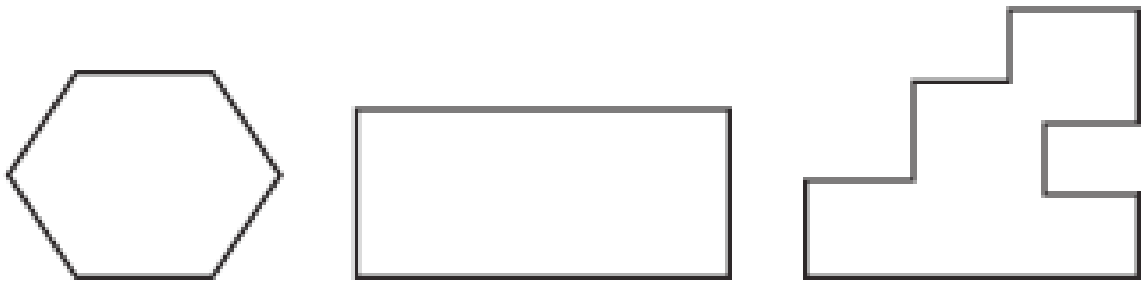
Lesson 15

Introduction to Polygons

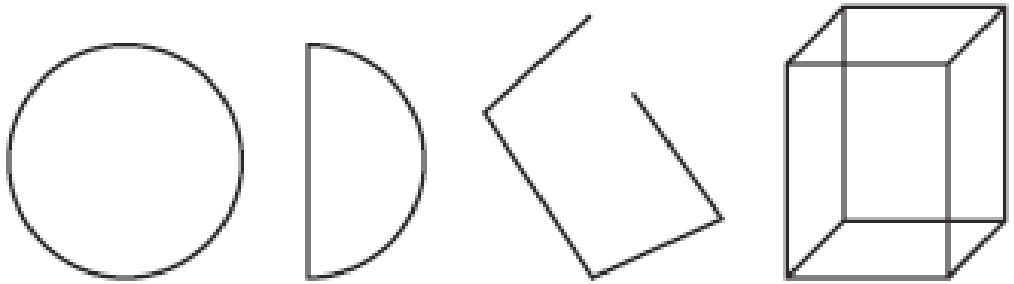
Polygon – A closed plane figure formed by three or more segments. Each segment intersects exactly two other segments only at their endpoints. No two segments with a common endpoint are collinear.

Sides – The segments that form a polygon.

Vertex of a Polygon – The intersection of two of its sides.



Polygons



Not Polygons

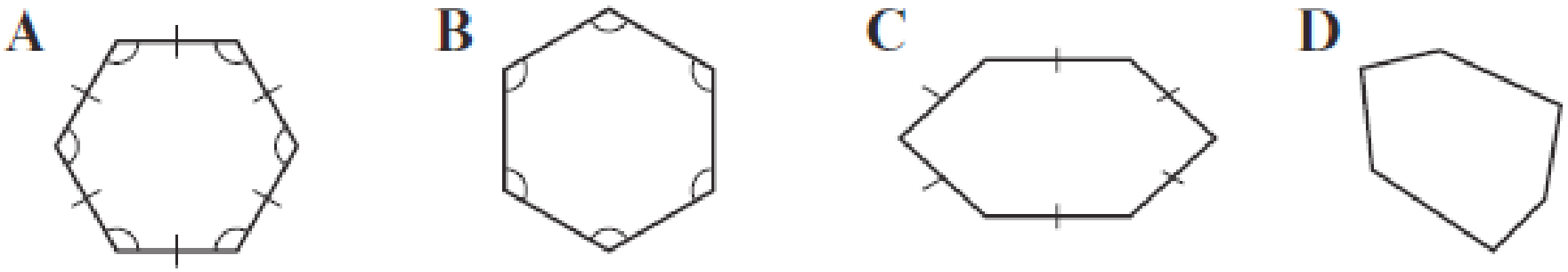
Equiangular Polygon – A polygon in which all angles are congruent.

