

Lesson 16

Finding Slope and Equations of Lines

Linear Equation – An equation whose graph is a line. Some examples are:

$$y = 3x - 1$$

$$10 = 2x$$

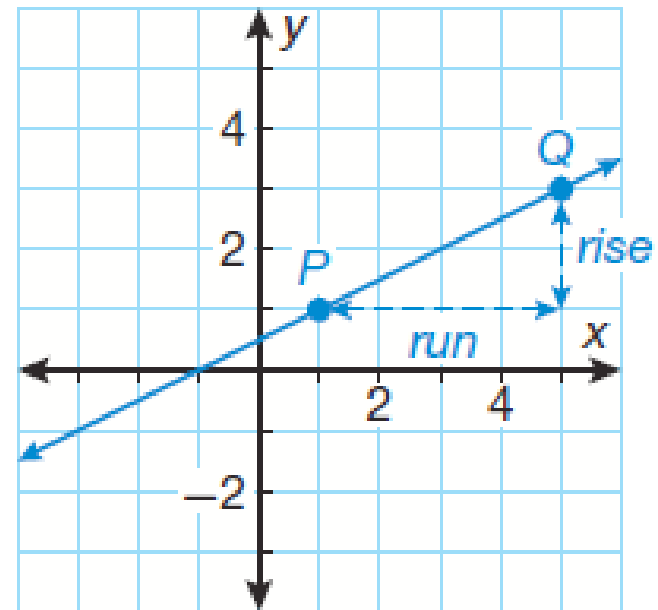
$$2x + 5y = 7$$

$$\frac{x}{4} + \frac{y}{13} = 1$$

The variables in linear equations never have exponents other than 1. Linear equations connect algebra (equations in x and y) to geometry (lines in a coordinate plane).

The rise from P to Q is the *vertical* change between P and Q , and equals $y_2 - y_1$.

The run from P to Q is the *horizontal* change between P and Q , and equals $x_2 - x_1$.



Slope – The ratio of the vertical change (rise) between two points on a line to the horizontal change (run).

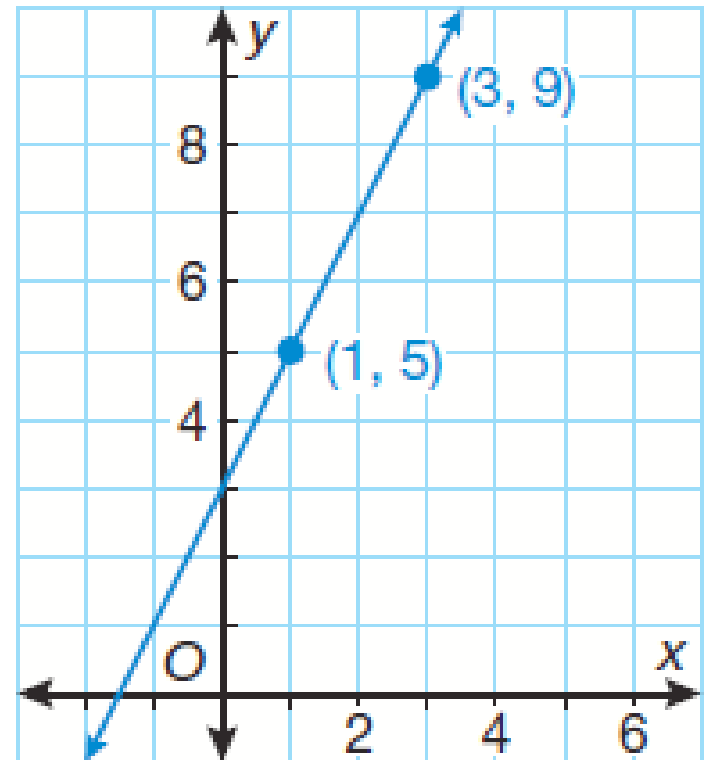
$$\text{slope} = m = \frac{y_2 - y_1}{x_2 - x_1}$$

Example 1 Finding the Slope of a Line

SOLUTION

Use the points (1, 5) and (3, 9) to calculate the slope. The rise is 4 units and the run is 2 units.

$$\text{The slope is } \frac{\text{rise}}{\text{run}} = \frac{4}{2} = 2.$$



Horizontal line – The rise is always zero, so the slope is 0.

Vertical line – The run is zero, so the slope is undefined because division by zero is undefined.

The slope–intercept form of a linear equation is a way of writing a linear equation using the slope (m) and the y –intercept (b) of the line. This way of writing the equation has the form $y = mx + b$.

