Lesson 38

Perpendicular and Angle Bisector of Triangles

An angle bisector divides an angle into two congruent angles.

Incenter of the Triangle – The point of concurrency when all three angles of a triangle are bisected.

The incenter of the triangle is equidistant from all of the sides of the triangle.

Example 1 Finding an Incenter in the Coordinate Plane

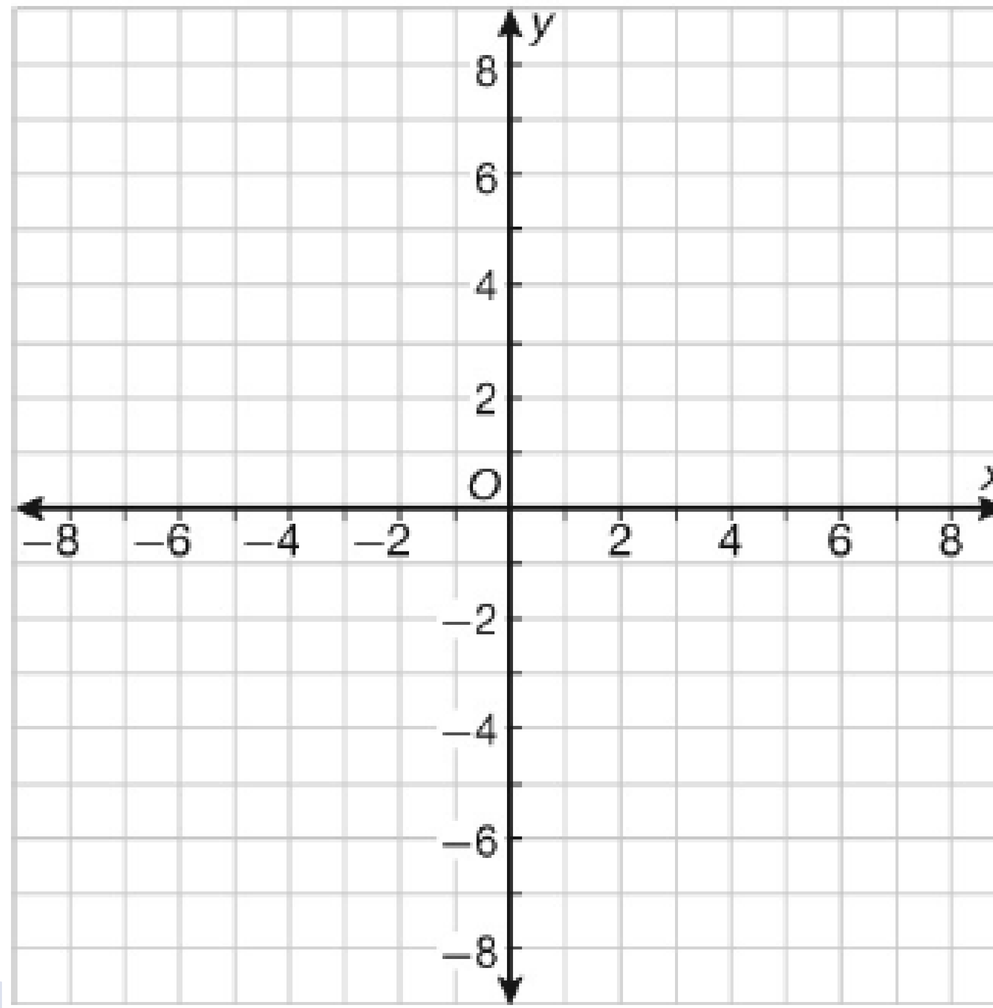
Use a compass and a straightedge to find the incenter of a triangle whose vertices are at (−2, 1), (1, 2), and (2, −2) in a coordinate plane.

SOLUTION

To find the incenter, bisect each of the triangle's angles using the methods described in Construction Lab 3. Once all three angle bisectors have been drawn, the central point where they intersect is the incenter.

