Lesson 27

Two-Column Proofs

In a proof, deductive reasoning is used to develop a logical argument from given information to prove a conclusion. Proofs in geometry must be done step by step, and each step must have a justification. These justifications can include the given information, definitions, postulates, theorems, and properties, as seen in the two-column proofs in this lesson.

Example 1 Justifying Statements in a Two-Column Proof, Part 1

Fill in the justifying statements to support the proof of Theorem 4-2: If there is a line and a point not on the line, then exactly one plane contains them.

Given: Point C is not on $\overleftrightarrow{AB}$.

Prove: Exactly one plane contains $\overleftrightarrow{AB}$ and C.

Example 1 Justifying Statements in a Two-Column Proof, Part 1

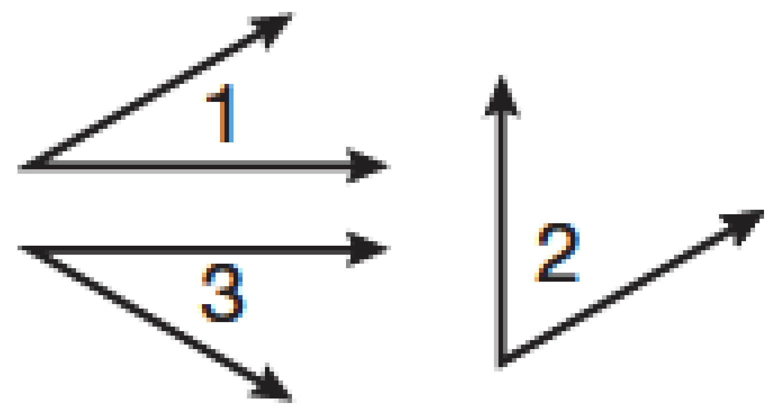
Statements	Reasons
1. Point *C* is noncollinear with $\overleftrightarrow{AB}$.	1. Given
2. Exactly one plane contains points *A*, *B*, and *C*.	2. Through any three noncollinear points there exists exactly one plane. (Postulate 6)
3. Exactly one plane contains $\overleftrightarrow{AB}$ and *C*.	3. If two points lie in a plane, then the line containing the points lies in the plane. (Postulate 8)

Example 2 Justifying Statements in a Two-Column Proof, Part 2

Prove Theorem 6-1: If two angles are complementary to the same angle, then they are congruent.

Given: $\angle 1$ is complementary to $\angle 2$. $\angle 3$ is complementary to $\angle 2$.

Prove: $\angle 1 \cong \angle 3$



Example 2 Justifying Statements in a Two-Column Proof, Part 2

Statements	Reasons
1. $\angle 1$ is complementary to $\angle 2$. $\angle 3$ is complementary to $\angle 2$.	1. Given
2. $m\angle 1 + m\angle 2 = 90°$ $m\angle 3 + m\angle 2 = 90°$	2. Definition of complementary angles
3. $m\angle 1 + m\angle 2 = m\angle 3 + m\angle 2$	3. Substitution Property of Equality
4. $m\angle 1 + m\angle 2 - m\angle 2 = m\angle 3 + m\angle 2 - m\angle 2$	4. Subtraction Property of Equality
5. $m\angle 1 = m\angle 3$	5. Simplify
6. $\angle 1 \cong \angle 3$	6. Definition of congruent angles

Two-column proofs have a format that is composed of five parts.

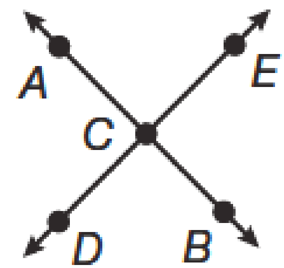
1. Given statement(s): The information that is provided.
2. Prove statement: The statement indicating what is to be proved.
3. Diagram: A sketch that summarizes the provided information. Sometimes you will need to draw the sketch yourself based on given information.
4. Statements: The specific steps that are written in the left-hand column.
5. Reasons: Postulates, theorems, definitions, or properties written in the right-hand column, which justify each statement.

Example 3 Writing a Two-Column Proof, Part 1

Prove Theorem 6-4: If two angles are vertical angles, then they are congruent. (Vertical Angles Theorem)

Given: $\overleftrightarrow{AB}$ and $\overleftrightarrow{DE}$ intersect at point C

Prove: $\angle ACD \cong \angle BCE$



SOLUTION

Statements	Reasons
1. $\overleftrightarrow{AB}$ and $\overleftrightarrow{DE}$ intersect at point C	1. Given
2. $m\angle BCD + m\angle ACD = 180°$	2. Linear Pair Theorem
3. $m\angle BCD + m\angle BCE = 180°$	3. Linear Pair Theorem
4. $m\angle BCD + m\angle ACD = m\angle BCD + m\angle BCE$	4. Transitive Prop of Equality
5. $m\angle ACD = m\angle BCE$	5. Sub Prop of Equality
6. $\angle ACD \cong \angle BCE$	6. Def of congruent angles