Example 2 Writing the Equation of a Line

a. Use this graph of a line to write its equation.

SOLUTION

First, determine the slope m using the points $(2, 5)$ and $(4, 11)$.

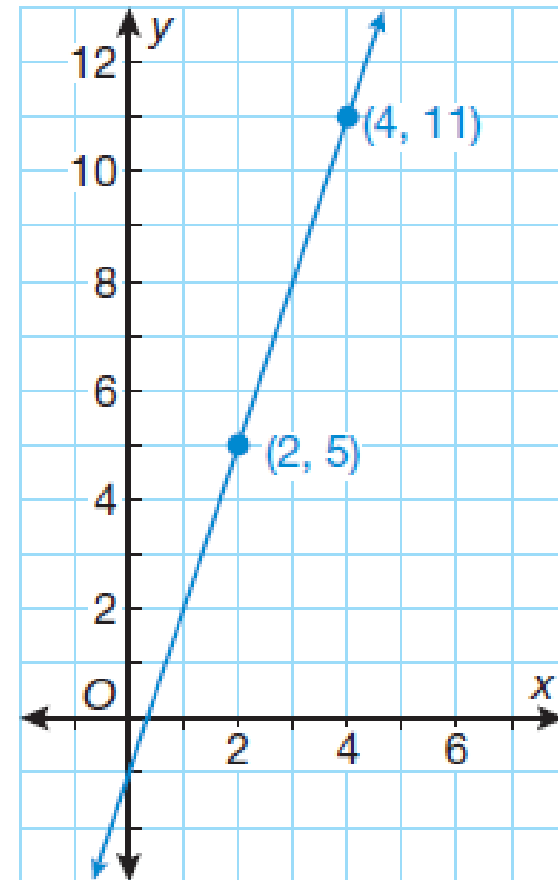
$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$m = \frac{11 - 5}{4 - 2}$$
$$m = 3$$

Read the y -intercept b directly from the graph. $b = -1$

Substitute for m and b in the slope-intercept form.

$$y = mx + b$$

$$y = 3x - 1$$



Example 2 Writing the Equation of a Line

b. Write the equation of the line that has slope $\frac{2}{3}$ and passes through $(-2, 4)$.

SOLUTION

Since $m = \frac{2}{3}$, substitute it into the slope-intercept form using the given point $(-2, 4)$.

