Lesson 28

Triangle Congruence: SAS

Included Angle – The angle formed by two adjacent sides of a polygon.

Included Side – The common side of two consecutive angles of a polygon.

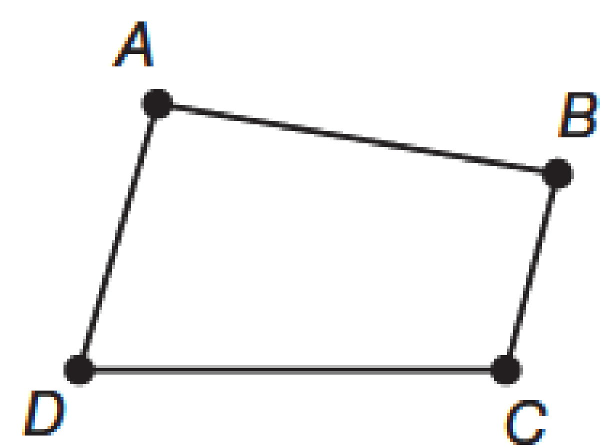
Example 1 Identifying Included Angles and Sides

What is the included side of $\angle A$ and $\angle B$? What is the included angle of $\overline{BC}$ *and* $\overline{CD}$?

SOLUTION

Angles A and B share the side $\overline{AB}$, so $\overline{AB}$ is the included side.

The angle between $\overline{BC}$ *and* $\overline{CD}$ is $\angle C$, so $\angle C$ is the included angle.



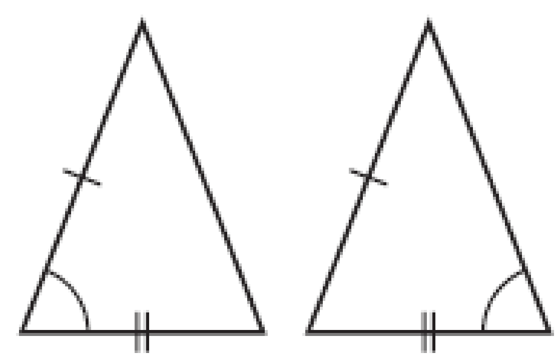
Side-Angle-Side (SAS) Triangle Congruence Postulate – If two sides and the included angle of one triangle are congruent to two sides and the included angle of another triangle, then the triangles are congruent by side-angle-side congruence.

Example 2 Using the SAS Postulate to Determine Congruency

Determine whether the pair of triangles is congruent by the SAS Postulate.

SOLUTION

The two indicated triangles are not necessarily congruent, even though they have two congruent sides and one congruent angle. In the second triangle, the angle that is congruent is not the included angle of the two congruent sides.



Example 3 Finding Missing Angle Measures

Find the value of x that makes the triangles congruent.

SOLUTION

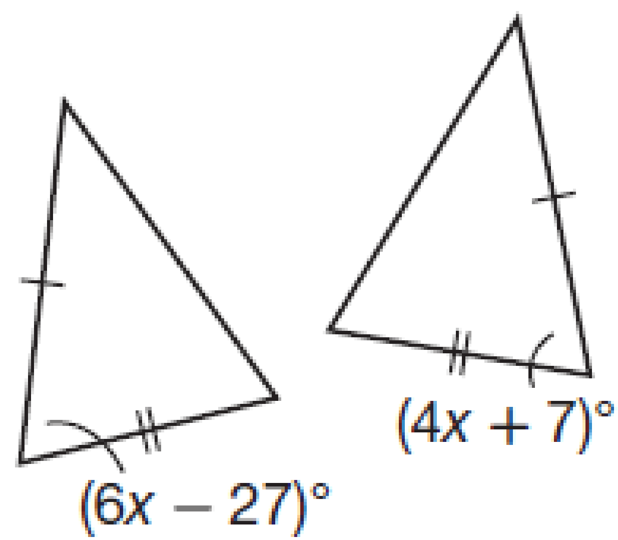
For the two triangles to be congruent, the measures of the included angles must be equal. Therefore,

$6x - 27 = 4x + 7$

$6x = 4x + 34$

$2x = 34$

$x = 17$

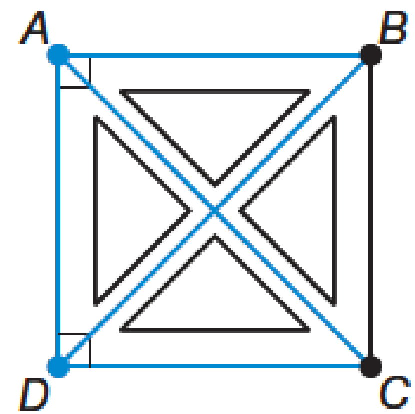


Example 4 Using the SAS Postulate in a Proof

Triangles make an "X" design on this barn door. Use the SAS Postulate to write a two-column proof.

