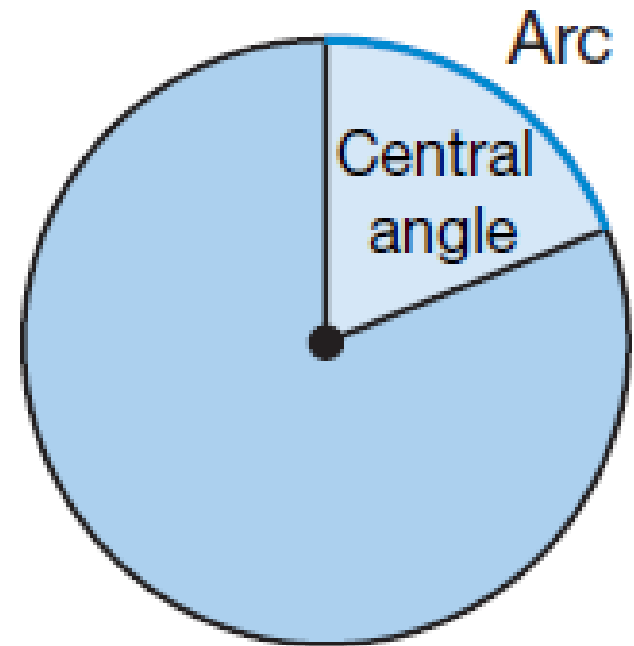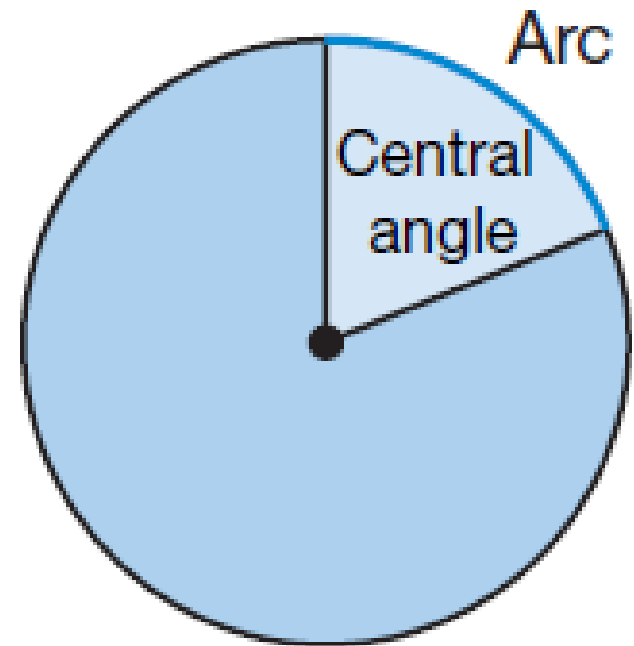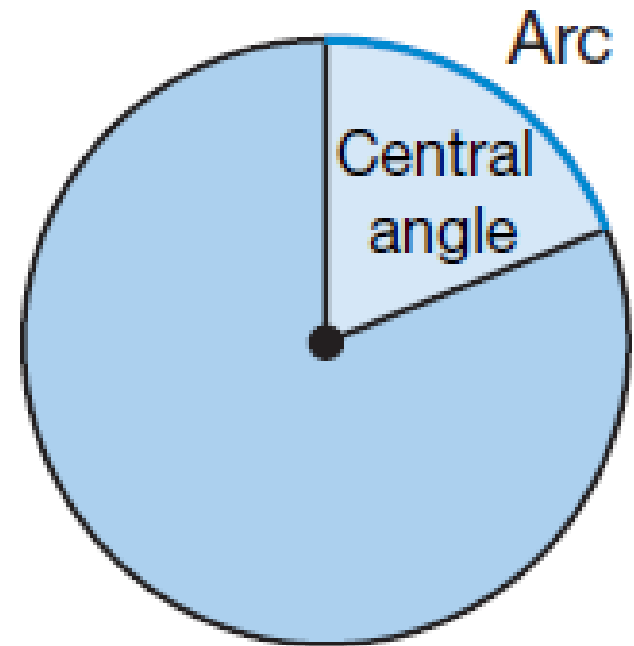# Lesson 26

Central Angles and Arc Measure

Arc – A part of a circle consisting of two points on the circle, called endpoints, and all the points on the circle between them.