Equilateral polygon – A polygon in which all sides are congruent.

Regular Polygon – If a polygon is both equiangular and equilateral.

Irregular Polygon – If a polygon is not equiangular and equilateral.

In the diagram, polygons A and B are equiangular. Polygons A and C are equilateral. Since polygon A is both equiangular and equilateral, it is a regular polygon. Polygons B, C and D are all irregular.



Polygons are named by the number of sides they have. The chart below shows some common polygons and their names.

Name	Sides	Regular Polygon	Irregular Polygon
Triangle	3		
Quadrilateral	4		
Pentagon	5		
Hexagon	6		
Heptagon	7		
Octagon	8		
Nonagon	9		
Decagon	10		
Hendecagon	11		
Dodecagon	12		

Example 1 Classifying Polygons

Classify each polygon. Determine whether it is equiangular, equilateral, regular, irregular, or more than one of these.

SOLUTION

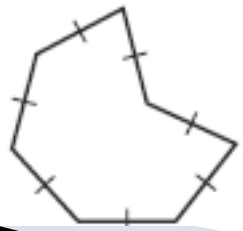
Polygon A has 5 sides, so it is a pentagon. It is equiangular but not equilateral, so it is irregular.

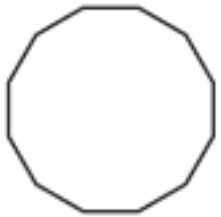
Polygon B has 7 sides, so it is a heptagon. It is equilateral and irregular.

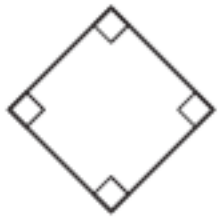
Polygon C is a dodecagon. It is irregular.

Polygon D is a quadrilateral. It is equilateral and equiangular, so it is regular.

A 

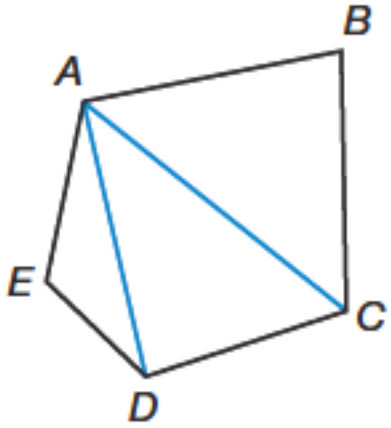
B 

C 

D 

Diagonal of a Polygon – A segment that connects two nonconsecutive vertices of a polygon.

Example: Pentagon *ABCDE* has two diagonals, $\overline{AC}$ and $\overline{AD}$, from vertex *A*. Three other diagonals could be drawn: $\overline{BD}, \overline{BE}, \text{ and } \overline{CE}$.

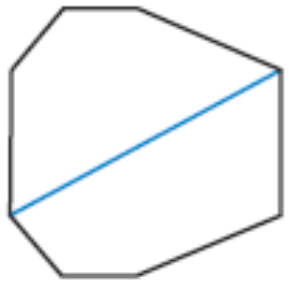


Diagonals can help determine whether a polygon is concave or convex.

Convex Polygon – Every diagonal of the polygon lies inside it, except for the endpoints.

Concave Polygon – At least one diagonal can be drawn so that part of the diagonal contains points in the exterior of the polygon.