$$y = mx + b$$

$$4 = \frac{2}{3}(-2) + b$$

$$4 = \frac{-4}{3} + b$$

$$4 + \frac{4}{3} = -\frac{4}{3} + b + \frac{4}{3}$$

$$5\frac{1}{3} = b$$

$$y = \frac{2}{3}x + 5\frac{1}{3}$$

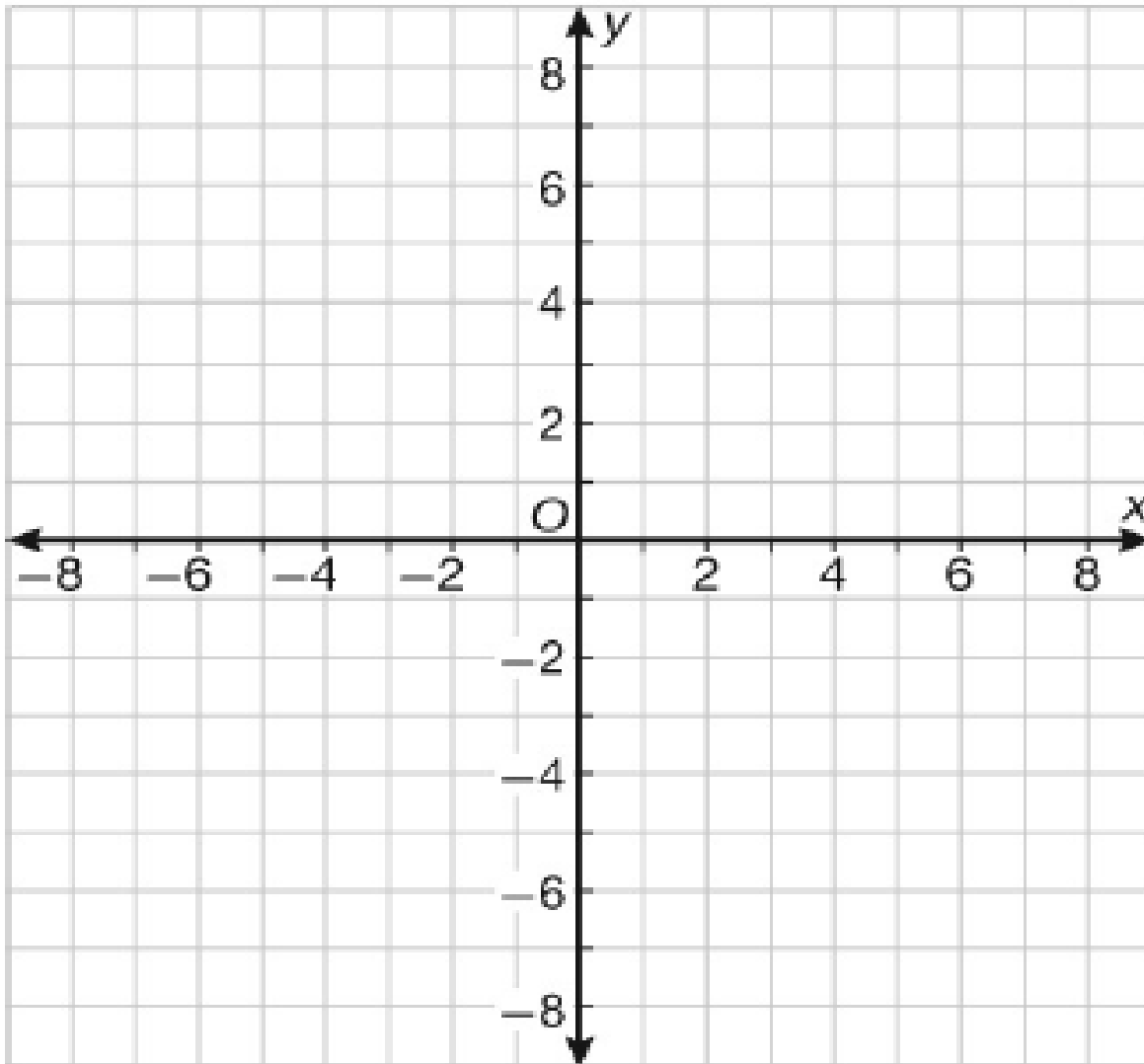
Example 3 Graphing a Linear Equation

a. Graph the line that has the equation

$$y = -5x + 3.$$

SOLUTION

The equation is in slope–intercept form. Since the y -intercept is 3, the line passes through 3 on the y -axis. The slope is -5 . For each run of 1 unit, the line has a rise of -5 , so it drops 5 units. Use this fact to plot points starting at $(0, 3)$.



Example 3 Graphing a Linear Equation

b. Graph the line that has the equation $2y - 4x = 7$.

SOLUTION

The equation is not in slope-intercept form. Convert it to slope-intercept form.

$$2y - 4x = 7$$

Add $4x$ to each side.

