

Lesson 24

Algebraic Proofs

Proof – An argument that uses logic to show that a conclusion is true.

Properties of Equality

Property

Example

Addition Property of Equality

If $a = b$, then $a + c = b + c$.

Subtraction Property of Equality

If $a = b$, then $a - c = b - c$.

Multiplication Property of Equality

If $a = b$, then $ac = bc$.

Division Property of Equality

If $a = b$ and $c \neq 0$, then $\frac{a}{c} = \frac{b}{c}$.

Symmetric Property of Equality

If $a = b$, then $b = a$.

Reflexive Property of Equality

$a = a$

Transitive Property of Equality

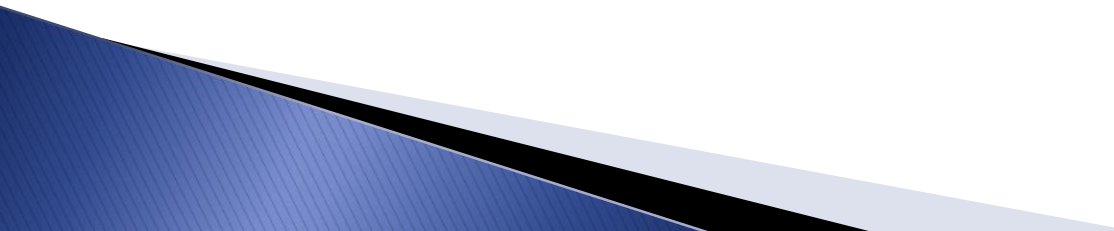
If $a = b$ and $b = c$, then $a = c$.

Substitution Property of Equality

If $a = b$, then b can be substituted for a in any expression.

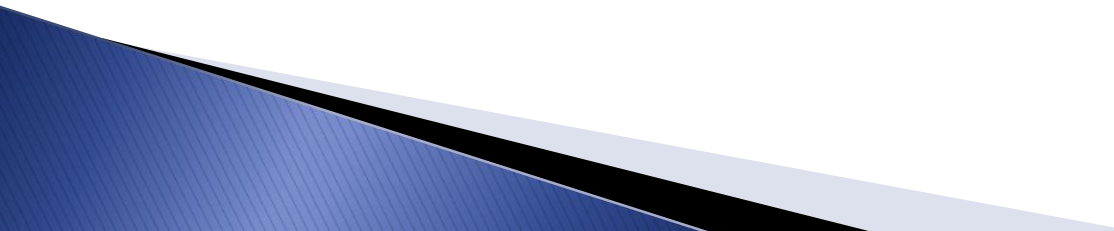
An algebraic proof shows step-by-step how a problem is solved. Each step has to be justified with one of the properties above, or by a property of arithmetic.

For example, if a step required adding a number to both sides of an equation, it would be justified by the Addition Property of Equality.



Whenever a step requires that you perform basic mathematical operations on a single side of the equation (like addition, subtraction, multiplication, or division), the step is justified by the term, “Simplify.”

Most proofs begin by presenting the facts of a problem. The first line restates what you have already been told, and is justified as “Given.”



Example 1 Writing an Algebraic Proof

a. Solve this equation. Provide a justification for each step.

$$2(x + 1) = x + 9$$

SOLUTION

$$2(x + 1) = x + 9$$

$$2x + 2 = x + 9$$

$$2x + 2 - 2 = x + 9 - 2$$

$$2x = x + 7$$

$$2x - x = x + 7 - x$$

$$x = 7$$

Given

Distributive Property

Sub Prop of Equality

Simplify.

Sub Prop of Equality

Simplify.

