Lesson 3

Angles

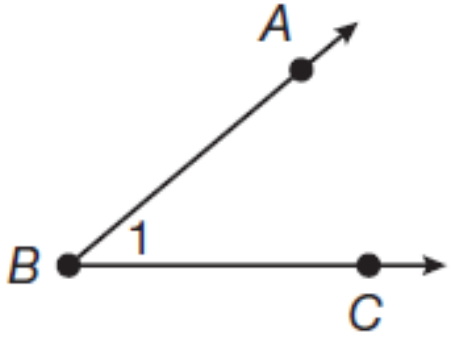
A ray is a part of a line that starts at an endpoint and extends infinitely in one direction.

A ray is named by its endpoint and any other point on the ray. For example, the ray in the diagram is called $\overrightarrow{AB}$, which is read "ray *AB*."

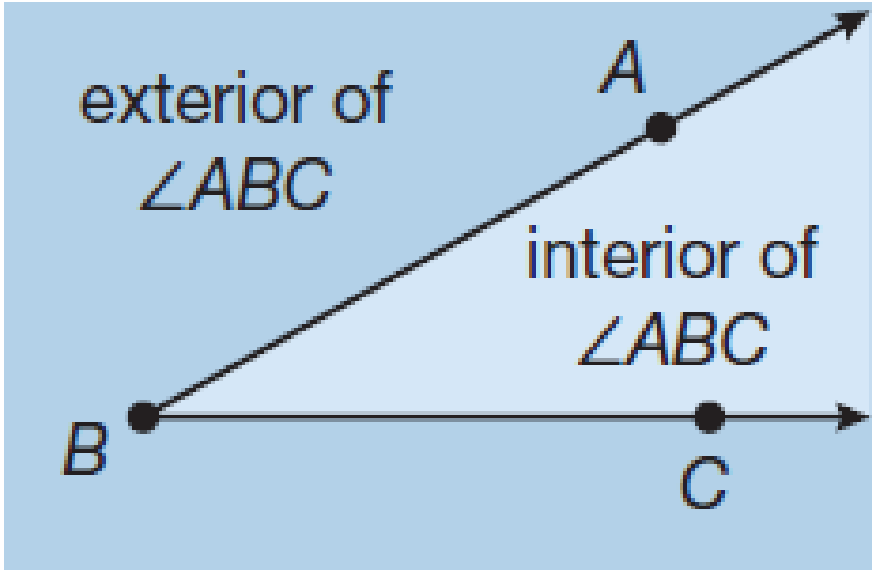
Two rays that have a common endpoint and form a line are called opposite rays. $\overrightarrow{YX}$ and $\overrightarrow{YZ}$ are opposite rays.

An angle is a figure formed by two rays with a common endpoint. The common endpoint is the angle's vertex. The rays are the sides of the angle. The sides of this angle are $\overrightarrow{BA}$ and $\overrightarrow{BC}$. The vertex is *B*.

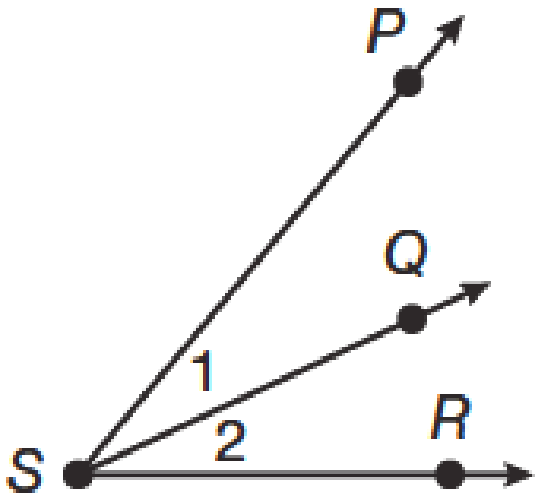
An angle can be named in several different ways: by its vertex, by a point on each ray and the vertex, or by a number. For example, the angle in the diagram could be called $\angle B$, $\angle ABC$, $\angle CBA$, or $\angle 1$.



The exterior of an angle is the set of all points outside the angle. The interior of an angle is the set of all points between the sides of an angle.



Example 1 Naming Angles and Rays



a. Name three rays in the diagram.

SOLUTION

$$\overrightarrow{SP}, \overrightarrow{SQ}, and\ \overrightarrow{SR}$$

b. Name three angles in the diagram.

SOLUTION

$\angle PSQ$ or $\angle 1$, $\angle QSR$ or $\angle 2$, and $\angle PSR$.

c. Could $\angle PSQ$ also be referred to as $\angle S$?

