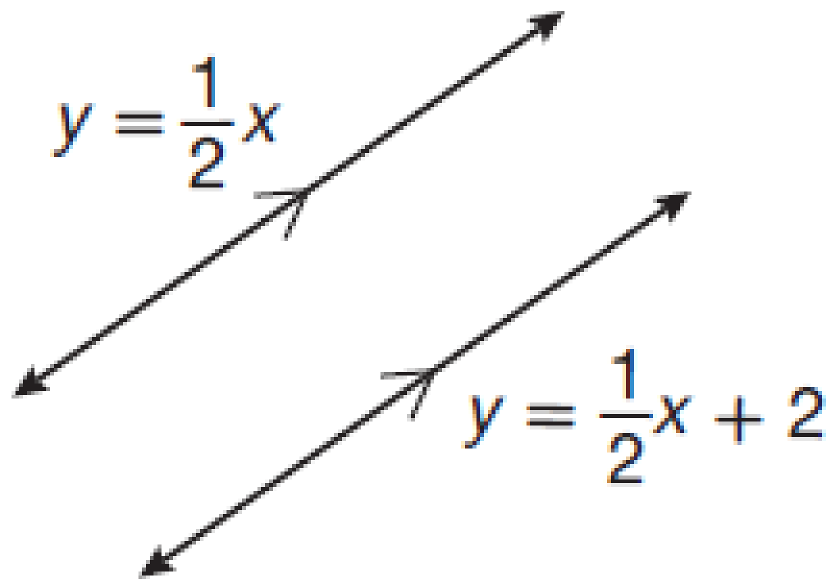
Lesson 37

Writing Equations of Parallel and Perpendicular Lines

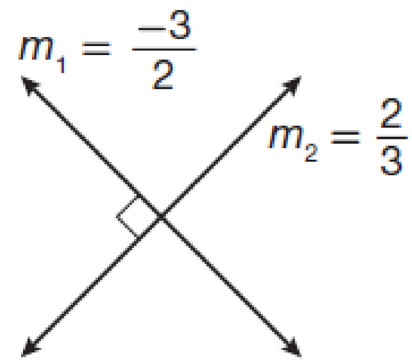
The coordinate plane provides a connection between algebra and geometry. Postulates 17 and 18 establish a simple way to find lines that are parallel or perpendicular on the coordinate plane.

Postulate 17: Parallel Lines Postulate – If two lines are parallel, then they have the same slope. All vertical lines are parallel to each other.



Perpendicular lines can also be found by looking at the slope.

Postulate 18: Perpendicular Lines Postulate – If two nonvertical lines are perpendicular, then the product of their slopes is –1. Vertical and horizontal lines are perpendicular to each other.



$$m_1 \times m_2 = \frac{-3}{2} \cdot \frac{2}{3} = -1$$

Opposite Reciprocal – The reciprocal of that number with the sign reversed.

Whenever two lines have slopes that are opposite reciprocals of each other, they are perpendicular lines.

Example 1 Finding the Slopes of Parallel and Perpendicular Lines

a. Find the slope of line *a*.

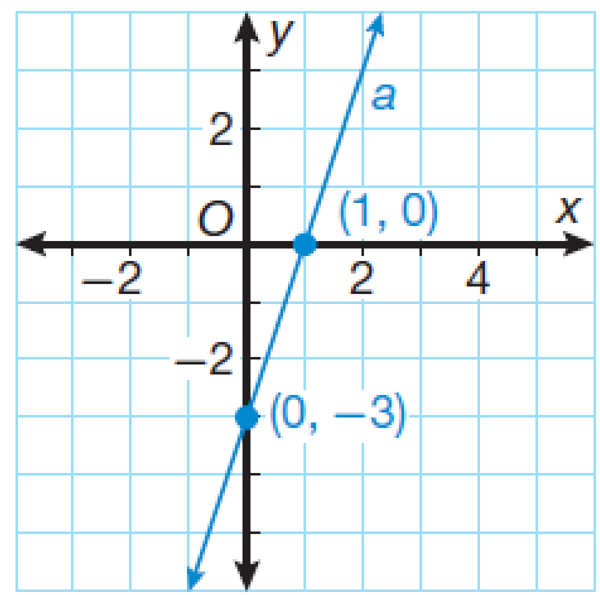
SOLUTION

Use the slope formula. $m = \frac{y_1 - y_1}{x_2 - x_1}$

Choose any two points on the line, for example (1, 0) and (0, −3). Substitute the coordinates into the slope formula.

$$m = \frac{0 - (-3)}{1 - 0}$$

$$m = 3$$



Example 1 Finding the Slopes of Parallel and Perpendicular Lines

b. Find the slope of a line parallel to line *a*.