Example 4 Writing a Two-Column Proof, Part 2

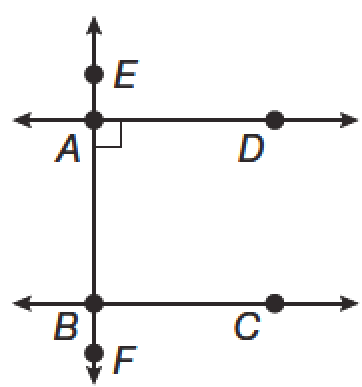
Prove Theorem 5-3: If a transversal is perpendicular to one of two parallel lines, then it is perpendicular to the other one.

Given: $\overleftrightarrow{AD} \parallel \overleftrightarrow{BC}$ *and* $\overleftrightarrow{EB} \perp \overleftrightarrow{AD}$

Prove: $\overleftrightarrow{EB} \perp \overleftrightarrow{BC}$

SOLUTION

Statements	Reasons
1. $\overleftrightarrow{AD} \parallel \overleftrightarrow{BC}$	1. Given
2. $\angle EAD \cong \angle ABC$	2. Postulate 11: corresponding angles are congruent
3. $m\angle EAD = 90°$	3. Def of perpendicular
4. $m\angle ABC = m\angle EAD$	4. Def of congruent angles
5. $m\angle ABC = 90°$	5. Transitive Prop of Equality
6. $\overleftrightarrow{EB} \perp \overleftrightarrow{BC}$	6. Definition of perpendicular



You Try!!!

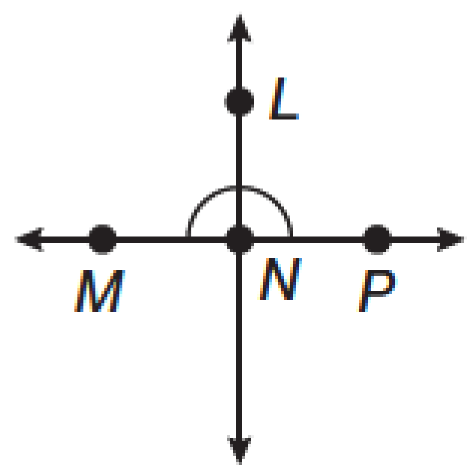
a. If a triangle is obtuse, what can you conclude about the measures of its two non-obtuse angles? Justify your answer.

b. Fill in the reasons of the proof of Theorem 5-5: If two lines form congruent adjacent angles, then they are perpendicular.

Given: $\angle LNM \cong \angle LNP$

Prove: $LN \perp MP$

Statements	Reasons
1. $\angle LNM$ _ $\angle LNP$	1.
2. $m\angle LNM = m\angle LNP$	2.
3. $m\angle MNP = 180°$	3.
4. $m\angle LNM + m\angle LNP = m\angle MNP$	4.
5. $2m\angle LNM = 180°$	5.
6. $m\angle LNM = 90°$	6.
7. $LN \perp MP$	7.

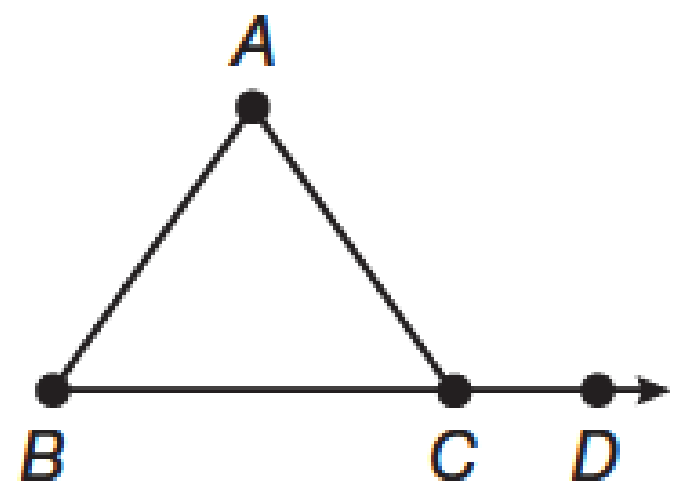


You Try!!!

c. Given ΔABC with exterior angle $\angle ACD$, write a two-column proof to prove the Exterior Angle Theorem.

Given: $\angle ACD$ is an exterior angle of ΔABC

Prove: $m\angle ACD = m\angle CAB + m\angle ABC$



Assignment

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Lesson Practice (Ask Mr. Heintz)

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Practice 1–30 (Do the starred ones first)