Adjacent Arcs – When two arcs on a circle share exactly one endpoint.

Central Angle – An angle whose vertex is at the center of a circle.

Arcs of a Circle

Minor Arc - An arc that is smaller than half a circle.

The measure of a minor arc is the same as the measure of its central angle. The measure of a minor arc must be greater than 0° and less than 180°.

All minor arcs are named using the two endpoints of the arc. $\overset{\frown}{AB}$

Major Arc - An arc that is larger than half a circle.

The measure of a major arc is the difference of 360° and the measure of the associated minor arc.

The measure of a major arc must be greater than 180°and less than 360°.

All major arcs are named using the two endpoints of the arc and a point on the circle between the endpoints. $\overset{\frown}{ABC}$

Semicircle – An arc equal to half a circle.

The measure of a semicircle is 180°.

Like major arcs, semicircles can be named with the two endpoints of the semi-circle and a point on the circle between the endpoints.

Example 1 Identifying Arcs and Angles

Identify a central angle, minor arc, major arc, and semicircle in $\odot P$.
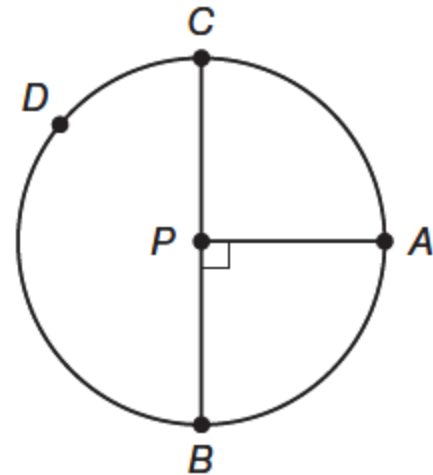
SOLUTION

Two central angles are pictured: $\angle APB$ and $\angle APC$.

Each central angle forms a minor arc: $\widehat{AB}$ and $\widehat{AC}$.

There are also two major arcs: $\widehat{ABC}$ and $\widehat{ACB}$.

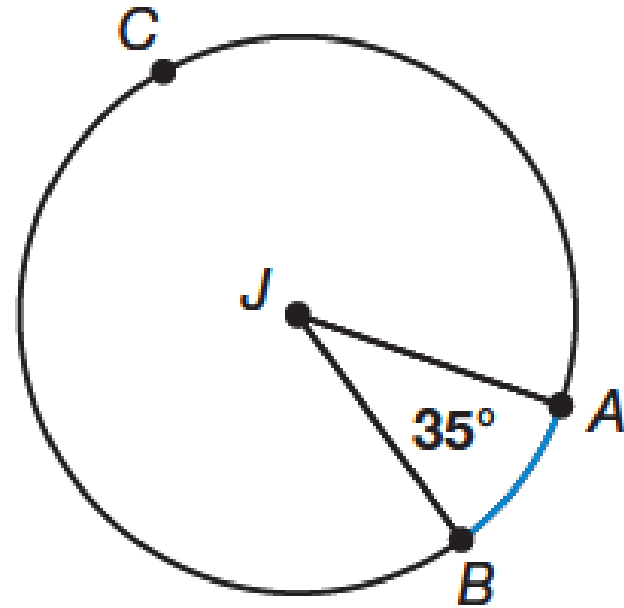Finally, there are two semicircles: $\widehat{BAC}$ and $\widehat{BDC}$.

Example 2 Finding Arc Measures

What is m $\widehat{AB}$?

SOLUTION

The central angle's measure is 35°, so the measure of the arc is also 35°.

Congruent Arcs – Two arcs that are in the same circle or in congruent circles and that have the same measure.