Concave polygon



Convex polygon

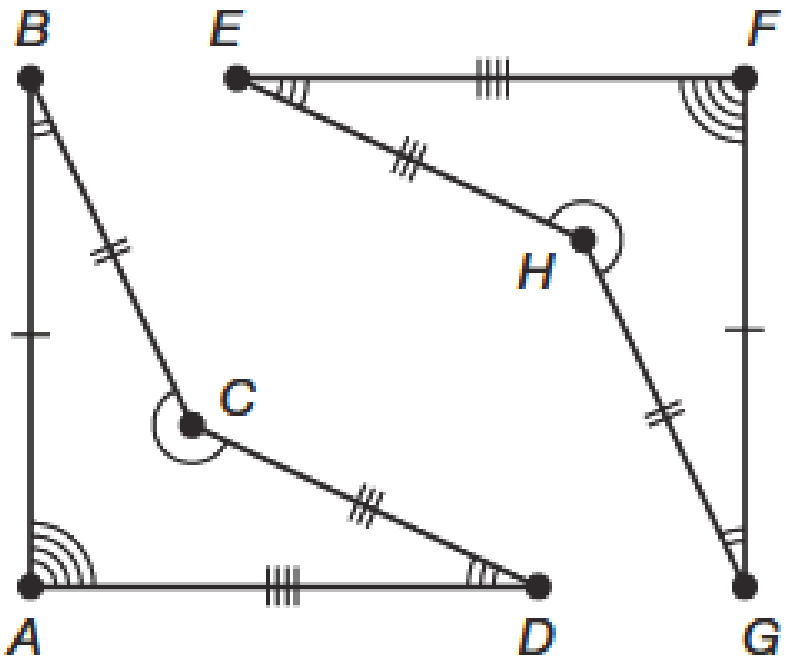
If two polygons have the same size and shape, they are congruent polygons.

Example 2 Identifying Polygon Properties

a. Find a diagonal that contains points in the exterior of polygon *ABCD*.

SOLUTION

Diagonal $\overline{BD}$ lies outside polygon *ABCD*, except for its endpoints.

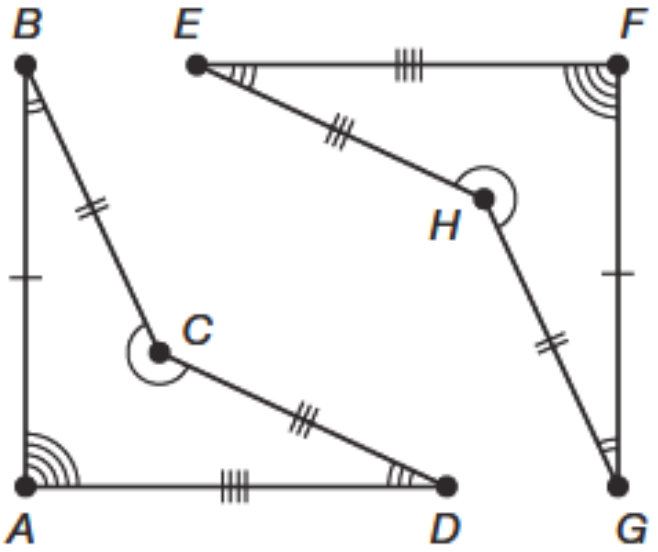


Example 2 Identifying Polygon Properties

b. Determine whether polygon *EFGH* is convex or concave. Explain.

SOLUTION

Diagonal $\overline{EG}$ contains points in the exterior of polygon *EFGH*. Therefore, polygon *EFGH* is concave.

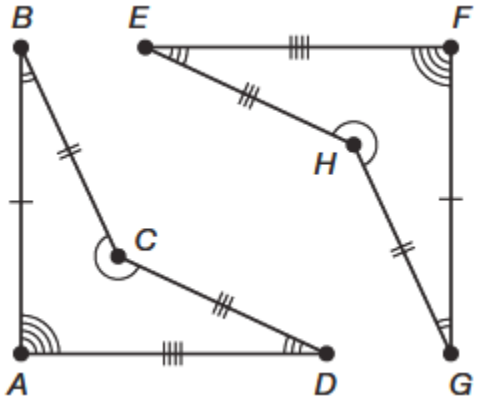


Example 2 Identifying Polygon Properties

c. Are polygons *ABCD* and *FGHE* congruent? Justify your answer.

