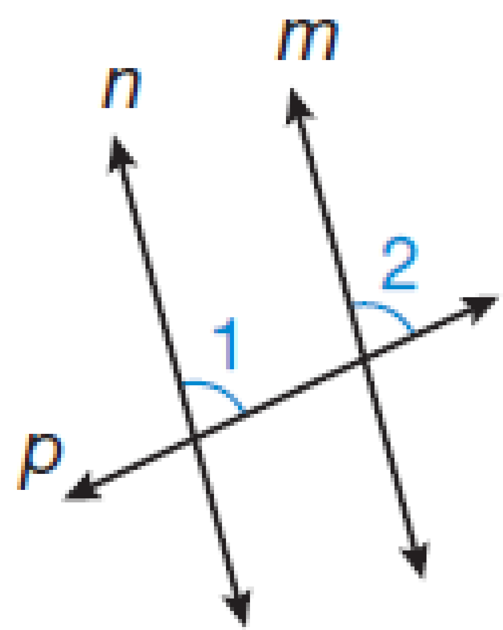
Lesson 12

Proving Lines Parallel

In Investigation 1, you learned Postulate 11: if two parallel lines are cut by a transversal, the corresponding angles formed are congruent. The converse of Postulate 11 is also true, and can be used to show that two lines are parallel.

Postulate 12: Converse of the Corresponding Angles Postulate – If two lines are cut by a transversal and the corresponding angles are congruent, then the lines are parallel.

If $\angle 1 \cong \angle 2, then\ m \parallel n.$



Hint

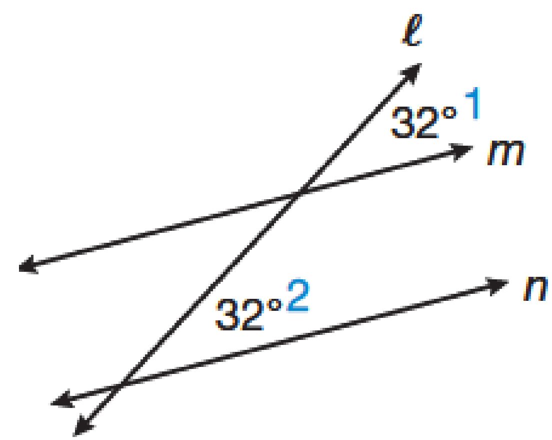
The postulate and theorems in Investigation 1 refer to two parallel lines cut by a transversal to prove that certain angles are congruent or supplementary. The postulate and theorems in this lesson work conversely. That is, they use known angle relationships to prove that lines are parallel.

Example 1 Proving Parallelism: Corresponding Angles

Prove that lines *m* and *n* in this diagram are parallel.

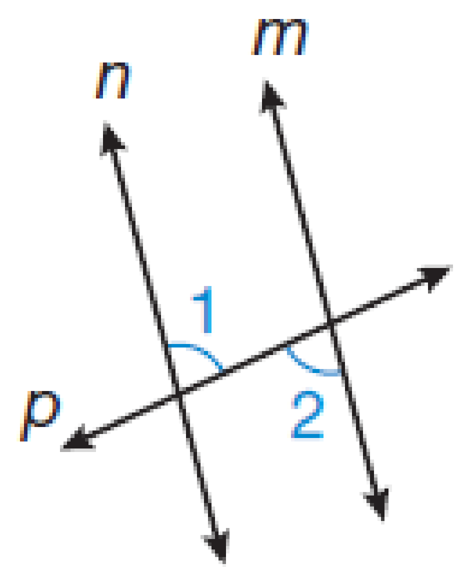
SOLUTION

Angles 1 and 2 both measure 32°, so by the definition of congruent angles, ∠1 and ∠2 are congruent. Since ∠1 and ∠2 are corresponding congruent angles, lines *m* and *n* are parallel by Postulate 12.