SOLUTION

By the Parallel Lines Postulate, parallel lines have the same slope. The slope of a line that is parallel to line *a* is 3.

Example 1 Finding the Slopes of Parallel and Perpendicular Lines

c. Find the slope of a line perpendicular to line *a*.

SOLUTION

By the Perpendicular Lines Postulate, the slopes of perpendicular lines are opposite reciprocals.

The reciprocal of 3 is $\frac{1}{3}$. Changing the sign gives the opposite reciprocal, $-\frac{1}{3}$.

Example 2 Identifying Parallel and Perpendicular Lines

a. Are the lines $y = 2x + 4$ and $y = -3 + 2x$ parallel, perpendicular, or neither?

SOLUTION

By looking at the equations we can see that the slope of both lines is 2. Lines with the same slope are parallel, so these two lines are parallel to each other.

b. Are the lines $y = \frac{2}{3}x - 1$ and $y = \frac{3}{2}x$ parallel, perpendicular, or neither?

SOLUTION

The slope of the first line is $\frac{2}{3}$. The slope of the second line is $\frac{3}{2}$. These slopes are reciprocals of each other. They are not, however, opposite reciprocals, since both are positive. These lines are neither perpendicular nor parallel.

The point-slope formula for a line: $y - y_1 = m(x - x_1)$. Sometimes it is helpful to find a line passing through a given point that is parallel or perpendicular to another line. The point-slope formula can be used to solve problems like this, once you have discovered the slope of the parallel or perpendicular line.

Example 3 Graphing a Line Parallel to a Given Line

a. Find a line that is parallel to $y = x + 2$ and passes through point (3, 8).

SOLUTION

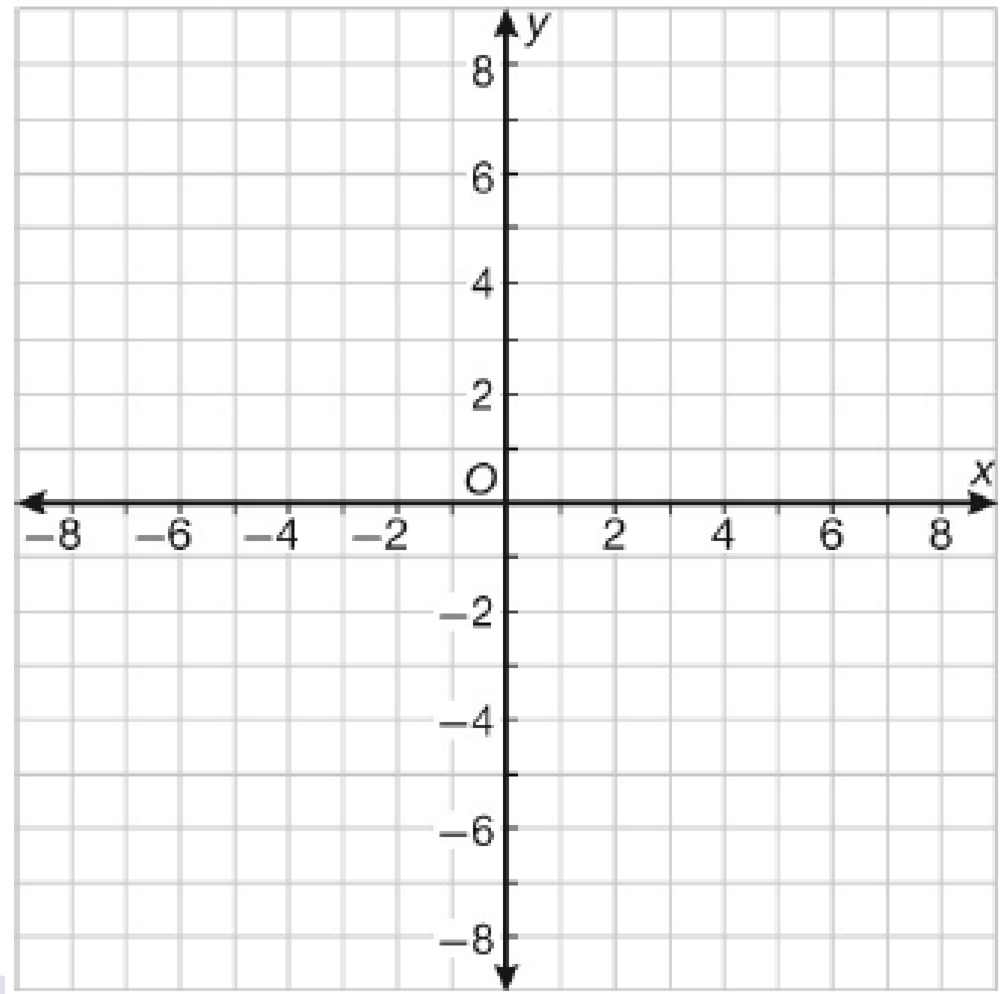
The slope of the given line is 1. Substitute the slope and the given point into the point-slope formula.

