Lesson 20

Interpreting Truth Tables

A conditional statement has the form, "If p, then q." Recall that several other statements can be constructed from the conditional statement.

- the converse statement, "If q, then p"
- the inverse statement, "If $\sim p$, then $\sim q$"
- the contrapositive statement, "If $\sim q$, then $\sim p$"

Biconditional Statement – The combination of a conditional statement and its converse. It is true only when both the original statement and its converse are true.

The biconditional of "If p, then q" and "if q, then p" can be written as "p if and only if q."

Example 1 Analyzing Conditional Statements

a. State the converse of this statement: *If* $x^2 \leq 4$, *then* x $\leq$ *2.*

SOLUTION Switch the hypothesis and conclusion: *If* x $\leq$ *2, then* $x^2 \leq 4$.

Example 1 Analyzing Conditional Statements

b. Determine if the statement and converse from part a are true.

SOLUTION Suppose the hypothesis is true: $x^2 \leq 4$. Solving for x shows that x must be 2 or –2. Values greater than –2 and less than 2 also satisfy this equation. In fact, all the possible solutions are less than or equal to 2. Therefore, the statement is true.

For the converse, you can find a counterexample. For example, if $x = -3$, the conclusion is not true.

$$x^2 \leq 4$$
$$(-3)^2 \leq 4,$$
$$9 \leq 4$$
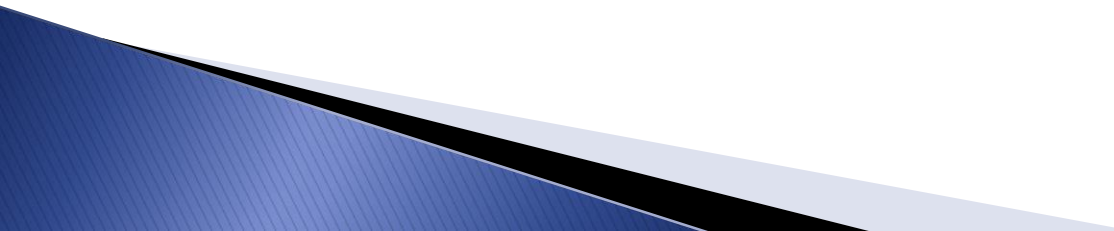
So, the converse is not true.

Example 1 Analyzing Conditional Statements

c. Write the biconditional of the statement. "If $x^2 \leq 4$, then $x \leq 2$." Is it true? Explain why or why not.

SOLUTION The biconditional is "$x^2 \leq 4$ if and only if $x \leq 2$."

For the biconditional to be true, both the statement and its converse must be true. In this case, the converse is false, so the biconditional is not true.

A truth table is a table that lists all possible combinations of truth values for a hypothesis, a conclusion, and the conditional statement or statements they form. Truth tables are useful tools because they show all the true/false possibilities at a glance:

Which row is the statement false? Why?

Hypothesis: p	Conclusion: q	Statement: If p, then q
T	T	T
T	F	F
F	T	T
F	F	T

Example 2 Using a Truth Table

a. Complete a truth table for the statement in Example 1.

Hypothesis: $x^2 \leq 4$	Conclusion: $x \leq 2$	Statement: If $x^2 \leq 4$, then $x \leq 2$
T	T	T
T	F	T
F	F	T

Example 2 Using a Truth Table

a. Complete a truth table for the statement in Example 1.

Notice that the statement is only false when the hypothesis is true but the conclusion is false (the second row of the table). For the statement

"If $x2 \leq 4$, then $x \leq 2$," this is impossible. Therefore, the statement is always true.

Example 2 Using a Truth Table

b. Add columns to your truth table for the statement's converse and its biconditional. Complete the table for these two statements.

Hypothesis: $x^2 \leq 4$	Conclusion: $x \leq 2$	Statement: If $x^2 \leq 4$, then $x \leq 2$	Converse: If $x \leq 2$, then $x^2 \leq 4$	Biconditional: $x^2 \leq 4$ if and only if $x \leq 2$
T	T	T	T	T
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

Compound Statement - A statement that combines two statements using *and* or *or*. It is similar to a conditional statement, except that p and q are related by "and" or "or" rather than by "if" and "then".

