Lesson 29

Using the Pythagorean Theorem

Pythagorean Triples – A Pythagorean triple is a set of three nonzero whole numbers *a*, *b*, and *c* such that:

$$a^2 + b^2 = c^2$$

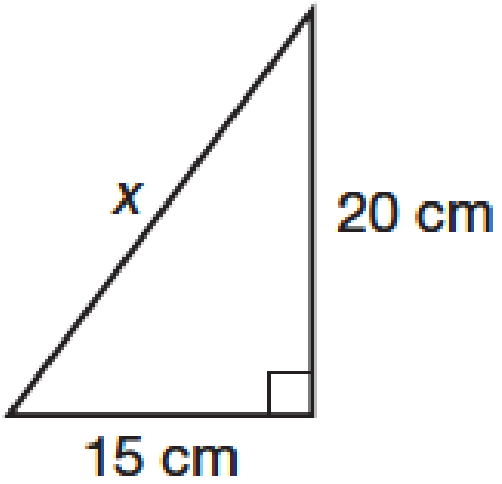
Two of the most well-known sets of Pythagorean triples are (3, 4, 5) and (5, 12, 13). An easy way to find Pythagorean triples is to multiply one of these two sets by a whole number.

Example 1 Finding Pythagorean Triples

Find the unknown length in the triangle. Do the side lengths form a Pythagorean triple?

SOLUTION

$$a^2 + b^2 = c^2$$
$$15^2 + 20^2 = x^2$$
$$225 + 400 = x^2$$
$$625 = x^2$$
$$\sqrt{625} = \sqrt{x^2}$$
$$25 = x$$

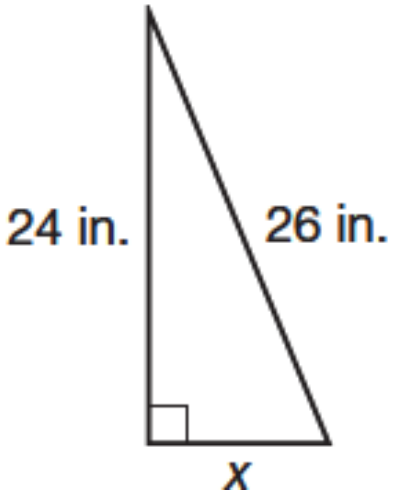


Therefore, the length of the hypotenuse is 25 centimeters. The set (15, 20, 25) , which gives the side lengths of this triangle, is the Pythagorean triple (3, 4, 5) multiplied by 5.

Example 2 Using Pythagorean Triples To Find the Legs

Find the unknown length in the triangle.
Do the side lengths form a Pythagorean triple?
SOLUTION

$$a^2 + b^2 = c^2$$
$$24^2 + x^2 = 26^2$$
$$576 + x^2 = 676$$
$$x^2 = 100$$
$$\sqrt{x^2} = \sqrt{100}$$
$$x = 10$$



The set which gives the side lengths of this triangle (10, 24, 26) , is the Pythagorean triple (5, 12, 13) multiplied by 2.

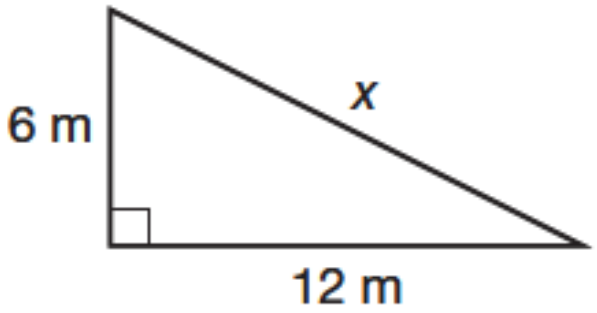
However, not all right triangles are composed of side lengths that are nonzero whole numbers. In such cases, one or more side lengths may be written as a radical expression.

Radical Expression – Any expression that contains a root. Typically, a radical expression should be reduced to simplified radical form.