$y - y_1 = m(x - x_1)$	Point-slope formula
$y - 8 = 1(x - 3)$	Substitute.
$y = x + 5$	Solve.

Example 3 Graphing a Line Parallel to a Given Line

b. Graph the parallel lines from part a.

SOLUTION



Example 4 Graphing a Line Perpendicular to a Given Line

a. Find a line that is perpendicular to $y = \frac{2}{3}x$ and passes through the point (2, 4).

SOLUTION

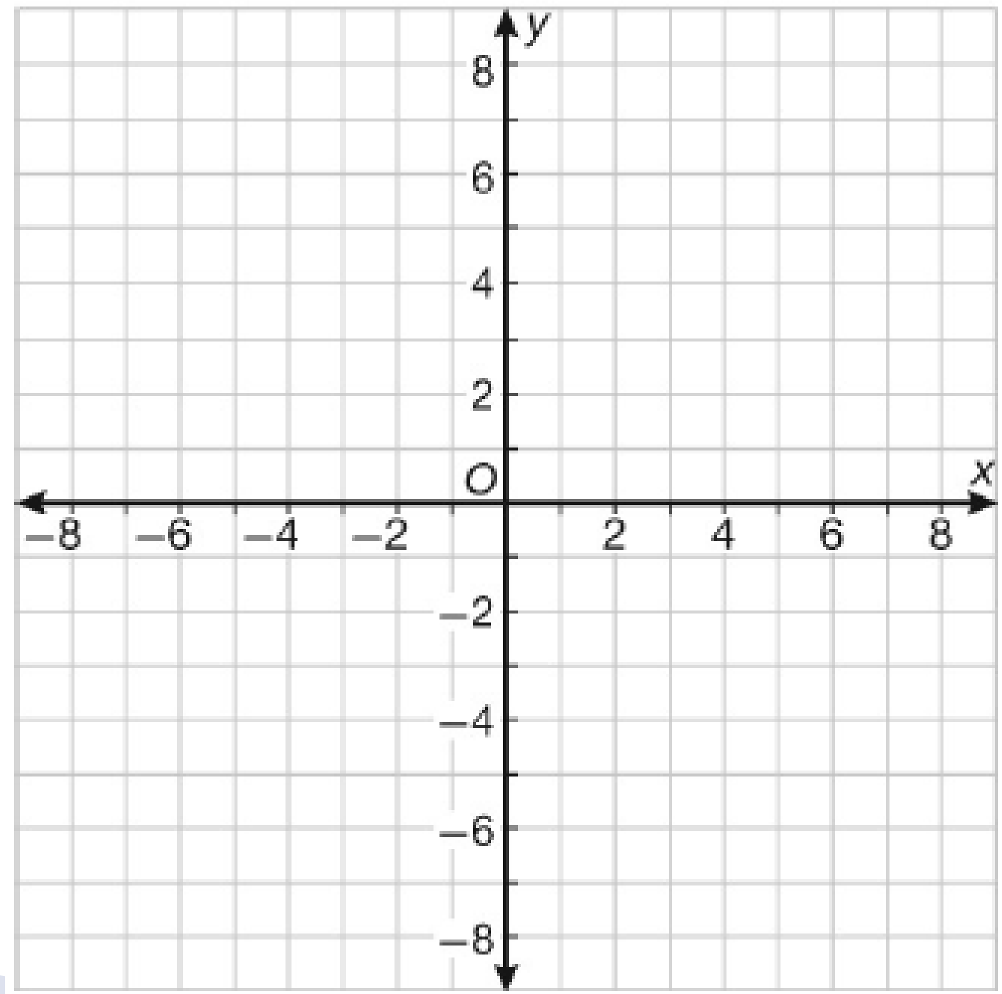
The slope of the given line is $\frac{2}{3}$. A perpendicular line will have a slope that is the opposite reciprocal, or $-\frac{3}{2}$. Substitute this slope and the given point into the point-slope formula.