SOLUTION

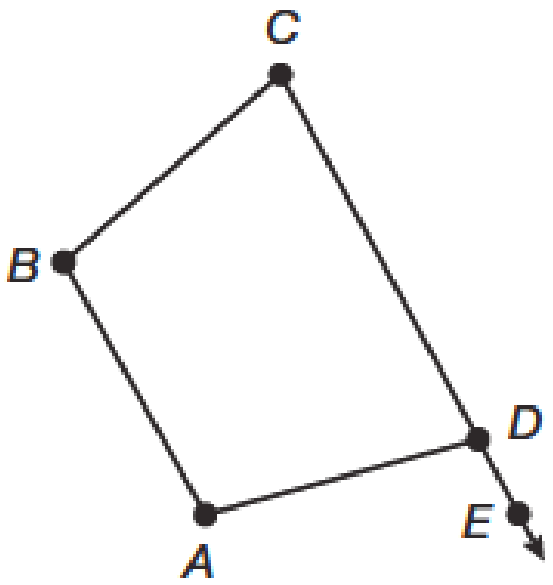
Write a congruency statement for all corresponding sides and angles. Angle pairs $\angle A \cong \angle F, \angle B \cong \angle G, \angle C \cong \angle H, and\ \angle D \cong \angle E$. Sides $\overline{AB} \cong \overline{FG}, \overline{BC} \cong \overline{GH}, \overline{CD} \cong \overline{HE}, and\ \overline{DA} \cong \overline{EF}$. Therefore, $ABCD \cong FGHE$.



Interior angle of a polygon – An angle formed by two sides of a polygon with a common vertex.

Exterior angle of a polygon – An angle formed by one side of a polygon and the extension of an adjacent side.

In the diagram, $\angle CDA$ is an interior angle and $\angle ADE$ is an exterior angle.

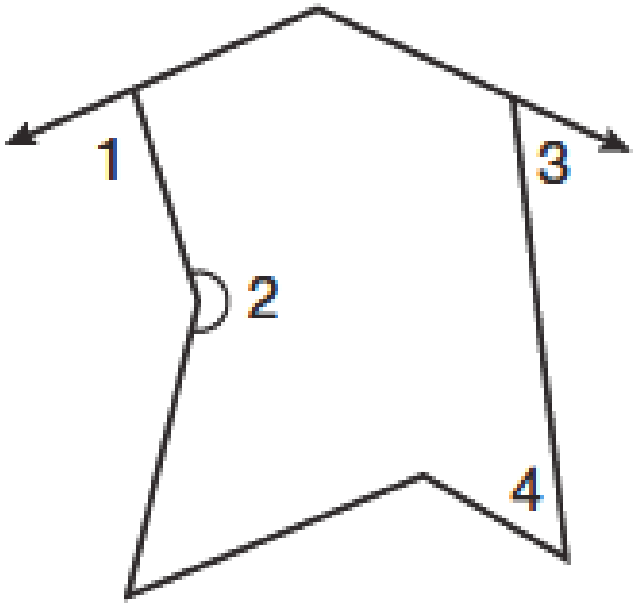


Example 3 Identifying Interior and Exterior Angles of Polygons

For each numbered angle in the polygon, determine whether it is an interior angle or an exterior angle.

SOLUTION

Angles 2 and 4 are interior. Angles 1 and 3 are exterior.



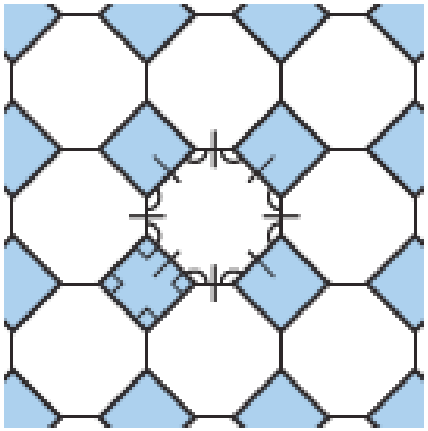
Example 4 Application: Tile Patterns

This floor tile pattern uses polygonal tiles that fit together exactly.

a. Name the two types of polygons used in the pattern. Are they regular or irregular? Explain.

