Lesson 30

Triangle Congruence: ASA and AAS

Postulate 16: Angle-Side-Angle (ASA) Congruence Postulate - If two angles and the included side of one triangle are congruent to two angles and the included side of another triangle, then the triangles are congruent.

Example 1 Using the ASA Postulate

Use ASA congruence to determine the measure of the sides of ΔDEF.

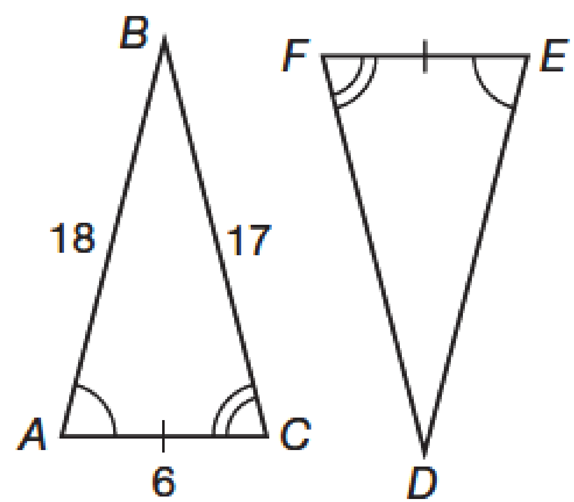
SOLUTION

Since two angles and the included side of ΔBAC are congruent to two angles and the included side of ΔDEF, by the ASA Postulate, $\Delta BAC \cong \Delta DEF$. Therefore, since

$\overline{AC} \cong \overline{EF}, EF = 6$

$\overline{CB} \cong \overline{FD}, FD = 17$

$\overline{AB} \cong \overline{ED}, ED = 18$

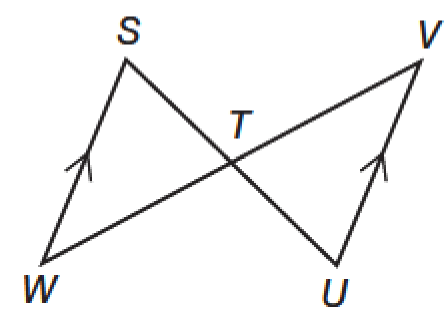


Example 2 Using the ASA Postulate in a Proof

Prove that $\Delta SWT \cong \Delta UVT$, given that T is the midpoint of $\overline{WV}$ and $\overline{VU} \parallel \overline{WS}$.

SOLUTION

1. T is the midpoint of $\overline{WV}$	1. Given
2. $\overline{WT} \cong \overline{VT}$	2. Def of midpoint
3. $\angle SWT \cong \angle TVU$	3. If two parallel lines are cut by a transversal, then alternate interior angles are congruent.
4. $\angle WTS \cong \angle VTU$	4. Vertical angles
5. $\Delta SWT \cong \Delta UVT$	5. ASA Cong Postulate



Theorem 30-1: Angle-Angle-Side (AAS) Triangle Congruence Theorem – If two angles and a non-included side of one triangle are congruent to two angles and the corresponding non-included side of another triangle, then the triangles are congruent.

Example 3 Using the AAS Congruence Theorem

Given that $\overline{DE} \cong \overline{LK}$, find the area of each triangle shown below.

SOLUTION

Since two angles and a non-included side of ΔDEF are congruent to the corresponding angles and non-included side of ΔLKM, $\Delta DEF \cong \Delta LKM$ by the AAS Congruence Theorem.

Therefore, solve for x using CPCTC: $EF = KM$

$4x - 2 = 3x + 1$

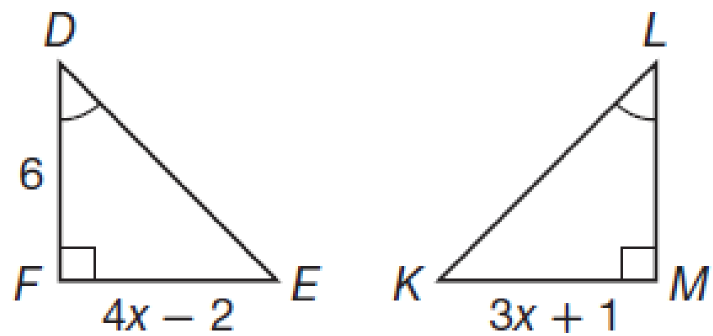
$x = 3$

EF$= 4 \cdot 3 - 2 = 10$

$A = \frac{1}{2}bh$

$A = \frac{1}{2}10 \cdot 6$

$A = 30$



Therefore, the area of each triangle is 30 square units.

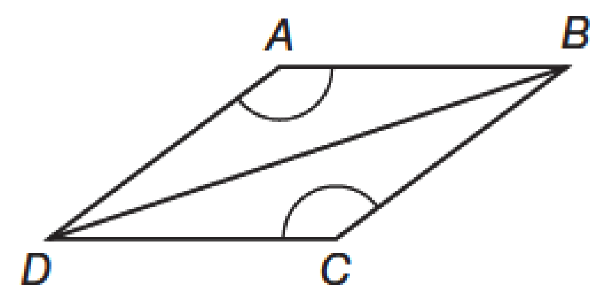
Example 4 Using the AAS Theorem in a Proof

Given: $\overline{BD}$ bisects $\angle ADC$ and $\angle A \cong \angle C$.