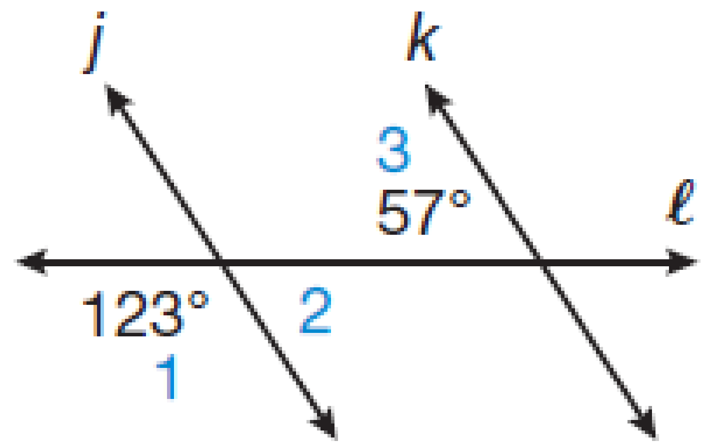
Theorem 12-1: Converse of the Alternate Interior Angles Theorem – If two lines are cut by a transversal and the alternate interior angles are congruent, then the lines are parallel.

$$\boldsymbol{If\ \angle 1 \cong \angle 2, then\ m \parallel n.}$$



Example 2 Proving Parallelism: Alternate Interior Angles

Prove that lines j and k in this figure are parallel.



SOLUTION

Angles 1 and 2 form a linear pair, which means they are supplementary.

$m\angle 1 + m\angle 2 = 180^\circ$

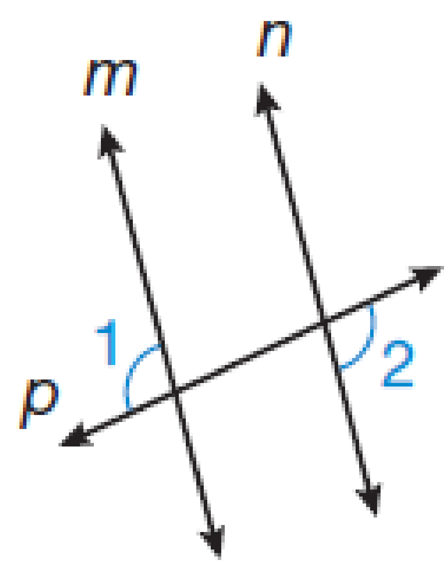
$123^\circ + m\angle 2 = 180^\circ$

$m\angle 2 = 57^\circ$

Since $m\angle 2 = m\angle 3$, $\angle 2 \cong \angle 3$. Angles 2 and 3 are congruent alternate interior angles, so by Theorem 12-1, lines j and k are parallel.

Theorem 12–2: Converse of the Alternate Exterior Angles Theorem – If two lines are cut by a transversal and the alternate exterior angles are congruent, then the lines are parallel.

$$\boldsymbol{If\ \angle 1 \cong \angle 2, then\ m \parallel n.}$$



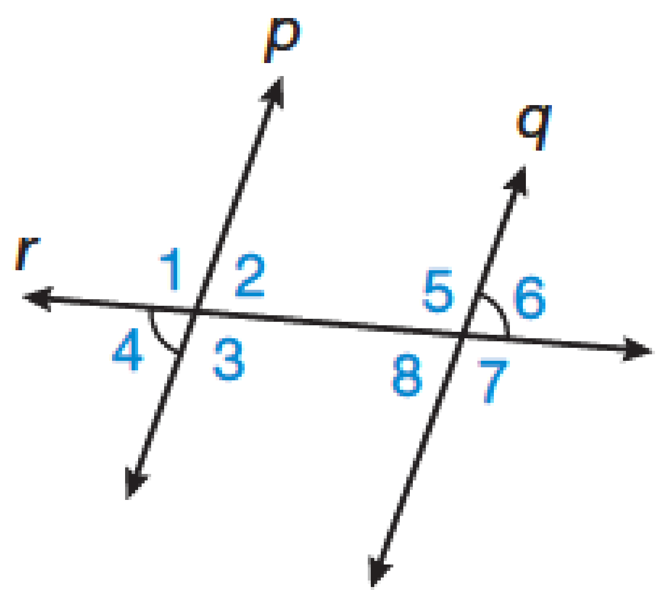
Example 3 Proving Parallelism: Alternate Exterior Angles

a. Identify both pairs of alternate exterior angles in this figure.