SOLUTION

Square and octagon; both types are regular, because they have all sides and all angles congruent, respectively.

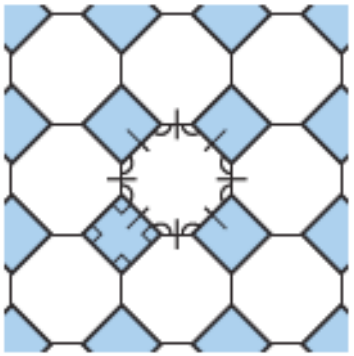


Example 4 Application: Tile Patterns

b. Pick any pair of unshaded polygons. Are they congruent? Are they convex or concave? Explain.

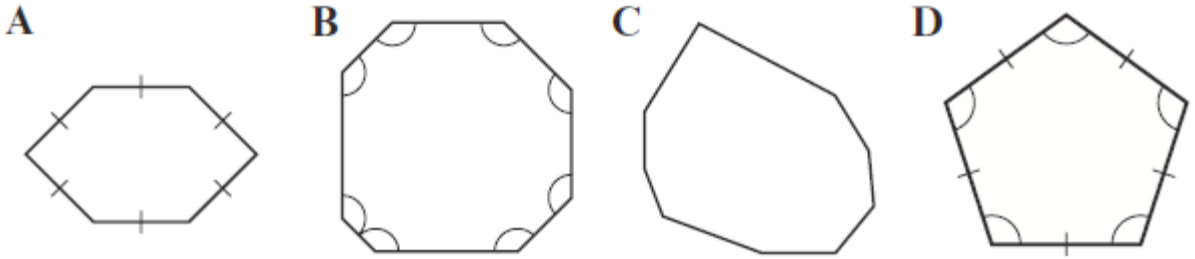
SOLUTION

All pairs of unshaded polygons are congruent, because corresponding sides and angles are congruent. Each unshaded polygon is convex, because none of the polygon's diagonals contain points in its exterior.

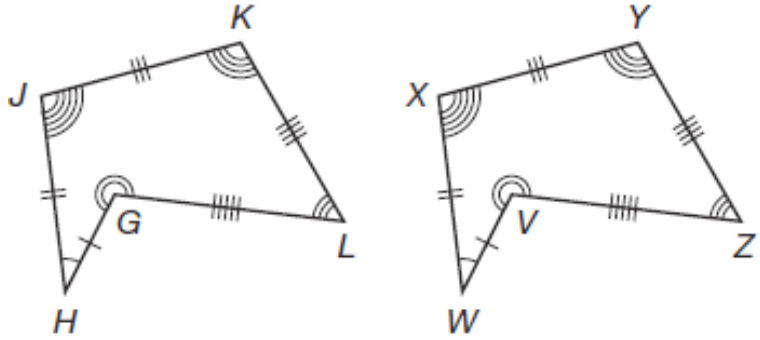


You Try!!!!

a.Name each polygon. Determine whether it is equiangular, equilateral, regular, irregular, or more than one of these.

