Lesson 14

Disproving Conjectures with Counterexamples

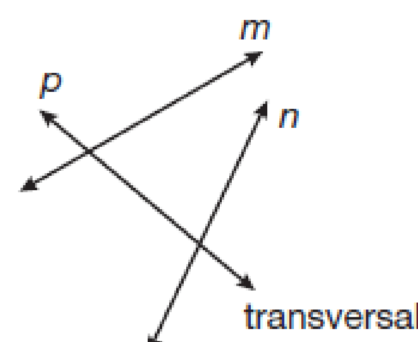
Consider the simple conjecture given below.

If two lines are both intersected by a transversal, then they are parallel.

This conjecture is false: two lines do not have to be parallel to be intersected by a transversal. A simple way to prove that this statement is not true is to use a counterexample.

Counterexample – An example that proves a conjecture or statement false.

For example, the diagram shows a pair of lines that are not parallel, but they are intersected by a transversal. It disproves the statement given above because it gives a specific example where the statement is *not* true. To construct a counterexample, find a situation where the hypothesis of the statement is true but the conclusion is false.



Example 1 Finding a Counterexample to a Geometric Conjecture

Use the conjecture to answer a and b.

If a triangle is isosceles, then it is acute.

a. What is the hypothesis of the conjecture? What is its conclusion?

SOLUTION

Hypothesis: *The triangle is isosceles*.

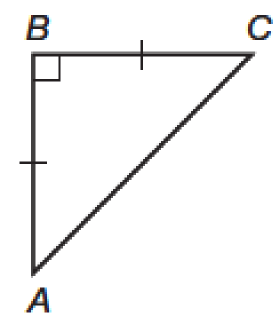
Conclusion: *The triangle is acute*.

Example 1 Finding a Counterexample to a Geometric Conjecture

b. Find a counterexample to the conjecture.

SOLUTION

A counterexample would be an example of a triangle for which the hypothesis is true, but the conclusion is false; that is, a triangle that is isosceles but not acute. Consider this right triangle, *ABC*. Since $\overline{BC}$ and $\overline{AB}$ are congruent, ΔABC is isosceles. Since $\angle B$ is a right angle, ΔABC is not an acute triangle. Therefore, ΔABC Is a counterexample to the conjecture.



Not all conjectures are geometric. Counterexamples can be used to disprove algebraic conjectures or any other kind of conjecture.

Example 2 Finding a Counterexample to an Algebraic Conjecture

a. Find a counterexample to the conjecture.

Every quadratic equation has either no solution or two solutions.

SOLUTION

You probably remember that sometimes quadratic equations can have only one solution. Such an equation would have to have only one x-intercept. A simple example is the parent function for quadratic equations.

$0 = x^2$

This equation can be solved by graphing, using the quadratic formula, or factoring. The answer is $x = 0$. Since this is a quadratic equation that has only one solution, the statement is proven false by this counterexample.

Example 2 Finding a Counterexample to an Algebraic Conjecture

b. Find a counterexample to the conjecture.

If $5x - 10 = 15$, *then* $2x + y > 9$.

SOLUTION

First, solve the hypothesis of this statement. We find that for the hypothesis to be true, $x = 5$. Then substitute $x = 5$ into the conclusion to solve for y.

$$2x + y > 9$$

$$2(5) + y > 9$$

$$y > -1$$

So for the conclusion to be true, y must be greater than -1. A counterexample to the statement is any value of y that is less than -1. Only one counterexample is needed, so a possible answer is $y = -2$.

Example 3 Application: Astronomy

Use the data in the table to prove the conjecture false.

If a planet orbits our Sun, its orbital period (*year*) *is proportional to its distance from the Sun*.

SOLUTION

The hypothesis of the conjecture, *the planet orbits our Sun*, is true for all three planets in the table. If the conclusion were true, the ratio $\frac{Orbital\ Period}{Distance\ from\ Sun}$ should be the same for all three planets. Extend the table by calculating this proportion for each planet:

Planet	Orbital Period (days)	Distance from Sun (million miles)	Proportion
Earth	365	93.0	$\frac{365}{93.0} \approx 3.92$
Mars	687	142	$\frac{687}{142} \approx 4.84$
Saturn	10,760	888	$\frac{10{,}760}{888} \approx 12.12$

By looking at the fourth column of the table, it is clear that the proportion is not always the same. Any two of these planets provide a counterexample that proves the statement false.

You Try!!!!

Use the conjecture below to answer a and b.

If line a is perpendicular to line b and to line c, then lines b and c are perpendicular.

a. What is the hypothesis of the conjecture? What is its conclusion?

b. Find a counterexample to the conjecture.

You Try!!!!

Use the conjecture below to answer c and d.

If x 2 = 9, then x = 3.

c. What is the hypothesis of the conjecture? What is its conclusion?

d. Find a counterexample to the conjecture.

e.The masses of two sedimentary rocks are 327 grams and 568 grams, respectively. Their volumes are 275 cm 3 and 501 cm 3, respectively.

Explain how this data disproves the conjecture below.

If a rock is sedimentary, then its mass is proportional to its volume.

Assignment

Page 86

Lesson Practice (Ask Mr. Heintz)

Page 87

Practice 1–30 (Do the starred ones first)