Lesson 17

More Conditional Statements

In Lesson 10, you learned that a conditional statement has the form, “If p, then q.” It is formed from two other statements: the hypothesis, p, and the conclusion, q. If you switch the statements, the result is the converse of the conditional statement, “If q, then p.”

Example 1 Analyzing the Truth Value and Converse of Conditional Statements

Consider the conditional statement, “If Sylvester walks to work, then it is a Wednesday.”

a. State the hypothesis and conclusion of this statement, and write its converse.

SOLUTION

Hypothesis: *Sylvester walks to work.*

Conclusion: *It is a Wednesday.*

Converse: *If it is a Wednesday, then Sylvester walks to work.*

Example 1 Analyzing the Truth Value and Converse of Conditional Statements

b. If the original statement is true, is the converse true?

SOLUTION

The converse is not necessarily true. Though we know that Sylvester only walks to work on Wednesdays, we do not know that he walks to work every Wednesday. He might drive to work on some Wednesdays. Therefore, the converse is false.

Negation of a Statement – The opposite of that statement. The negation of a statement p is "not p," and is written as $\sim p$.

Example: The negation of "a pentagon is regular" is "a pentagon is not regular."

Example 2 Examining the Negation of Conditional Statements

Identify the hypothesis and the conclusion in the statement below. Then, write the negation of each.

If a pentagon is regular, then it is equiangular.

SOLUTION

The hypothesis is: *A pentagon is regular.*

The conclusion is: *It is equiangular.* The negation of the hypothesis is: *A pentagon is not regular.*

The negation of the conclusion is: *It is not equiangular.*

Inverse of a Conditional Statement – When the hypothesis AND conclusion are both negated. The inverse of “If p, then q” is “If $\sim p$, then $\sim q$.”

NOTE: The converse of a conditional statement and the inverse of the same conditional statement always have the same truth value: either both are true or both are false.

Logically Equivalent Statements – When two related conditional statements have the same truth value.

Example 3 Examining the Inverse of Conditional Statements

Write the inverse of the statement below. Is the statement true? Is the inverse of the statement true?

For two lines that are cut by a transversal, if alternate interior angles are congruent, then the lines are parallel.

SOLUTION

The inverse of the statement is, "For two lines that are cut by a transversal, if alternate interior angles are not congruent, then the lines are not parallel."

Since the converse and the inverse of a statement have the same truth value, the converse can be used to determine the truth value of the inverse.

The original statement is the converse of Theorem 10-1. Since the converse of the statement is known to be true, the inverse of the statement is also true.

Contrapositive of a Conditional Statement – When both the hypothesis and conclusion are exchanged and negated. The contrapositive of, "If p, then q," is, "if $\sim q$, then $\sim p$." A conditional statement and its contrapositive are logically equivalent statements: either both are true or both are false.

We summarize the different types of conditional statements in this table.

	Form
Statement	If p, then q
Converse	If q, then p
Inverse	If $\sim p$, then $\sim q$
Contrapositive	If $\sim q$, then $\sim p$

Example 4 Examining the Contrapositive of Conditional Statements

a. Determine the contrapositive of the statement.

If Mai finishes school at 1 p.m., then it is a Thursday.

SOLUTION

In this statement, p is "Mai finishes school at 1 p.m." and q is "it is a Thursday." Therefore, the contrapositive statement "If $\sim q$, then $\sim p$" is "If it is not a Thursday, then Mai does not finish school at 1 p.m."

Example 4 Examining the Contrapositive of Conditional Statements

b. Determine the contrapositive of the solution to part a. What do you notice?

SOLUTION

The new p and q are “it is not a Thursday” and “Mai does not finish school at 1 p.m.” Therefore, its contrapositive is “If $\sim$(Mai does not finish school at 1 p.m.), then $\sim$(it is not a Thursday),” which is the same as “If Mai finishes school at 1 p.m., then it is a Thursday.” This is the original statement.

You Try!!!!

State the hypothesis and conclusion of this statement and its converse.

"*If a polygon is regular, then it is convex*"

If the statement in problem a is true, is the converse true?

You Try!!!!

Identify the hypothesis and the conclusion in the statement below.

Then write the negation of each.

If Durrell buys juice, then he buys pretzels.

Write the inverse of the statement below. Is the statement true? Is the inverse of the statement true?

For two lines that are cut by a transversal, if the lines are parallel, then the same-side interior angles are congruent.

You Try!!!!

Determine the contrapositive of the statement.

If two angles are complementary, then the sum of their measures is 90°.

Determine the *converse* of the solution to problem e. What do you notice?

Assignment

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Lesson Practice (Ask Mr. Heintz)

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Practice 1-30 (Do the starred ones first)