SOLUTION

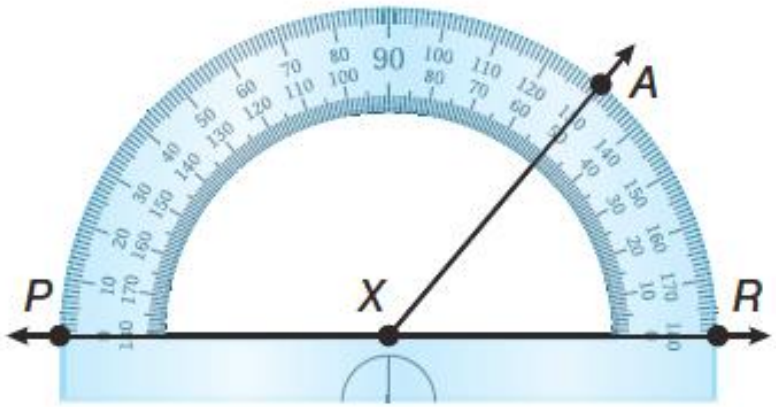
No, there are three different angles with S as a vertex.

A protractor is a tool used to measure angles. Unlike segments, angles are measured in degrees. One degree is a unit of angle measure that is equal to $\frac{1}{360}$ of a circle.

Postulate 3: Protractor Postulate – Given a point X on $\overleftrightarrow{PR}$, consider rays $\overrightarrow{XP}$ and $\overrightarrow{XR}$, as well as all the other rays that can be drawn with X as an endpoint, on one side of $\overleftrightarrow{PR}$. These rays can be paired with the real numbers from 0 to 180 such that:

1. $\overrightarrow{XP}$ is paired with 0, and $\overrightarrow{XR}$ is paired with 180.

2. If $\overrightarrow{XA}$ is paired with a number c and $\overrightarrow{XB}$ is paired with a number d then $m\angle AXB = |c - d|$.



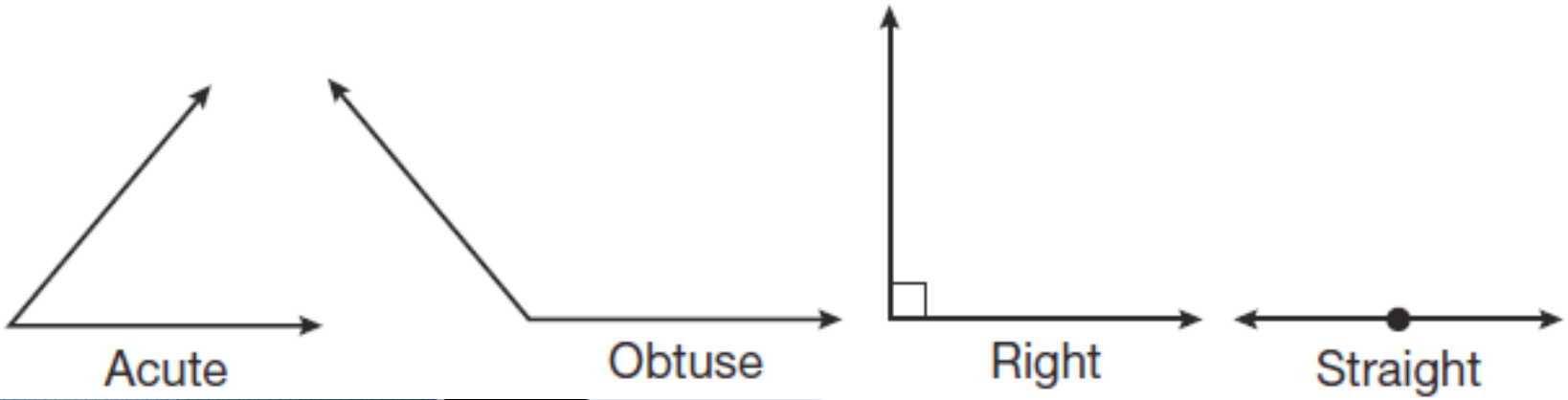
Angles are classified according to their angle measure.

An acute angle measures greater than 0° and less than 90°.

An obtuse angle measures greater than 90° and less than 180°.

A right angle measures exactly 90°. A box drawn at the vertex of an angle shows that it is a right angle, as shown in the diagram.

A straight angle measures exactly 180°.

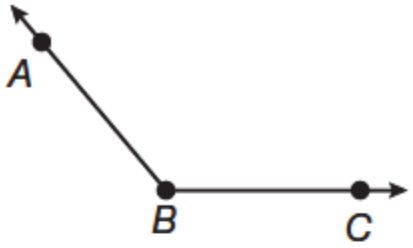


Example 2 Measuring and Classifying Angles

a. Use a protractor to measure $\angle ABC$, then classify the angle.