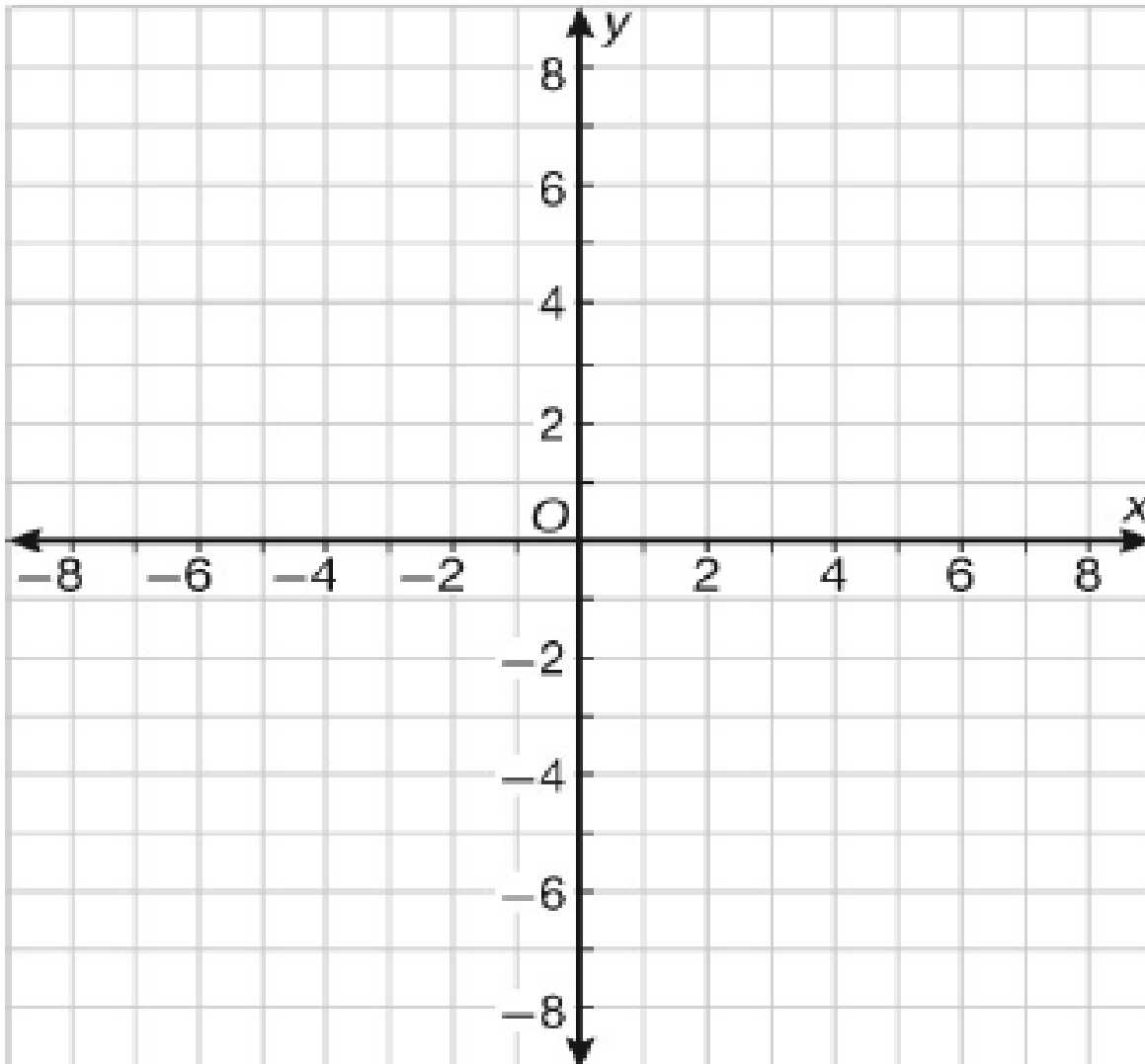
$$2y = 7 + 4x$$

Divide each side by 2.

$$y = 2x + 3.5$$

Simplify.

The slope of the line is 2 and the y -intercept is 3.5. Graph the line on the coordinate plane.



Example 4 Application: Meteorology

Kim believes that there is a linear relationship between the average July temperature in the city of Brightdale in a particular year, and the number of days of sunshine Brightdale enjoys that year. She defines x to be the average July temperature (daily high, in Fahrenheit), and y to be the number of days of sunshine.

Kim's model is shown on this graph.

a. Determine the slope of the graph. What does the slope represent?

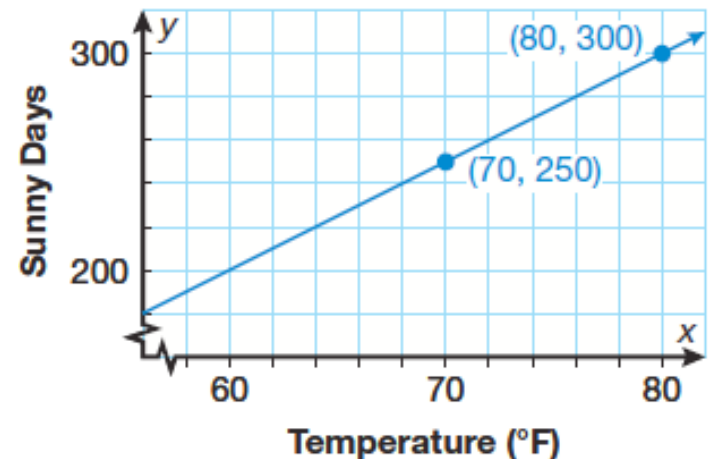
SOLUTION

Use the formula to find the slope.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{300 - 250}{80 - 70}$$

$$m = 5$$



Example 4 Application: Meteorology

b. Write an equation for Kim's model.

SOLUTION

Find the y -intercept using the slope and a point on the line.

$$y = mx + b$$

$$250 = 5(70) + b$$

Substitute.

$$250 = 350 + b$$

Simplify.

$$250 - 350 = 350 + b - 350$$

Sub. 350 from each side.

$$b = -100$$

Simplify.

Kim's model has the equation $y = 5x - 100$.

Example 4 Application: Meteorology

c. Use the equation to predict the average July temperature if there are 280 days of sunshine.