$y - y_1 = m(x - x_1)$	Point-slope formula
$y - 4 = -\frac{3}{2}(x - 2)$	Substitute.
$y = -\frac{3}{2}x + 7$	Solve.

Example 4 Graphing a Line Perpendicular to a Given Line

b. Graph the perpendicular lines from part a.

SOLUTION



Example 5 Application: Swimming

In a race, one swimmer is swimming at a rate of 2 meters per second. Another swimmer gets a 5-meter head start, and also swims at 2 meters per second. What is the equation that will model the distance, y, that each swimmer has gone after x seconds? Will the first swimmer ever catch up to the second?

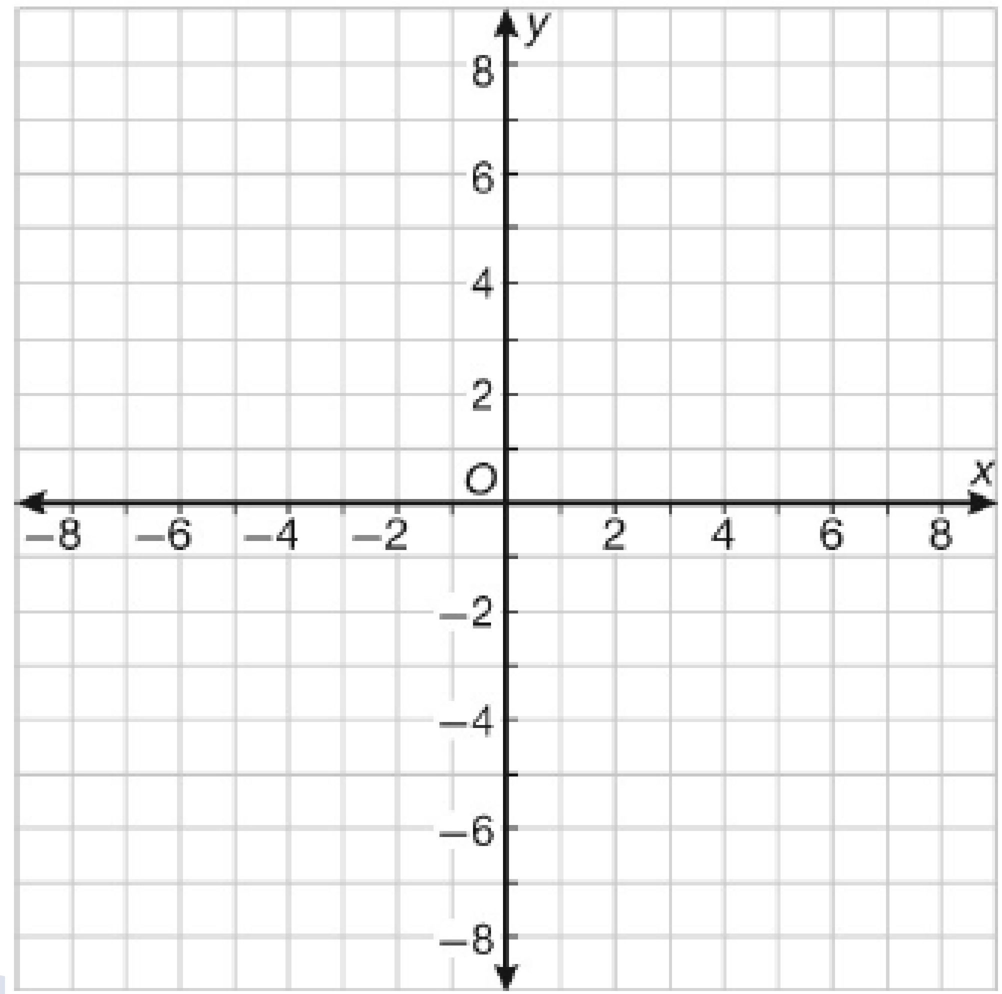
SOLUTION

Solve: Since we know that the slope of each line is 2, and the second swimmer has a y-intercept of 5, the equations for each line are given below. 1st swimmer: $y = 2x$ 2nd swimmer: $y = 2x + 5$

These lines both have a slope of 2, so they will never intersect. This tells us that the first swimmer will never catch up to the second swimmer.

