

## Geometry Lesson 27

Objective: TSW use a two-column proof.

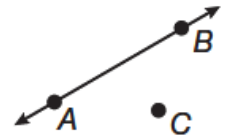
In a proof, deductive reasoning is used to develop a logical argument from given information to prove a conclusion. Proofs in geometry must be done step by step, and each step must have a justification. These justifications can include the \_\_\_\_\_ information, definitions, \_\_\_\_\_, theorems, and \_\_\_\_\_, as seen in the two-column proofs in this lesson.

### Example 1 Justifying Statements in a Two-Column Proof, Part 1

Fill in the justifying statements to support the proof of Theorem 4-2: If there is a line and a point not on the line, then exactly one plane contains them.

Given: Point  $C$  is not on  $\overleftrightarrow{AB}$ .

Prove: Exactly one plane contains  $\overleftrightarrow{AB}$  and  $C$ .



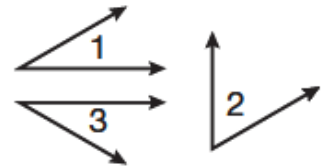
Statements	Reasons
1. Point $C$ is noncollinear with $\overleftrightarrow{AB}$ .	1.
2. Exactly one plane contains points $A$ , $B$ , and $C$ .	2.
3. Exactly one plane contains $\overleftrightarrow{AB}$ and $C$ .	3.

### Example 2 Justifying Statements in a Two-Column Proof, Part 2

Prove Theorem 6-1: If two angles are complementary to the same angle, then they are congruent.

Given:  $\angle 1$  is complementary to  $\angle 2$ .  $\angle 3$  is complementary to  $\angle 2$ .

Prove:  $\angle 1 \cong \angle 3$



Statements	Reasons
1. $\angle 1$ is complementary to $\angle 2$ . $\angle 3$ is complementary to $\angle 2$ .	1. Given
2. $m\angle 1 + m\angle 2 = 90^\circ$ $m\angle 3 + m\angle 2 = 90^\circ$	2.
3. $m\angle 1 + m\angle 2 = m\angle 3 + m\angle 2$	3.
4. $m\angle 1 + m\angle 2 - m\angle 2 = m\angle 3 + m\angle 2 - m\angle 2$	4.
5. $m\angle 1 = m\angle 3$	5.
6. $\angle 1 \cong \angle 3$	6.

Two-column proofs have a format that is composed of five parts.

- \_\_\_\_\_ statement(s): The information that is provided.
- \_\_\_\_\_ statement: The statement indicating what is to be proved.
- \_\_\_\_\_ : A sketch that summarizes the provided information. Sometimes you will need to draw the sketch yourself based on given information.
- \_\_\_\_\_ : The specific steps that are written in the left-hand column.
- \_\_\_\_\_ : Postulates, theorems, definitions, or properties written in the right-hand column, which justify each statement.

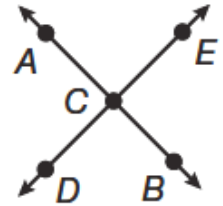
Example 3 Writing a Two-Column Proof, Part 1

Prove Theorem 6-4: If two angles are vertical angles, then they are congruent. (Vertical Angles Theorem)

Given:  $\overleftrightarrow{AB}$  and  $\overleftrightarrow{DE}$  intersect at point C

Prove:  $\angle ACD \cong \angle BCE$

SOLUTION



Proof:

Statements	Reasons
1.	1.
2.	2.
3.	3.
4.	4.
5.	5.
6.	6.

Example 4 Writing a Two-Column Proof, Part 2

Prove Theorem 5-3: If a transversal is perpendicular to one of two parallel lines, then it is perpendicular to the other one.

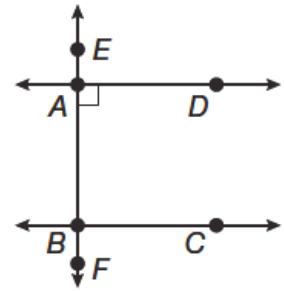
Given:  $\overleftrightarrow{AD} \parallel \overleftrightarrow{BC}$  and  $\overleftrightarrow{EB} \perp \overleftrightarrow{AD}$

Prove:  $\overleftrightarrow{EB} \perp \overleftrightarrow{BC}$

SOLUTION

Proof:

Statements	Reasons
1.	1.
2.	2.
3.	3.
4.	4.
5.	5.
6.	6.



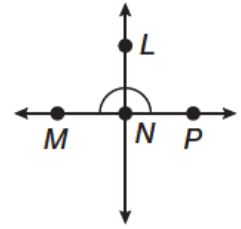
You Try!!!!

- a. If a triangle is obtuse, what can you conclude about the measures of its two non-obtuse angles? Justify your answer.

b. Fill in the reasons of the proof of Theorem 5-5: If two lines form congruent adjacent angles, then they are perpendicular.

Given:  $\angle LNM \cong \angle LNP$

Prove:  $LN \perp MP$

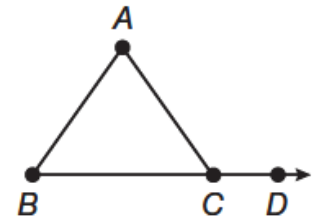


Statements	Reasons
1. $\angle LNM \cong \angle LNP$	1.
2. $m\angle LNM = m\angle LNP$	2.
3. $m\angle MNP = 180^\circ$	3.
4. $m\angle LNM + m\angle LNP = m\angle MNP$	4.
5. $2m\angle LNM = 180^\circ$	5.
6. $m\angle LNM = 90^\circ$	6.
7. $LN \perp MP$	7.

c. Given  $\triangle ABC$  with exterior angle  $\angle ACD$ , write a two-column proof to prove the Exterior Angle Theorem.

Given:  $\angle ACD$  is an exterior angle of  $\triangle ABC$

Prove:  $m\angle ACD = m\angle CAB + m\angle ABC$



- 1.
- 2.
- 3.
- 4.
- 5.
- 6.