Example 3 Simplifying Radicals

a. Find the value of x. Give your answer in simplified radical form.
SOLUTION

$$a^2 + b^2 = c^2$$
$$6^2 + 12^2 = x^2$$
$$36 + 144 = x^2$$
$$180 = x^2$$
$$\sqrt{180} = \sqrt{x^2}$$



To write the answer in simplified radical form, you must factor out all perfect square factors of the number under the radical sign. The largest perfect square that is a factor of 180 is 36, so 180 is factored out as 36×5.

$$\sqrt{36 \cdot 5} = x$$
$$6\sqrt{5} = x$$

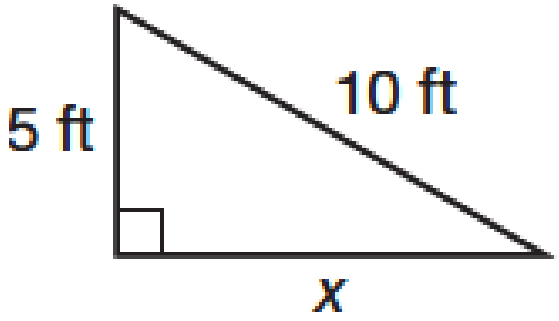
Therefore, the length of the hypotenuse is $6\sqrt{5}$ m.

Example 3 Simplifying Radicals

b. Find the value of x. Give your answer in simplified radical form.

SOLUTION

$$a^2 + b^2 = c^2$$
$$5^2 + x^2 = 10^2$$
$$25 + x^2 = 100$$
$$x^2 = 75$$
$$\sqrt{x^2} = \sqrt{75}$$
$$x = 5\sqrt{3}$$



Therefore, the length of the second leg is $5\sqrt{3}$ feet.

Example 4 Application: TV Aspect Ratios

The aspect ratio of a TV screen is the ratio of the width to the height of the image. Find the height and the width of a 42-inch TV screen with an aspect ratio of 4:3 to the nearest tenth of an inch. The length 42 inches refers to the diagonal distance across the screen.

SOLUTION

$$a^2 + b^2 = c^2$$
$$(3x)^2 + (4x)^2 = 42^2$$
$$9x^2 + 16x^2 = 42^2$$
$$25x^2 = 42^2$$
$$x^2 = \frac{42^2}{25}$$
$$x = \frac{42}{5} = 8.4$$

Example 4 Application: TV Aspect Ratios

Width: $= 4x$
$= 4\ (8.4)$
$= 33.6$ inches

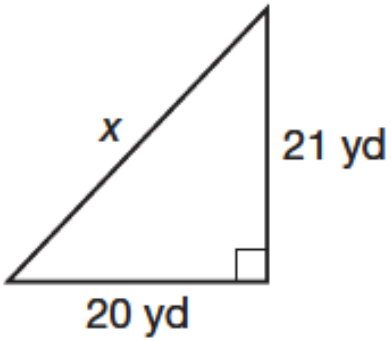
Height: $= 3x$
$= 3\ (8.4)$
$= 25.2$ inches

Check

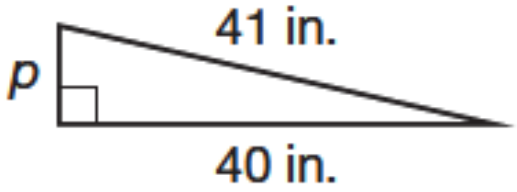
$$a^2 + b^2 = c^2$$
$$(33.6)^2 + (25.2)^2 = c^2$$
$$42 = c$$

You Try!!!!

a.Find the hypotenuse of the triangle. Do the side lengths form a Pythagorean triple?

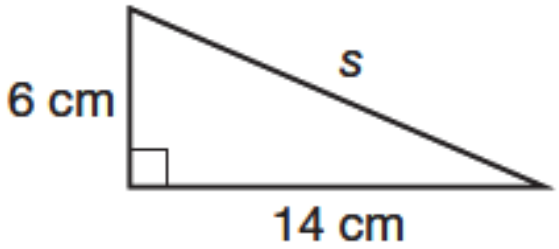


b.Find the value of *p* in the triangle at right. Do the side lengths form a Pythagorean triple?

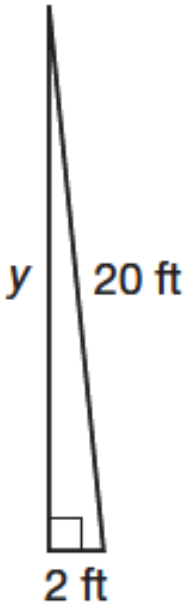


You Try!!!!

c.Find the value of *s* in the triangle at right. Give your answer in simplified radical form.



d.Find the value of *y* in the triangle at right. Give your answer in simplified radical form.



You Try!!!!

e.A ratio of a TV's width to its height is 16:9. If its width is 32 inches, what is the length of its diagonal?

Assignment

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Lesson Practice (Ask Mr. Heintz)

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Practice 1-30 (Do the starred ones first)