Given: $\overline{AB} \cong \overline{DC}$

Prove: $\Delta ABD \cong \Delta \mathrm{DCA}$



SOLUTION

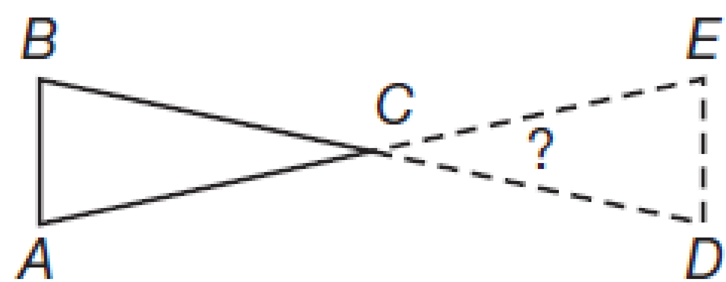
1. $\overline{AB} \cong \overline{DC}$ $\angle ADC$ and $\angle DAB$ are right angles	1. Given
2. $m\angle DAB = m\angle ADC$	2. All right angles are congruent.
3. $\overline{AD} \cong \overline{AD}$	3. Reflexive Prop of Congruence
4. $\Delta ABD \cong \Delta \mathrm{DCA}$	4. SAS Postulate

Example 5 Application: Design

An artist is designing patterned wallpaper made of congruent triangles. He starts by drawing ΔABC, shown below. He wants to design a mirror image of ΔABC, shown as ΔEDC below. How can he make sure that this new triangle is congruent to ΔABC using the SAS pattern of triangle congruence?

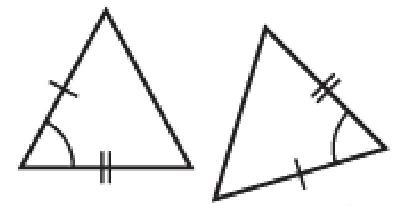
SOLUTION

To ensure that the two triangles are congruent, he should first measure $\overline{BC}$ and $\overline{AC}$. He can then extend both segments at points E and D, respectively, such that C is the midpoint of both $\overline{AE}$ and $\overline{BD}$. Since $\angle BCA$ and $\angle ECD$ are vertical angles, they are congruent, and the triangles must also be congruent by the SAS Postulate.



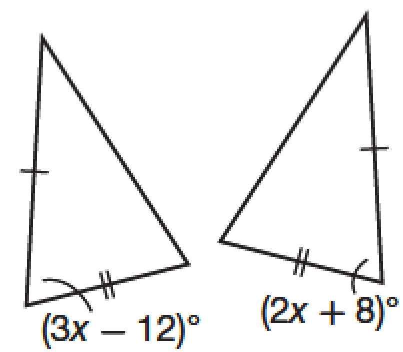
You Try!!!!!

Determine whether the pair of triangles is congruent by the SAS Postulate.



Yes

Find the value of x that makes the triangles congruent.



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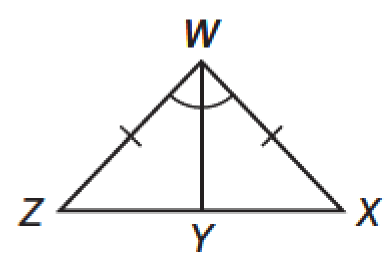
Use the SAS Postulate to prove $\Delta WXY \cong \Delta WZY$ if $\overline{WZ} \cong \overline{WX}$ and $\angle ZWY \cong \angle XWY$.



Assignment

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Lesson Practice (Ask Mr. Heintz)

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Practice 1–30 (Do the starred ones first)