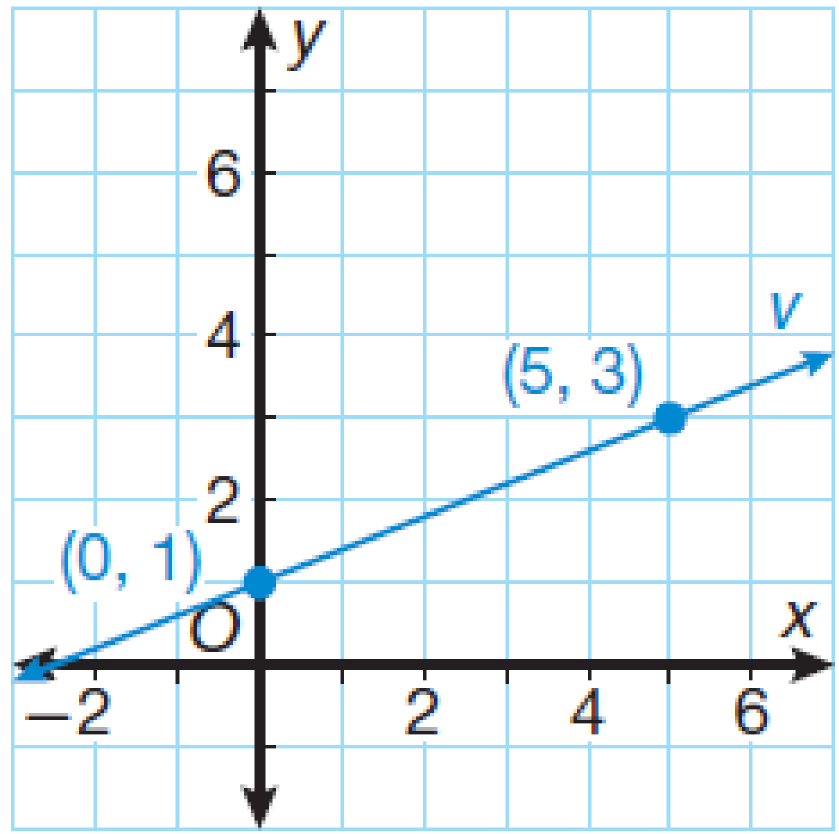
Example 5 Application: Swimming

Check Graph the lines $y = 2x$ and $y = 2x + 5$.



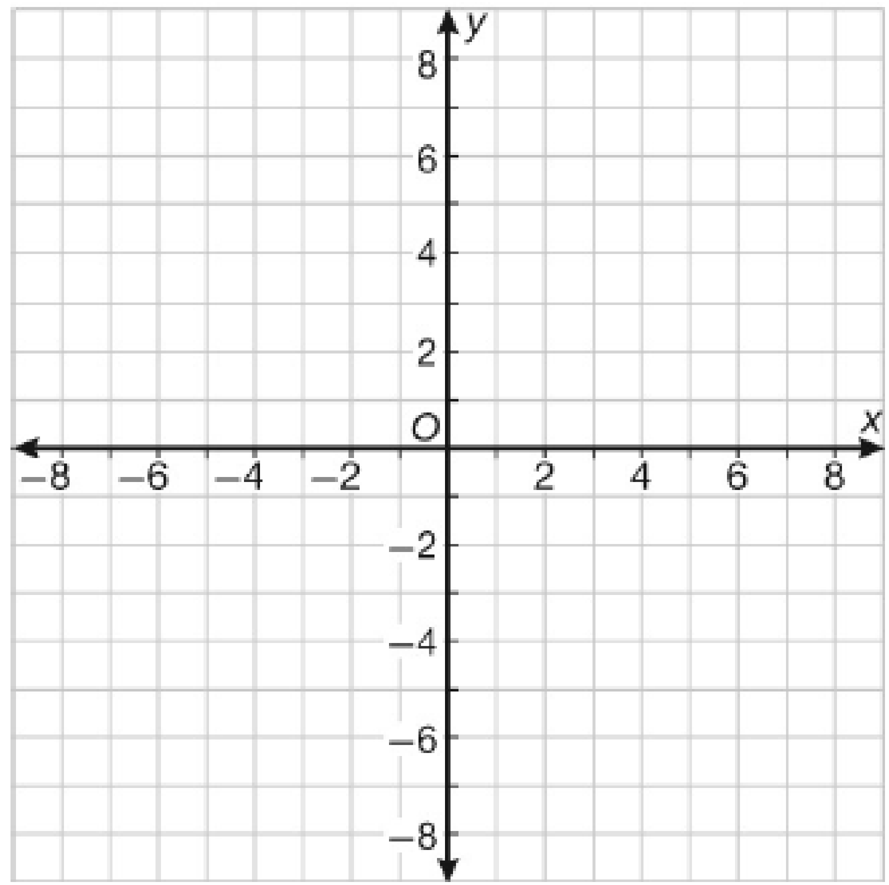
You Try!!