Conjunction – A compound statement that uses *and*. Conjunctions have the form "*p* and *q*."

For example, statement *p* stands for "I had bacon for breakfast," and *q* stands for "I had eggs for breakfast." If you have bacon and eggs for your breakfast, the conjunction, "I had bacon and eggs for breakfast," is true because *p* and *q* are both true. But if you have bacon and toast, the conjunction is false, because *p* is true but *q* is not.

Disjunction – A compound statement that uses *or*. Disjunctions have the form "*p* or *q*."

For example, suppose a lunch menu offers the choice of "soup or salad" as an appetizer. This is considered a disjunction because you can choose one or the other.

Example 3 Analyzing Compound Statements

A clothing store accepts cash or credit cards but not personal checks. It gives discounts on all cash purchases. Consider the statements, "a customer makes a credit−card purchase," and, "a customer gets a discount."

a. What is the conjunction of these statements? Use a truth table to assess its truth value.

SOLUTION

The conjunction is: "a customer makes a credit−card purchase *and* gets a discount."

Statement: *p*	Statement: *q*	Conjunction: *p* and *q*
T	T	T
T	F	F
F	T	F
F	F	F

The conjunction is false.

Example 3 Analyzing Compound Statements

b. What is the disjunction of these statements? Is it true or false?

SOLUTION

The disjunction is, "A customer makes a credit-card purchase *or* gets a discount." Extend your truth table:

Statement: p	Statement: q	Conjunction: p and q	Disjunction: p or q
T	T	T	T
T	F	F	T
F	T	F	T
F	F	F	F

The disjunction is true.

Example 4 Application: Astronomy

When stars run out of fuel, they either become black holes or degenerate stars. Consider the statements, "A star will become a degenerate star," and "A star will become a black hole." Form the conjunction and disjunction of these statements. Is the conjunction true? Is the disjunction true? Explain.

SOLUTION

Conjunction: *A star will become a degenerate star and a black hole.*

Disjunction: *A star will become a degenerate star or a black hole.*

The conjunction is false because a star cannot be both a degenerate star and a black hole. The disjunction is true, because all stars eventually become either degenerate stars or black holes.

You Try!!!!

For a–c, consider the statement, "If a quadrilateral is equiangular, then it is a rhombus."
c. Write the biconditional of the statement. Is it true? Explain.

You Try!!!!

Use the description of the restaurant to answer d–g.

Restaurants: The chef's special at a five-star restaurant offers its customers a complimentary appetizer based on their choice of entrée. If customers order a filet mignon entrée, they receive leek soup for the appetizer. If customers order grilled salmon for the entrée, they receive baby spinach salad for the appetizer.

d. Use truth tables to represent the statement, "If customers order grilled salmon for the entrée, they receive baby spinach salad for the appetizer." Interpret the tables for the statements.

e. Add columns to your truth tables to address the statements converses and biconditionals. Interpret the tables for these statements.

For f and g, consider the statements, "A customer orders a filet mignon entrée," and, "A customer receives a baby spinach salad appetizer."

You Try!!!!

Hypothesis: If customers order grilled salmon for the entrée	Conclusion: They receive baby spinach salad for the appetizer.	Statement: If customers order grilled salmon for the entrée, then they receive baby spinach salad for the appetizer.	Converse: If the customer receives baby spinach salad for the appetizer, then they order grilled salmon for the entrée.	Biconditional: Customers order grilled salmon if and only if they received baby spinach salad for the appetizer.
T	T	T	T	T
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

You Try!!!!

f. What is the conjunction of these statements? Use a truth table to assess its truth value.

g. What is the disjunction of these statements? Is it true or false?

You Try!!!!

A customer orders a filet mignon entrée.	A customer receives a baby spinach salad appetizer.	Conjunction: A customer orders a filet mignon entrée AND receives a baby spinach salad appetizer.	Disjunction: A customer orders a filet mignon entrée OR receives a bay spinach salad appetizer.
T	T	T	T
T	F	F	T
F	T	F	T
F	F	F	F

Assignment

Page 124

   Lesson Practice (Ask Mr. Heintz)

Page 124

   Practice 1-30 (Do the starred ones first)