SOLUTION

$\angle 1$ and $\angle 7$ are alternate exterior angles.

$\angle 4$ and $\angle 6$ are also alternate exterior angles.

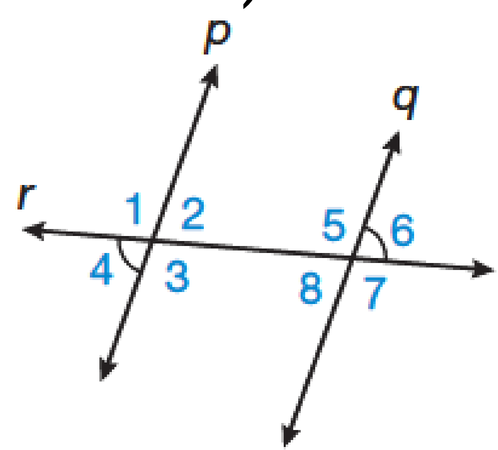


Example 3 Proving Parallelism: Alternate Exterior Angles

b. Prove that lines *p* and *q* are parallel.

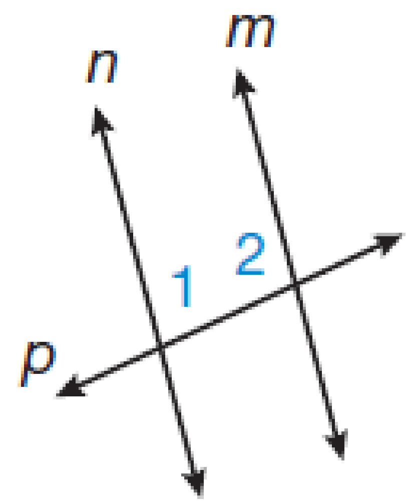
SOLUTION

The angle congruency marks show that the alternate exterior angles, $\angle 4$ and $\angle 6$, are congruent. Therefore lines *p* and *q* are parallel by the Converse of the Alternate Exterior Angles Theorem (Theorem 12-2).



Theorem 12-3: Converse of the Same-Side Interior Angles Theorem – If two lines are cut by a transversal and the same-side interior angles are supplementary, then the lines are parallel.

$$If\ m\angle 1 + m\angle 2 = 180°, then\ m \parallel n.$$

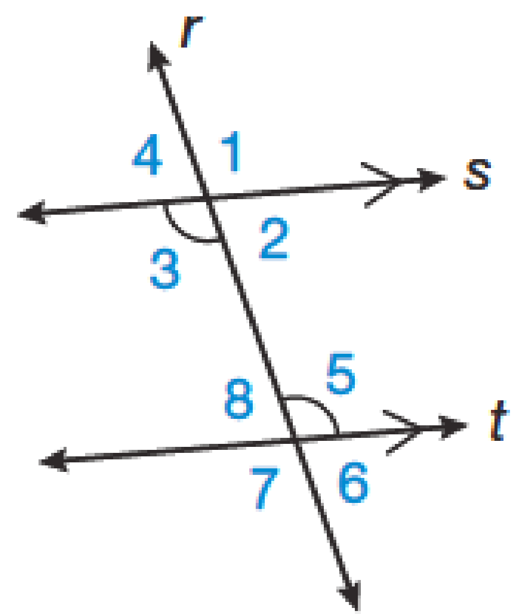


Example 4 Proving Parallelism: Same-Side Interior Angles

a. Identify both pairs of same-side interior angles in this figure.

SOLUTION

Angle 2 and $\angle 5$ are same-side interior angles. Angle 3 and $\angle 8$ are also same-side interior angles.