Example 3 Congruent Arcs

The measure of $\widehat{DE}$ is given by the expression $3x + 10$, and the measure of $\widehat{HJ}$ is given by the expression $5x - 40$. It is given that $\widehat{DE} \cong \widehat{HJ}$. Determine the value of x and the measure of each arc.

SOLUTION

Since the two arcs are congruent, the expressions for their measures must be equal. Therefore,

$3x + 10 = 5x - 40$

$3x + 10 - 10 = 5x - 40 - 10$

$3x = 5x - 50$

$3x - 5x = 5x - 50 - 5x$

$-2x = -50$

$\dfrac{-2X}{-2} = \dfrac{-50}{-2}$
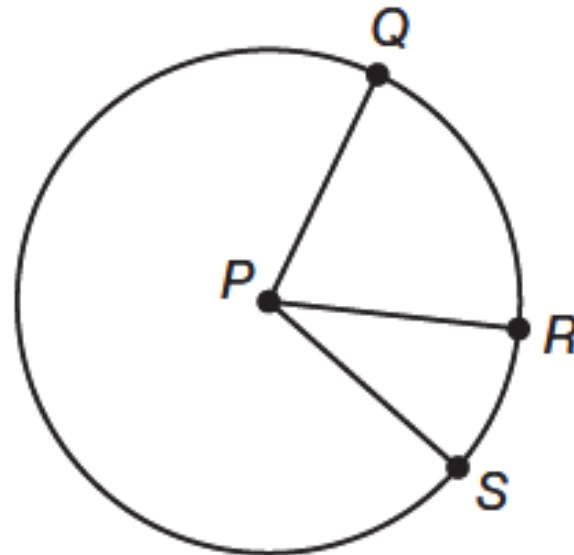
$x = 25$

Therefore, $m\widehat{DE} = 3(25) + 10$

$= 75 + 10$

$= 85°$

Since the arcs are congruent, $m\widehat{HJ} = 85°$.

Postulate 14: Arc Addition Postulate – The measure of an arc formed by two adjacent arcs is the sum of the measures of the two arcs.

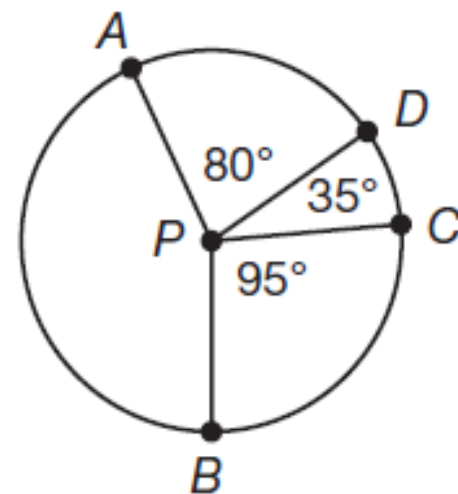Example: $m\widehat{QR} + m\widehat{RS} = m\widehat{QS}$

Example 4 Using the Arc Addition Postulate

a. Use the Arc Addition Postulate to write an expression that represents $m\widehat{AC}$.

SOLUTION

While there are several arcs indicated, the only two that are adjacent and make up the same arc as $\widehat{AC}$ are $\widehat{AD}$ and $\widehat{DC}$. Therefore,

$$m\widehat{AD} + m\widehat{DC} = m\widehat{AC}.$$

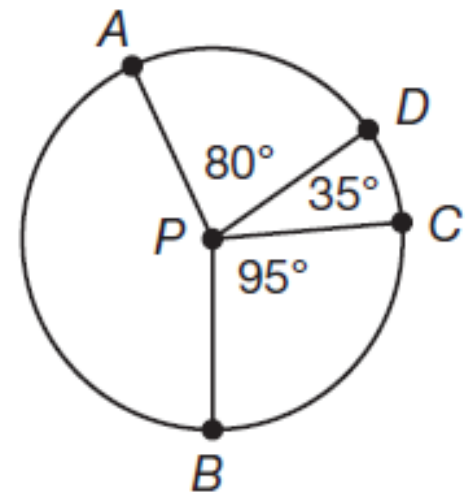# Example 4 Using the Arc Addition Postulate

b. Find $m\widehat{AC}$.