SOLUTION

Substitute 280 for y in your equation from part b:

$$280 = 5x - 100$$

$$280 + 100 = 5x - 100 + 100$$

$$380 = 5x$$

$$\frac{380}{5} = \frac{5x}{5}$$

$$76 = x$$

Substitute.

Add 100 to each side.

Simplify.

Divide each side by 5.

Simplify.

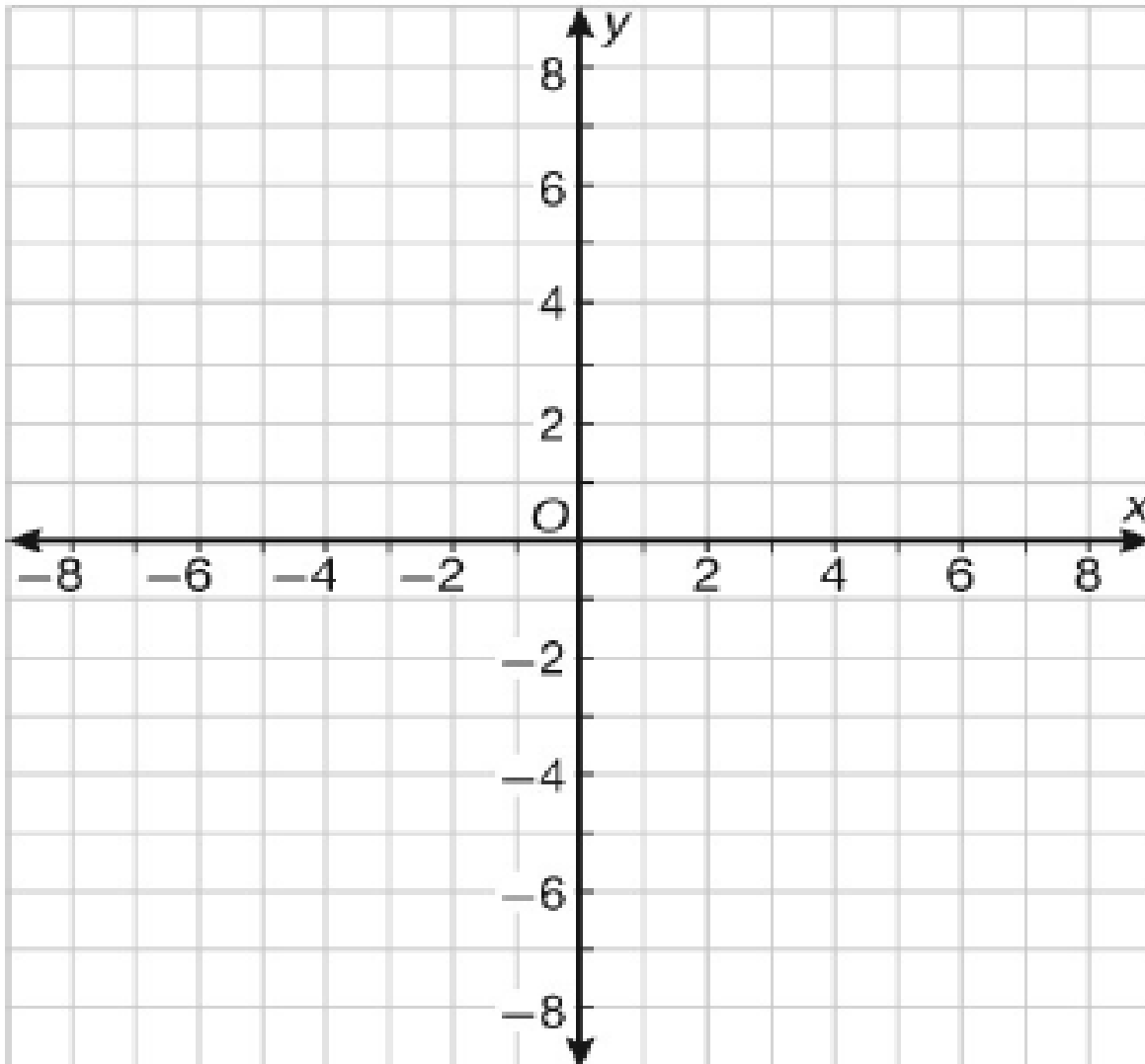
The average July temperature should be 76°F.

You Try!!!

e. Write the equation of the line that passes through $(0, -2)$ and $(5, 2)$.

$$y = \frac{4}{5}x - 2$$

g. Graph the line with the equation $3x + y = 6$.



Assignment

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Lesson Practice (Ask Mr. Heintz)

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Practice 1–30 (Do the starred ones first)