Example 4 Proving Parallelism: Same-Side Interior Angles

b. Use the Converse of the Same-Side Interior Angles Theorem (Theorem 12-3) to prove that lines *s* and *t* are parallel.

SOLUTION

It is shown in the drawing that $\angle 3 \cong \angle 5$. Angle 5 and $\angle 8$ are supplementary since they form a straight line. Therefore, by substitution, $\angle 3$ and $\angle 8$ are supplementary. Since $\angle 3$ and $\angle 8$ are also same-side interior angles, Theorem 12-3 proves that lines *s* and *t* are parallel.

Example 5 Application: City Planning

In San Francisco, California, Columbus Avenue crosses Stockton, Powell, Mason, and Taylor Streets as shown on the map. Columbus Avenue makes a 40˚ angle with each of these four streets.

a. What geometric term best describes Columbus Avenue?

SOLUTION

Columbus Avenue is a transversal

Example 5 Application: City Planning

b. Prove that Powell, Mason, and Taylor streets are all parallel to each other.

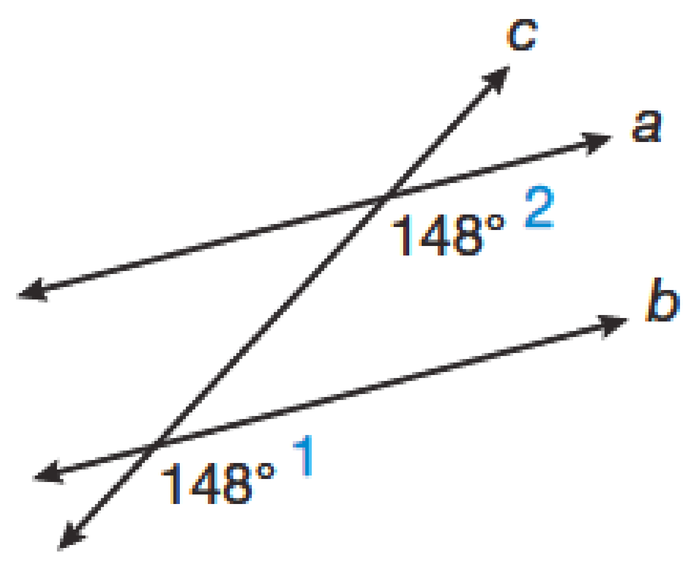
SOLUTION

The two 40° angles at the intersections of Columbus and Mason, and Columbus and Powell are congruent by definition. They are also corresponding angles. By Postulate 12, Mason and Powell are parallel. Taylor is parallel to Mason and Powell for the same reason. Since two lines that are parallel to the same line are also parallel to each other (Theorem 5-7), all three streets are parallel to one another.

You Try!!!!

Prove that lines *a* and *b* in this figure are parallel.

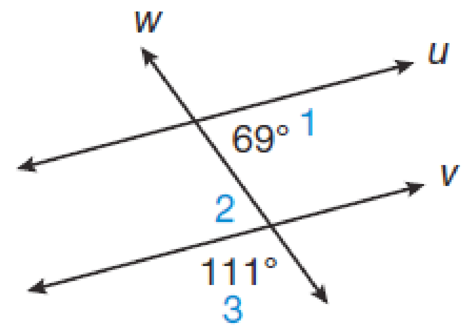
Angles 1 and 2 both measure 148°, so by the definition of congruent angles, ∠1 and ∠2 are congruent. Since ∠1 and ∠2 are corresponding congruent angles, lines a and b are parallel by Postulate 12.



You Try!!!!

Prove that lines *u* and *v* in this figure are parallel.

Angles 2 and 3 form a linear pair, which means they are supplementary.



$$m\angle 2 + m\angle 3 = 180°$$
$$111° + m\angle 2 = 180°$$
$$m\angle 2 = 69°$$

Since $m\angle 2 = m\angle 1$, $\angle 2 \cong \angle 1$. Angles 2 and 1 are congruent alternate interior angles, so by Theorem 12-1, lines *u* and *v* are parallel.

