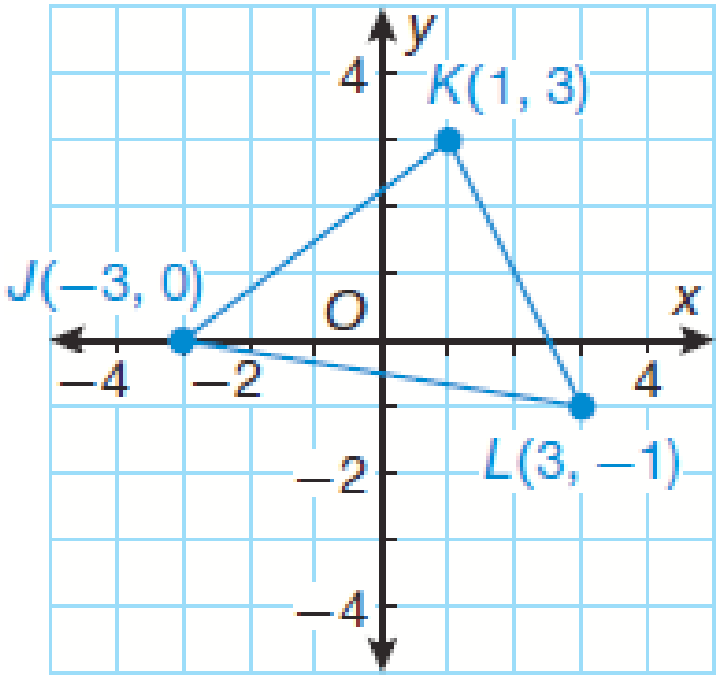
You Try!!!!

d. Determine the coordinates of the midpoint of each side of ΔJKL.

$A(-1,1.5)$

B$(2,1)$

C$(0,-0.5)$



Assignment

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Lesson Practice a-e (Ask Mr. Heintz)

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Practice 1-30 (Do the starred ones first)