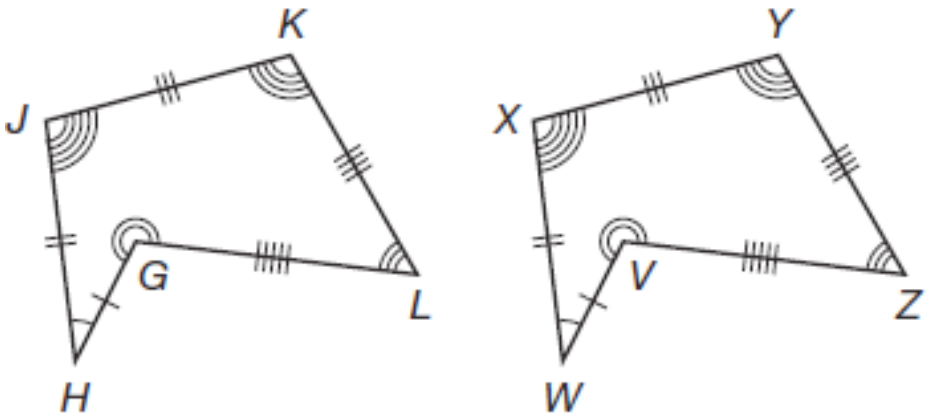


b.Find a diagonal in polygon *GHJKL* that contains points in the exterior of the polygon.



You Try!!!!

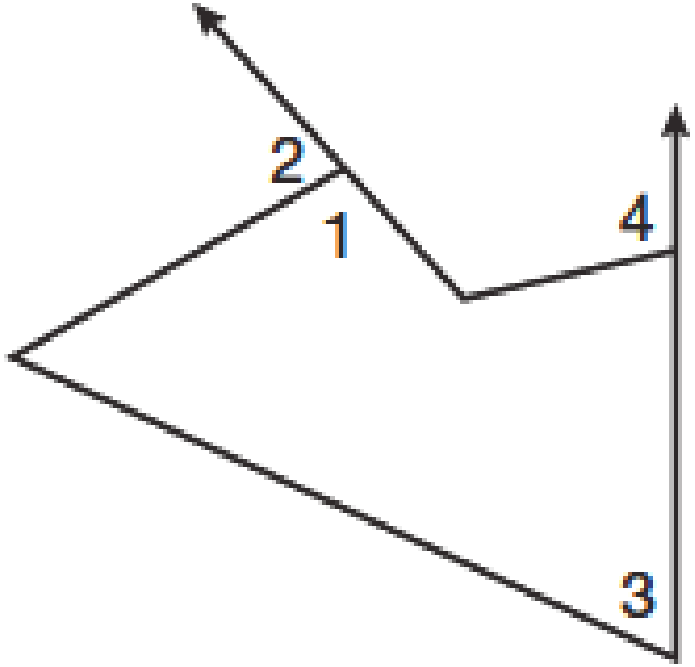
c.Determine whether polygon *VWXYZ* is convex or concave. Explain.



d.Are polygons *GHJKL* and *VWXYZ* congruent? Justify your answer.

You Try!!!!

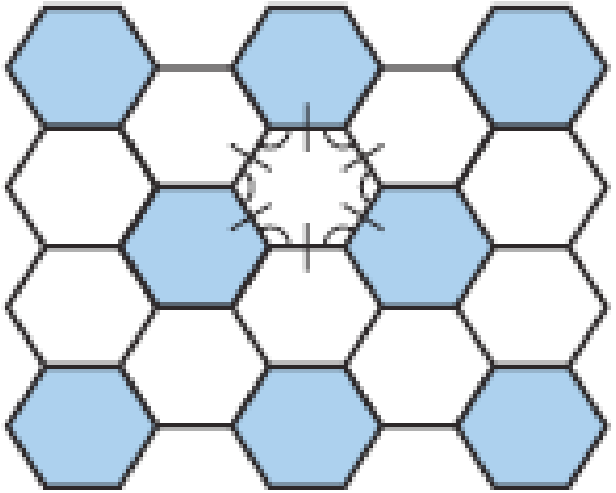
e. For each numbered angle in the polygon, determine whether it is an interior angle or an exterior angle.



You Try!!!!

f. Name the type of polygon used in this pattern. Are the polygons regular or irregular? Explain.

g.Pick any pair of polygons in this pattern. Are they congruent? Are they convex or concave? Explain.



Assignment

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Lesson Practice (Ask Mr. Heintz)

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Practice 1-30 (Do the starred ones first)