SOLUTION

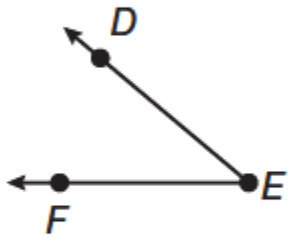
$\angle ABC$ measures 130°, so it is an obtuse angle.



b. Use a protractor to measure $\angle DEF$, then classify the angle.

SOLUTION

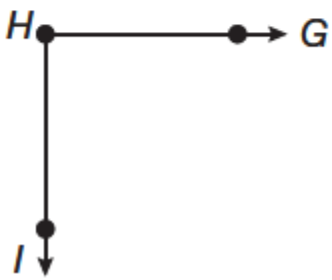
$\angle DEF$ measures 40°, so it is an acute angle.



c. Use a protractor to measure $\angle GHI$, then classify the angle.

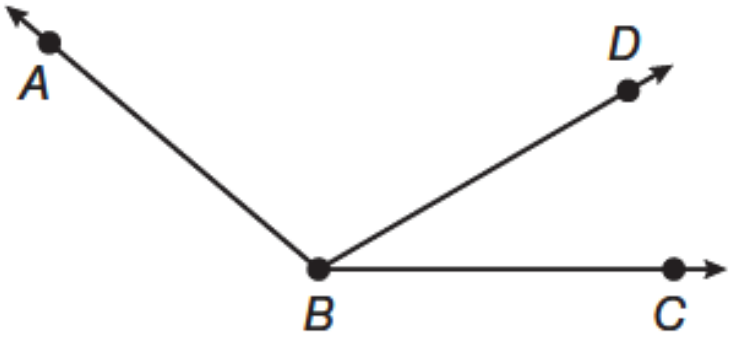
SOLUTION

$\angle GHI$ measures 90°, so it is a right angle.



Angles can be added in the same way that segments are added.

Postulate 4: The Angle Addition Postulate – If point D is in the interior of $\angle ABC$, then $m\angle ABD + m\angle DBC = m\angle ABC$.



Example 3 Using the Angle Addition Postulate

The measure of $\angle RST = 22°$ and $m\angle TSU = 69°$. Find $m\angle RSU$. Classify the angle.

SOLUTION

$m\angle RST + m\angle TSU = m\angle RSU$

Angle Addition Postulate

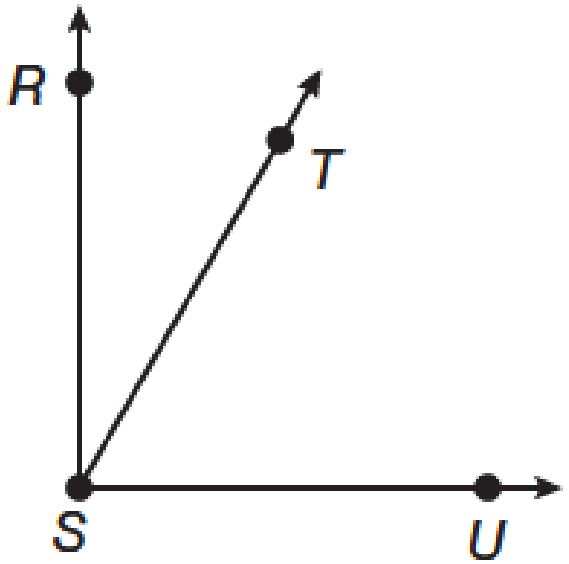
$22° + 69° = m\angle RSU$

Substitute.

$91° = m\angle RSU$

Simplify.

$\angle RSU$ is an obtuse angle.

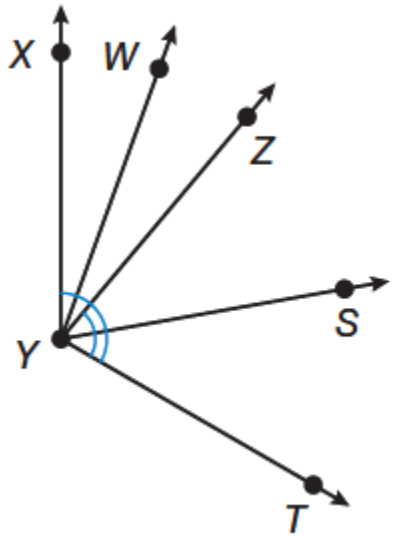


To bisect a figure is to divide it into two congruent parts.

An angle bisector is a ray that divides an angle into two congruent angles.

Congruent angles have the same measure.

They are marked with arc marks, as shown in the diagram.



Example 4 Using Angle Bisectors and Congruence Marks

The measure of $\angle ABC = 44°$. $\overrightarrow{BC}$ bisects $\angle ABD$.
The measure of $\angle EBF = 23°$. Find the measure of $\angle CBE$.

SOLUTION

Since $\overrightarrow{BC}$ bisects $\angle ABD$, it divides $\angle ABD$ into two congruent angles.

