Lesson 18

Triangle Theorems

Exploration: Developing the Triangle Angle Sum Theorem

In this exploration, you will use unlined paper to discover the relationship between the measures of the interior angles of a triangle.

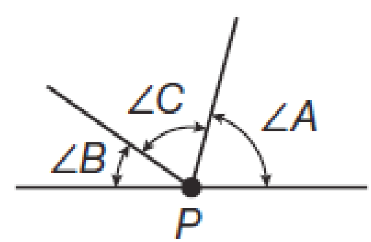
1. On a piece of unlined paper, draw a line and label a point on the line *P*.

2. Place the unlined paper on top of ΔABC. Align the papers so that $\overline{AB}$ is on the line you drew and *P* and *B* coincide. Trace $\angle B$. Rotate the triangle and trace $\angle C$ adjacent to $\angle B$. Rotate the triangle once more and trace $\angle A$ adjacent to $\angle C$. The diagram shows your final step.

3. What do you notice about the three angles of the triangle you traced?

4. Draw a new triangle and repeat the activity using the new triangle. What is the result?

5. Write an equation describing the relationship you found between the three angles of ΔABC.



Theorem 18-1: Triangle Angle Sum Theorem – The sum of the measures of the angles of a triangle is equal to 180°.

$$m\angle A + m\angle B + m\angle C = 180°$$

Example 1 Using the Triangle Angle Sum Theorem

In the right triangle ΔABC, $m\angle B = 35°$ and the right angle is at vertex A. Find the measure of $\angle C$.

SOLUTION

From the Triangle Angle Sum Theorem:

$m\angle A + m\angle B + m\angle C = 180°$

$90° + 35° + m\angle C = 180°$	Substitute.
$m\angle C = 55°$	Solve.

A corollary to a theorem is a statement that follows directly from that theorem. The Triangle Angle Sum Theorem has several useful corollaries.

Triangle Angle Sum Theorem Corollaries:

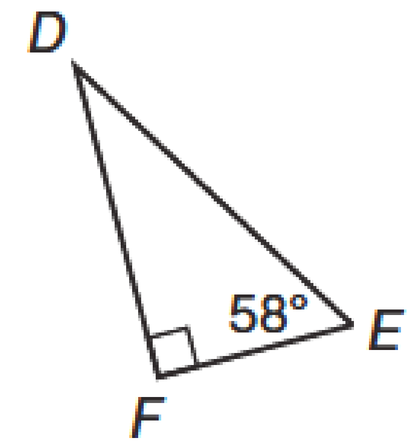
<u>Corollary 18-1-1: If two angles of one triangle are congruent to two angles of another triangle, then the third angles are congruent.</u>

<u>Corollary 18-1-2: The acute angles of a right triangle are complementary.</u>

<u>Corollary 18-1-3: The measure of each angle of an equiangular triangle is 60°.</u>

<u>Corollary 18-1-4: A triangle can have at most one right or one obtuse angle.</u>

Example 2 Finding Angle Measures in Right Triangles



a. Find the measure of $\angle D$ in ΔDEF.

SOLUTION

By Corollary 18-1-2, $\angle D$ and $\angle E$ are complementary.

$m\angle D + m\angle E = 90^\circ$

$m\angle D + 58^\circ = 90^\circ$ Substitute for $m\angle E$.

$m\angle D = 32^\circ$ Sub. 58° from each side.

Example 2 Finding Angle Measures in Right Triangles

b. In right $\Delta KLM, \angle K \cong \angle L$. Determine m$\angle K$.

SOLUTION

By Corollary 18-1-4, $\angle K$ and $\angle L$ cannot both be right angles. Therefore they are the acute angles of ΔKLM. Since they are congruent, $\angle K$ can be substituted for $\angle L$.

$m\angle K + m\angle L = 90^\circ$

$m\angle K + m\angle K = 90^\circ$ Substitute for m$\angle L$.

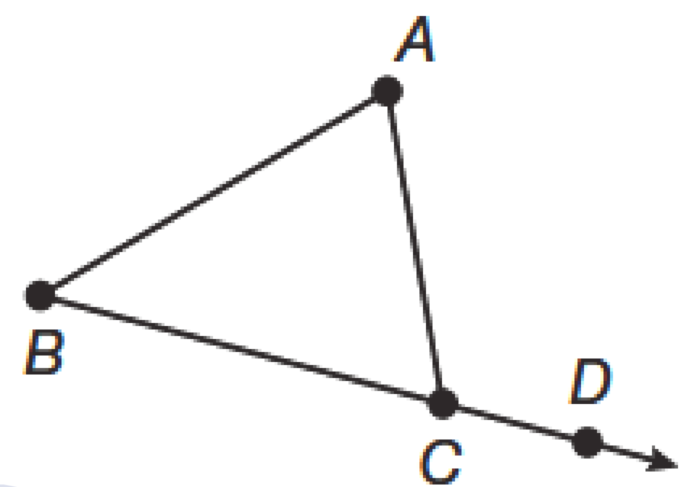
$2m\angle K = 90^\circ$ Simplify.

$m\angle K = 45^\circ$ Divide both sides by 2.

Remote Interior Angle – In any polygon, the interior angle that is not adjacent to a given exterior angle.

Theorem 18-2: Exterior Angle Theorem – The measure of each exterior angle of a triangle is equal to the sum of the measures of its two remote interior angles.

$m\angle DCA = m\angle A + m\angle B$



Example 3 Using the Exterior Angle Theorem

a. For ΔXYZ, determine the measure of $\angle WYZ$.