a. Find the slopes of lines that are parallel and perpendicular to line *v*.



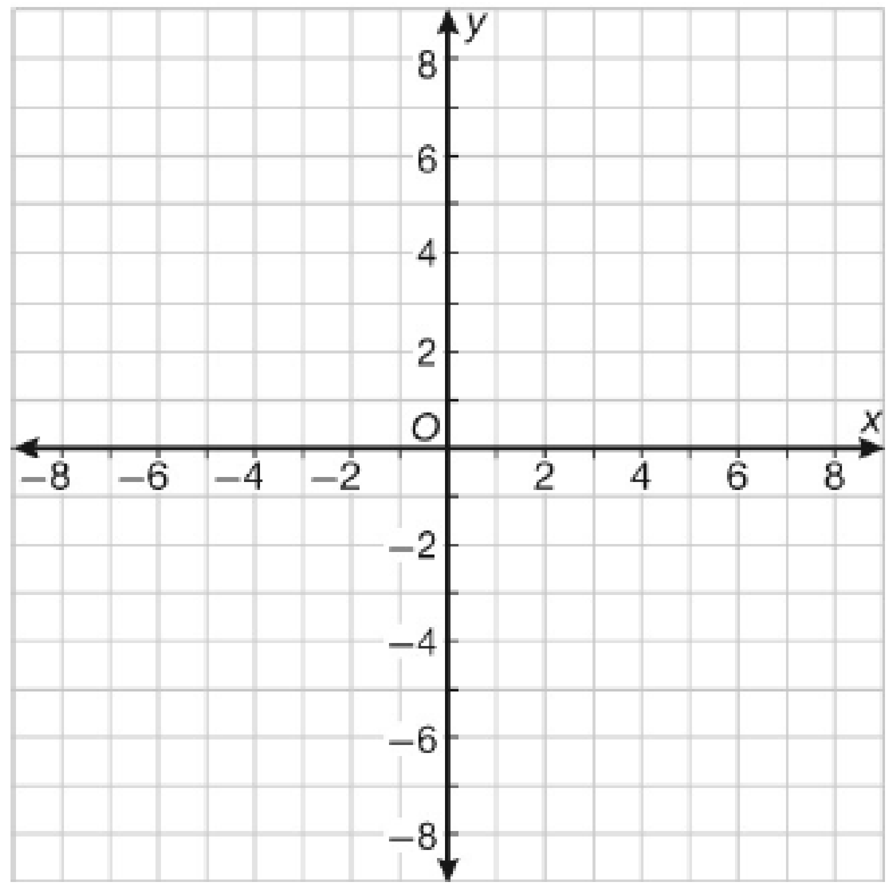
You Try!!

d. Find and graph a line that is parallel to $y = -2x + 7$ and passes through the origin.



You Try!!

e. Find and graph a line that is perpendicular to $y = -\frac{4}{3}x + 3$ and passes through the point (2, 3).



Assignment

Page 240

Lesson Practice (Ask Mr. Heintz)

Page 241

Practice 1–30 (Do the starred ones first)