Use the diagram to answer problems c through f.

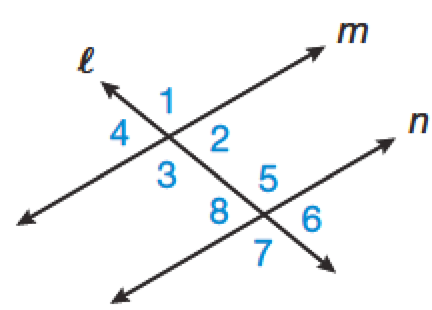
You Try!!!!

c. Identify both pairs of alternate exterior angles in this figure.

$\angle 1$ and $\angle 7$, $\angle 4$ and $\angle 6$

d. Given that $\angle 1 \cong \angle 7$, prove that lines *m* and *n* are parallel.

$\angle 1$ and $\angle 7$ are $\cong$ alternate exterior angles; lines m and n are parallel by Theorem 12-2.



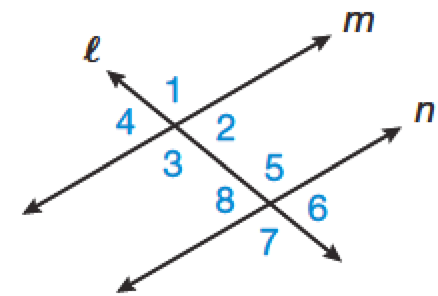
You Try!!!!

e. Identify both pairs of same-side interior angles in this figure.

$\angle 2$ and $\angle 5$, $\angle 3$ and $\angle 8$

f. Given that $\angle 2 \cong \angle 6$, use Theorem 12-3 to prove that lines *m* and *n* are parallel.

Angles 5 and 6 are supplementary; since $\angle 2 \cong \angle 6$, $\angle 2$ and $\angle 5$ are supplementary; lines m and n are parallel by Theorem 12-3.



You Try!!!!

g. City Planning: Speer Boulevard crosses Fox Street, Elati Street, and Delaware Street. Give the geometric term for Speer Boulevard.

Transversal

You Try!!!!

h. City Planning: Prove that Fox, Elati, and Delaware streets are all parallel to one another.

The two 33° angles at the intersections of Speer and Fox, and Speer and Elati are congruent by definition. They are also corresponding angles. By Postulate 12, Fox and Elati are parallel. Delaware is parallel to Fox and Elati for the same reason. Since two lines that are parallel to the same line are also parallel to each other (Theorem 5-7), all three streets are parallel to one another.

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Prove that lines *a* and *b* in this figure are parallel.

Angles 1 and 2 both measure 148°, so by the definition of congruent angles, $\angle 1$ and $\angle 2$ are congruent. Since $\angle 1$ and $\angle 2$ are corresponding congruent angles, lines a and b are parallel by Postulate 12.

Prove that lines *u* and *v* in this figure are parallel.

Angles 2 and 3 form a linear pair, which means they are supplementary.

$m\angle 2 + m\angle 3 = 180°$

$111° + m\angle 2 = 180°$

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Since $m\angle 2 = m\angle 1$, $\angle 2 \cong \angle 1$. Angles 2 and 1 are congruent alternate interior angles, so by Theorem 12-1, lines *u* and *v* are parallel.

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e. Identify both pairs of same-side interior angles in this figure.

$\angle 2$ and $\angle 5$, $\angle 3$ and $\angle 8$

f. Given that $\angle 2 \cong \angle 6$, use Theorem 12-3 to prove that lines *m* and *n* are parallel.

Angles 5 and 6 are supplementary; since $\angle 2 \cong \angle 6$, $\angle 2$ and $\angle 5$ are supplementary; lines m and n are parallel by Theorem 12-3.

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∠1 and ∠7 are ≅ alternate exterior angles; lines m and n are parallel by Theorem 12-2

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Assignment

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Practice 1–30 (Do the starred ones first)