

Geometry Lesson 18

Objective: TSW use triangle theorems.

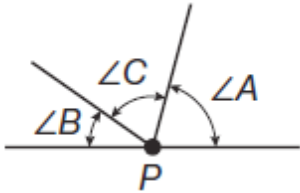
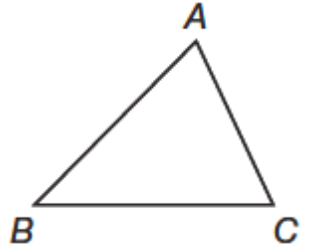
Exploration: Developing the Triangle Angle Sum Theorem

In this exploration, you will use unlined paper to discover the relationship between the measures of the interior angles of a triangle.

1. On a piece of unlined paper, draw a line and label a point on the line P .
2. Place the unlined paper on top of $\triangle ABC$. Align the papers so that \overline{AB} is on the line you drew and P and B coincide. Trace $\angle B$. Rotate the triangle and trace $\angle C$ adjacent to $\angle B$.

Rotate the triangle once more and trace $\angle A$ adjacent to $\angle C$.

The diagram shows your final step.



3. What do you notice about the three angles of the triangle you traced?

4. Draw a new triangle and repeat the activity using the new triangle. What is the result?

5. Write an equation describing the relationship you found between the three angles of $\triangle ABC$.

Theorem 18-1: Triangle Angle Sum Theorem - The sum of the measures of the angles of a triangle is equal to 180° .

Example 1 Using the Triangle Angle Sum Theorem

In the right triangle $\triangle ABC$, $m\angle B = 35^\circ$ and the right angle is at vertex A . Find the measure of $\angle C$.

SOLUTION

A _____ to a theorem is a statement that follows directly from that theorem. The Triangle Angle Sum Theorem has several useful corollaries.

Triangle Angle Sum Theorem Corollaries:

Corollary 18-1-1: If two angles of one triangle are congruent to two angles of another triangle, then the third angles are congruent.

Corollary 18-1-2: The acute angles of a right triangle are complementary.

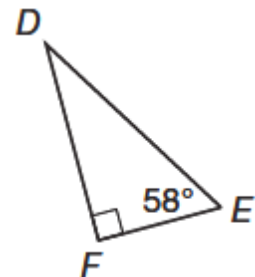
Corollary 18-1-3: The measure of each angle of an equiangular triangle is 60° .

Corollary 18-1-4: A triangle can have at most one right or one obtuse angle.

Example 2 Finding Angle Measures in Right Triangles

- a. Find the measure of $\angle D$ in $\triangle DEF$.

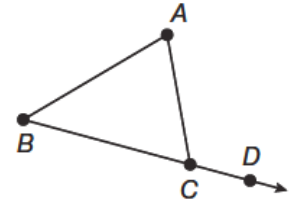
SOLUTION



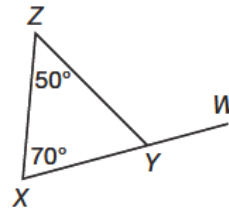
b. In right $\triangle KLM$, $\angle K \cong \angle L$. Determine $m\angle K$.
 SOLUTION

_____ Interior Angle - In any polygon, the interior angle that is not adjacent to a given exterior angle.

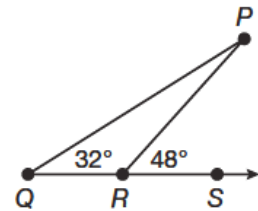
Theorem 18-2: Exterior Angle Theorem - The measure of each exterior angle of a triangle is equal to the sum of the measures of its two remote interior angles.



Example 3 Using the Exterior Angle Theorem
 a. For $\triangle XYZ$, determine the measure of $\angle WYZ$.
 SOLUTION

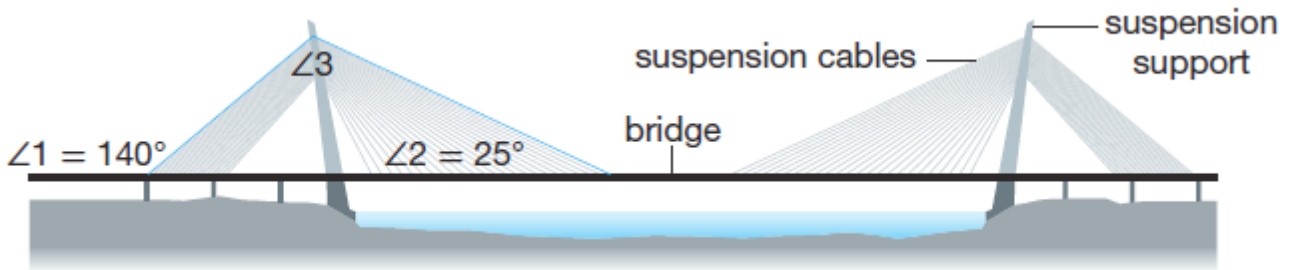


b. Determine the measure of $\angle P$ in $\triangle PQR$.
 SOLUTION



Example 4 Application: Civil Engineering

A bridge uses cables to support its 2000 foot span. Use the data in the image to determine the measure of the angle at the apex of the marked cable structure.



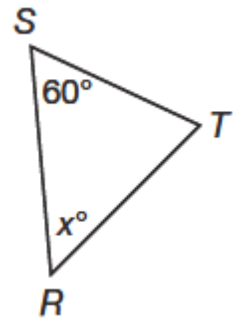
SOLUTION

You Try!!!!!!

Use this figure to answer a and b.

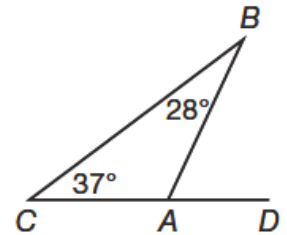
a. If $x = 50$, determine the measure of $\angle T$.

b. Determine $m\angle T$ if $x = 60$.



c. In right triangle ΔPQR , the measure of one acute angle is 20° . What is the measure of the other acute angle in ΔPQR ?

d. In ΔABC , determine the measure of $\angle DAB$.



e. In ΔJKL , $\angle K$ measures 60° and the exterior angle at vertex L measures 100° . Make a sketch of ΔJKL showing the given interior and exterior angle measures.

f. Determine the measure of $\angle J$ in ΔJKL in problem e.

g. Civil Engineering: A planned glass pyramid structure has four triangular faces. The angles at the base of each face are congruent. Each of the angles at the apex of the pyramid measures 68° . What are the measures of the congruent base angles?