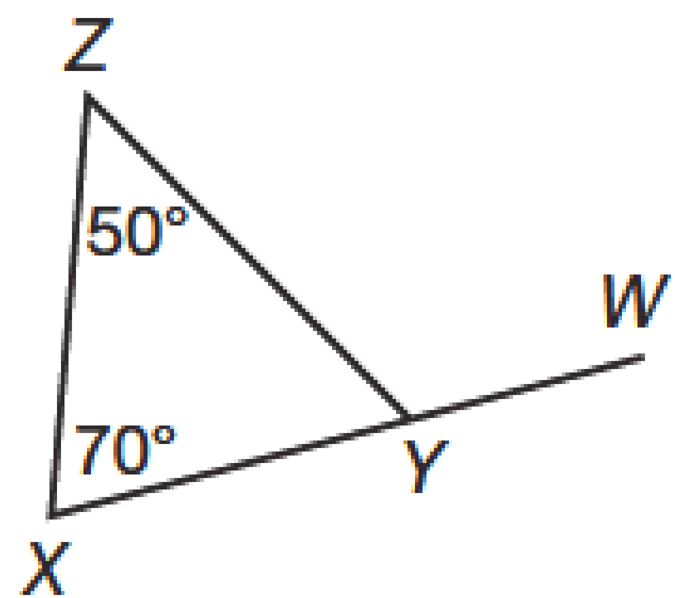
SOLUTION

The remote interior angles are at X and Z. Therefore,

$m\angle X + m\angle Z = m\angle WYZ$

$70^\circ + 50^\circ = m\angle WYZ$

$120^\circ = m\angle WYZ$



Example 3 Using the Exterior Angle Theorem

b. Determine the measure of $\angle P$ in ΔPQR.

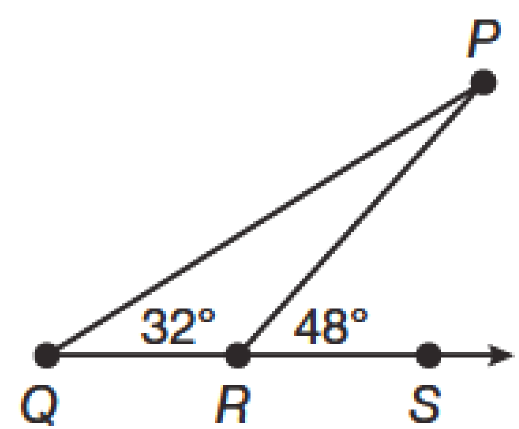
SOLUTION

Apply the Exterior Angle Theorem with the exterior angle at vertex R:

$m\angle P + m\angle Q = m\angle PRS$

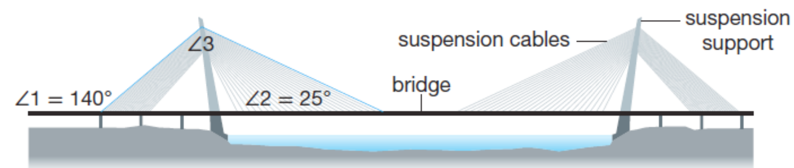
$m\angle P + 32^\circ = 48^\circ$ Substitute.

$m\angle P = 16^\circ$ Sub. 32° from each side.



Example 4 Application: Civil Engineering

A bridge uses cables to support its 2000 foot span. Use the data in the image to determine the measure of the angle at the apex of the marked cable structure.



SOLUTION

Apply the Exterior Angle Theorem.

$m\angle 2 + m\angle 3 = m\angle 1$

$25° + m\angle 3 = 140°$ Substitute.

$m\angle 3 = 115°$ Subtract 25° from each side.

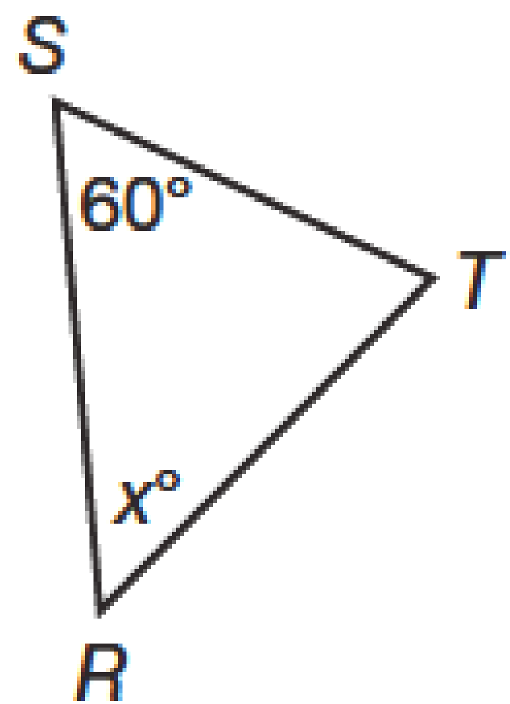
The angle at the apex measures 115°.

You Try!!!!!!

Use this figure to answer a and b.

If $x = 50$, determine the measure of $\angle T$.

Determine $m\angle T$ if $x = 60$.



You Try!!!!!!

In right triangle ΔPQR, the measure of one acute angle is 20°. What is the measure of the other acute angle in ΔPQR?

In ΔABC, determine the measure of $\angle DAB$.

You Try!!!!!!

In ΔJKL, $\angle K$ measures 60° and the exterior angle at vertex L measures 100°. Make a sketch of ΔJKL showing the given interior and exterior angle measures.

Determine the measure of $\angle J$ in ΔJKL in problem e.

You Try!!!!!!

Civil Engineering: A planned glass pyramid structure has four triangular faces. The angles at the base of each face are congruent. Each of the angles at the apex of the pyramid measures 68°. What are the measures of the congruent base angles?

Assignment

Page 112

Lesson Practice (Ask Mr. Heintz)

Page 112

Practice 1-30 (Do the starred ones first)