Prove: $\Delta ABD \cong \Delta CBD$

SOLUTION

Statements	Reasons
1. $\angle A \cong \angle C$	1. Given
2. $\angle ADB \cong \angle CDB$	2. Def of angle bisector
3. $\overline{DB} \cong \overline{DB}$	3. Reflexive Prop
4. $\Delta ABD \cong \Delta CBD$	4. AAS Theorem

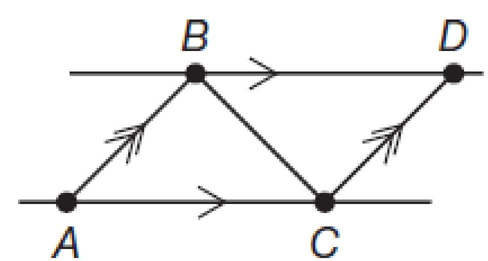


Example 5 Application: Bridges

A diagram of a portion of the truss system of a new bridge is shown below. Prove $\Delta ABC \cong \Delta DCB$.

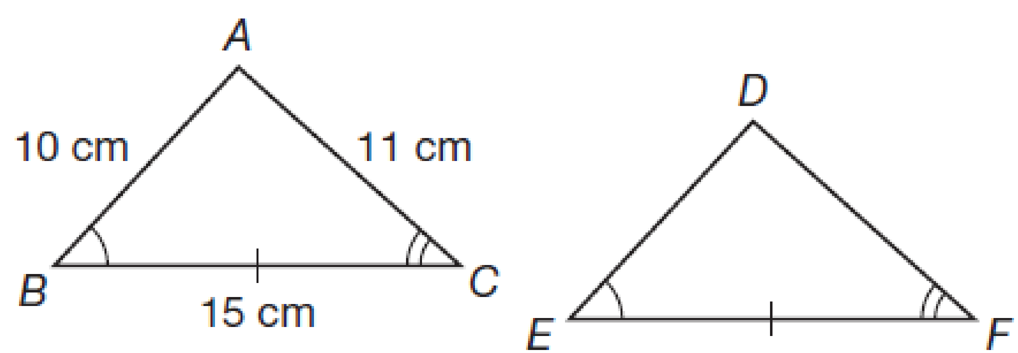
SOLUTION

Statements	Reasons
1. $\overline{BD} \parallel \overline{AC}$ $\overline{AB} \parallel \overline{CD}$	1. Given
2. $\angle DBC \cong \angle ACB$	2. If parallel lines are cut by a transversal, then alternate interior angles are congruent
(Theorem 10-1).	
3. $\angle ABC \cong \angle DCB$	3. Theorem 10-1
4. $\overline{BC} \cong \overline{BC}$	4. Reflexive Property of Congruence
5. $\Delta ABC \cong \Delta DCB$	5. ASA Theorem



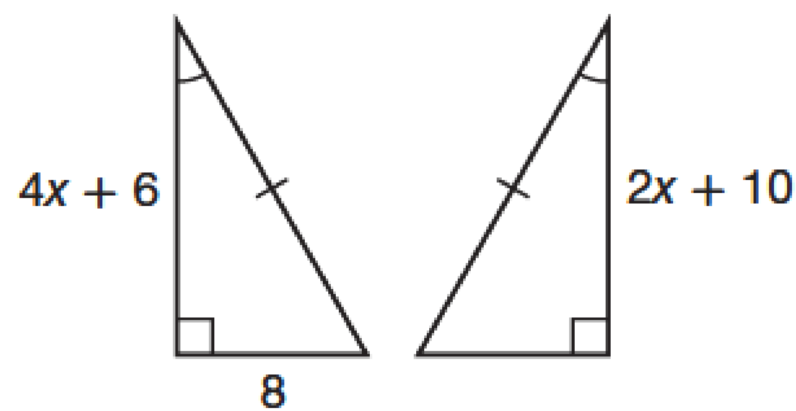
You Try!!!

a. State the postulate that can be used to prove the triangles congruent, and state the measure of the sides of ΔDEF.



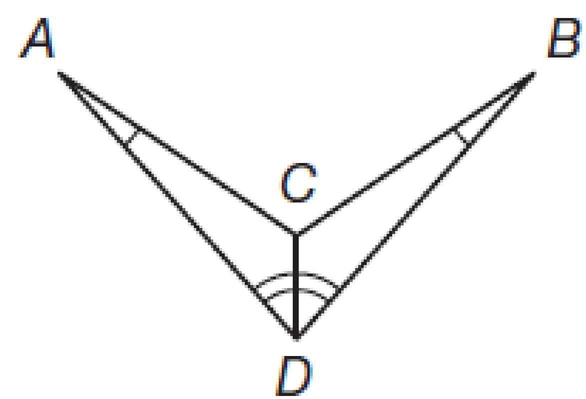
You Try!!!

c. If the two triangles are congruent by the AAS Theorem, what is the area of each triangle?



You Try!!!

d. Prove that $\Delta ADC \cong BDC$.



Assignment

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Lesson Practice (Ask Mr. Heintz)

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Practice 1–30 (Do the starred ones first)