Example 1 Writing an Algebraic Proof

b. Solve this equation. Provide a justification for each step.

$$\frac{3x-1}{5} = \frac{2x+3}{3}$$

SOLUTION

$$\frac{3x-1}{5} = \frac{2x+3}{3}$$

$$15 \left(\frac{3x-1}{5} \right)^3 = 15 \left(\frac{2x+3}{3} \right)^3$$

$$\frac{15}{5} (3x - 1) = \frac{15}{3} (2x + 3)$$

$$9x - 3 = 10x + 15$$

$$9x - 3 + 3 = 10x + 15 + 3$$

$$9x = 10x + 18$$

$$9x - 10x = 10x + 18 - 10x$$

$$-x = 18$$

$$\frac{-x}{-1} = \frac{18}{-1}$$

$$x = -18$$

Given

Multiplication Property of Equality

Associative Property of Multiplication

Distributive Property

Addition Property of Equality

Simplify.

Subtraction Property of Equality

Simplify.

Division Property of Equality

Simplify.

Example 2 Verifying Algebraic Reasoning

The steps of the algebraic proof for solving the equation $2(a + 1) = -6$ are given below in the correct order. However, the justifications for each step are out of order. Determine the correct order for the justifications.

SOLUTION

$$2(a + 1) = -6$$

Simplify.

Given

$$2a + 2 = -6$$

Distributive Prop

Distributive Prop

$$2a + 2 - 2 = -6 - 2$$

Div Prop of Equality

Sub Prop of Equality

$$2a = -8$$

Given

Simplify

$$\frac{2a}{2} = \frac{-8}{2}$$

Sub Prop of Equality

Div Prop of Equality

$$a = -4$$

Simplify.

Simplify

Example 3 Application: Finding Dimensions

The area of a rectangular patio is 28 square feet. The patio's length is $(3x + 1)$ feet and the patio's width is $2x$ feet. Find the dimensions of the patio. Provide a justification for each step.

SOLUTION

The formula for the area of a rectangle is $A = l/w$, so

$$A = 28, l = (3x + 1), w = 2x$$

$$A = l/w$$

$$28 = (3x + 1)(2x)$$

$$28 = 6x^2 + 2x$$

$$6x^2 + 2x = 28$$

$$\frac{6x^2 + 2x}{2} = \frac{28}{2}$$

$$3x^2 + x = 14$$

$$3x^2 + x - 14 = 14 - 14$$

$$3x^2 + x - 14 = 0$$

$$(3x + 7)(x - 2) = 0$$

Given

Area formula for a rectangle

Substitution Property of Equality

Distributive Property

Symmetric Property of Equality

Division Property of Equality

Simplify.

Subtraction Property of Equality

Simplify.

Factor.

Example 3 Application: Finding Dimensions

There are two solutions to this factorization, $3x + 7$, and $x - 2$. However, the solution to $3x + 7$ is negative. It does not make sense for a side of the rectangle to have a negative length, so that solution is thrown out. Therefore,

$$x - 2 = 0$$

Given

$$x - 2 + 2 = 0 + 2$$

Add Prop of Equality

$$x = 2$$

Simplify.

Example 3 Application: Finding Dimensions

Now, substitute $x = 2$ into the expressions for length and width of the rectangle to find the dimensions.

$$\begin{aligned}\text{length} &= 3x + 1 \\ &= 3(2) + 1 \\ &= 7\end{aligned}$$

$$\begin{aligned}\text{width} &= 2x \\ &= 2(2) \\ &= 4\end{aligned}$$

Therefore, the patio is 7 feet long and 4 feet wide.

You Try!!!!

c. The steps of the proof below are given in the correct order. However, the justifications for each step are out of order. Determine the correct order of the justifications.

$$\frac{2}{3}x + 6 = 4 - 2x$$

Given

$$2x + 18 = 12 - 6x$$

Subtraction Property of Equality

$$2x = -6 - 6x$$

Multiplication Property of Equality

$$8x = -6$$

Addition Property of Equality

$$x = -\frac{2}{3}$$

Division Property of Equality

d. The area of the rectangular floor of a shed is 40 yd^2 . The length of the shed is $(x + 2)$ yd and the width is $(x - 1)$ yd. Find the dimensions of the shed. Provide a justification for each step.

Assignment

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Lesson Practice (Ask Mr. Heintz)

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Practice 1–30 (Do the starred ones first)