SOLUTION

Since $m\widehat{AD} + m\widehat{DC} = m\widehat{AC}$

$= 80° + 35°$

$= 115°$

Therefore, $m\widehat{AC} = 115°$.

Example 5 Application: Surveillance Cameras

The cameras' overlap will be any number of degrees over 180° that they cover when their viewing angles are added together. How many surveillance cameras would be needed to cover a semicircle of a room, with minimal overlap of the area to be viewed? How much of an overlap would these cameras produce?

Example 5 Application: Surveillance Cameras

SOLUTION

The Arc Addition Postulate can be used to determine the number of cameras to be used. A circle is 360°, so a semicircle is 180°. To find the number of cameras needed, divide 180° by the viewing angle of a camera:
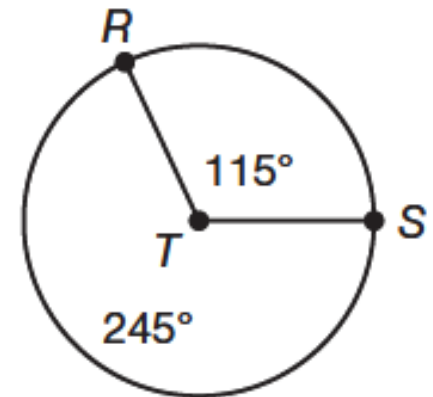
$$\frac{180°}{42°} = 4.3$$

Four cameras would not quite cover the area, so 5 cameras are needed. To find the overlap, multiply the number of cameras by the viewing angle and subtract 180°:

$$(42° \cdot 5) - 180° = 30°.$$

You Try!!!!!!

Draw a diagram of a circle, identifying a central angle, a minor arc, and a major arc.

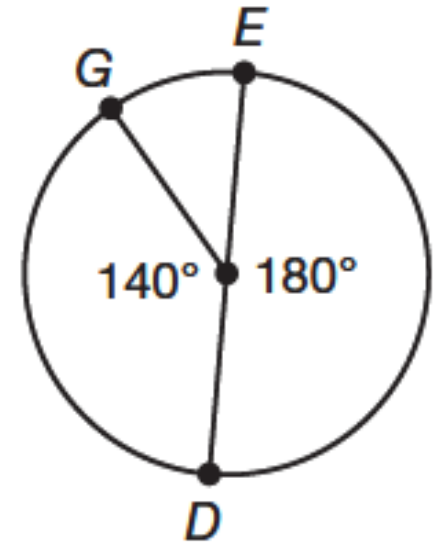Identify the measure of the minor arc.

You Try!!!!!!

The measure of $\widehat{JK}$ is given by the expression $2x - 15$, and the measure of $\widehat{LM}$ is given by the expression $x + 30$. It is given that $\widehat{JK} \cong \widehat{LM}$. Determine the value of x and the measure of each arc.

Use the Arc Addition Postulate to write an expression that represents $m\widehat{AB}$.
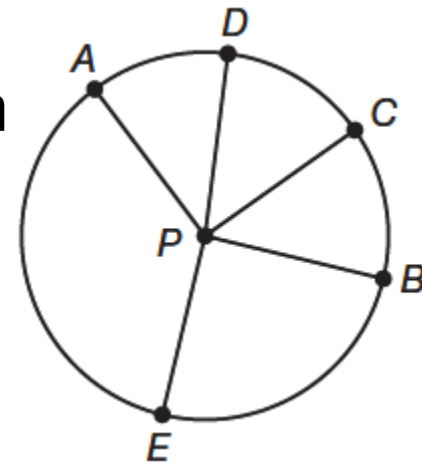
You Try!!!!!!

Find $m\widehat{DEG}$.

Outdoor Lighting A lamp projects a beam of light over a 100° arc. How many lamps facing outward from the center of a circle would be needed to form a full circle of light at the center of a park? What would be the overlap of these beams?

Assignment

Page 165

    Lesson Practice (Ask Mr. Heintz)

Page 166

    Practice 1-30 (Do the starred ones first)