So, $\angle ABC \cong \angle CBD$ and $m\angle ABC = m\angle CBD$.

Since $m\angle ABC = 44°$, $m\angle CBD = 44°$.

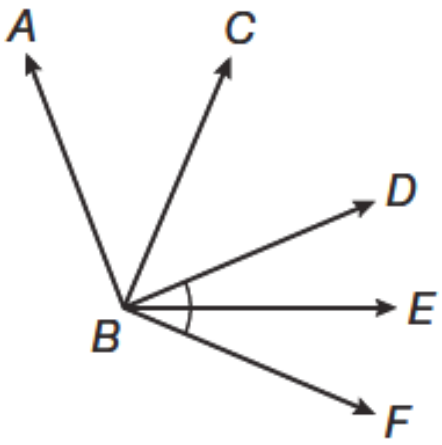
Using the congruence marks in the diagram, $\angle DBE \cong \angle EBF$, so $m\angle DBE = m\angle EBF$. Since $m\angle EBF = 23°$, $m\angle DBE = 23°$.

$m\angle CBE = m\angle CBD + m\angle DBE$ Angle Addition Postulate

$= 44° + 23°$ Substitute.

$= 67°$ Add.

The measure of $m\angle CBE$ is 67°.



Example 5 Application: Interpreting Statistics

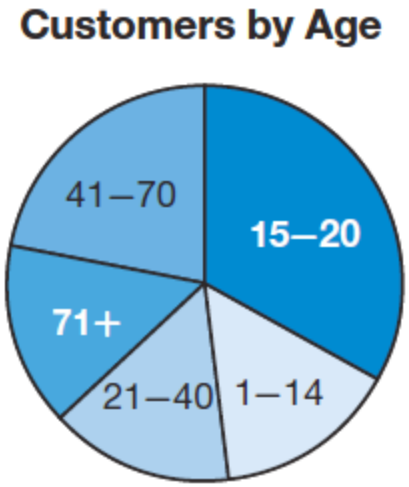
Louis runs a restaurant. He knows that he has about 900 customers a day. The circle graph in the diagram shows what percentage of his customers fall into the given age brackets. He wants to know exactly how many of his customers are between ages 15 and 20. Use a protractor to measure the angle and find the number of Louis's customers that fall into the 15–20 age bracket.

SOLUTION

Measure the angle of the sector that represents 15–20–year–old customers. The sector has an angle measure of 120°.

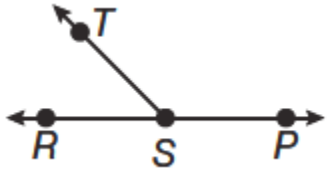
Since an entire circle is 360°, this is $\frac{120}{360} = \frac{1}{3}$ of the circle.

One third of Louis's customers is $\left(\frac{1}{3}\right)(900) = 300$ customers.

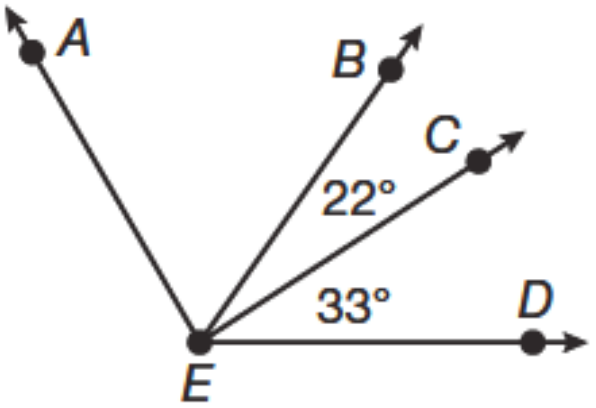


You Try!!!!

a. Name three rays and three angles in the diagram.

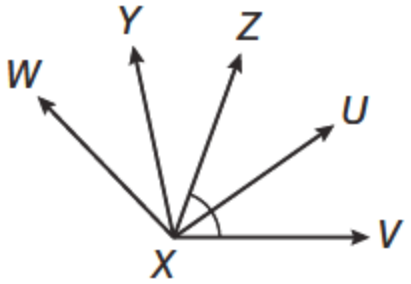


c.Determine m$\angle AEB$ if m$\angle AED = 120°$.



You Try!!!!

d.The measure of $\angle WXY = 32°$. $\overrightarrow{XY}$ bisects $\angle WXZ$. The measure of $\angle UXV = 35°$. Find the measure of $\angle YXU$.



e. A survey shows that 10% of students in a class did not eat lunch. What would be the degree measure of an angle indicating these students on a circle graph?

Assignment

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Lesson Practice a-f (Ask Mr. Heintz)

Page 17

Practice 1-30 (Do the starred ones first)