Example 1 Finding an Incenter in the Coordinate Plane

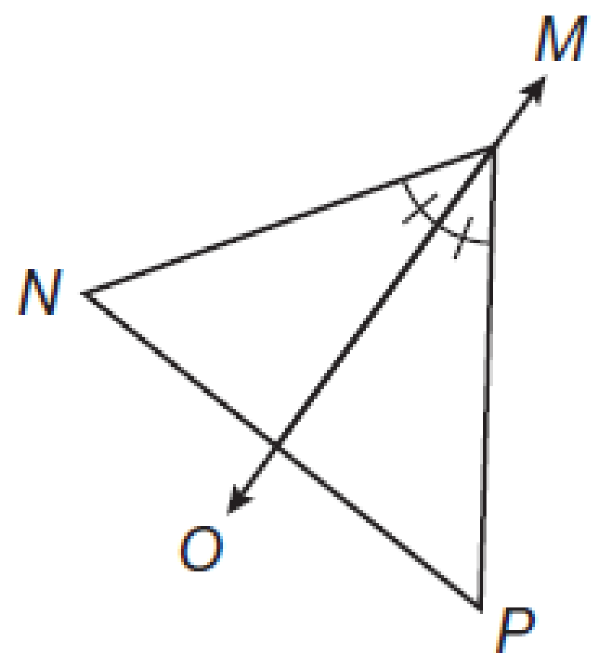
(–2, 1), (1, 2), and (2, –2)



In addition to finding the incenter of a triangle, angle bisectors can also be used to find the lengths of segments in the triangle. When an angle bisector intersects the side of a triangle, it makes a proportional relationship, given by Theorem 38-1.

Theorem 38–1: Triangle Angle Bisector Theorem – If a line bisects an angle of a triangle, then it divides the opposite side proportionally to the other two sides of the triangle.

In the diagram, $\frac{PM}{PO} = \frac{NM}{NO}$.



Example 2 Using the Triangle Angle Bisector Theorem

Using the diagram at the right, find *BC* if *AD* = 15, *DC* = 8, and *AB* = 20.

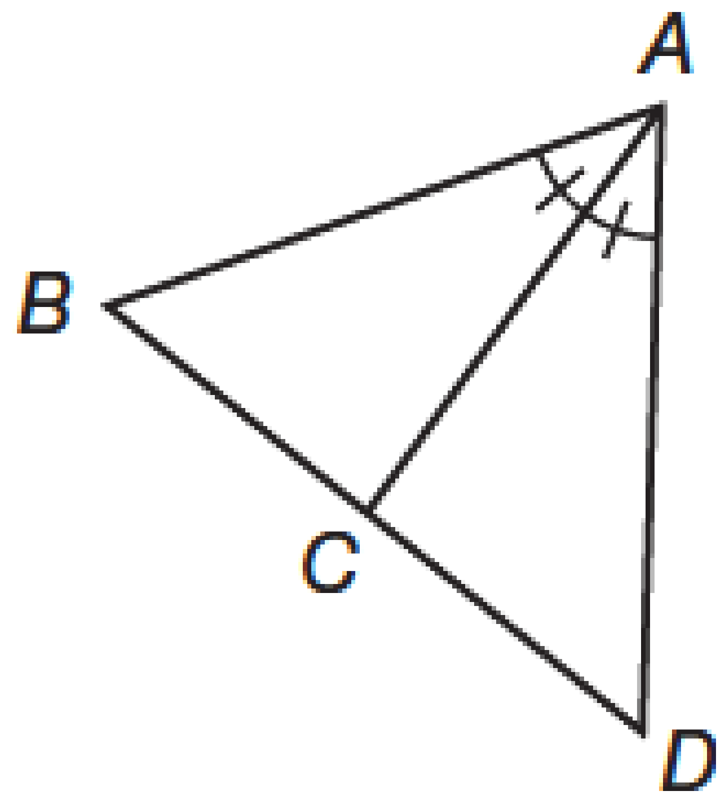
SOLUTION

$$\frac{AD}{DC} = \frac{AB}{BC}$$

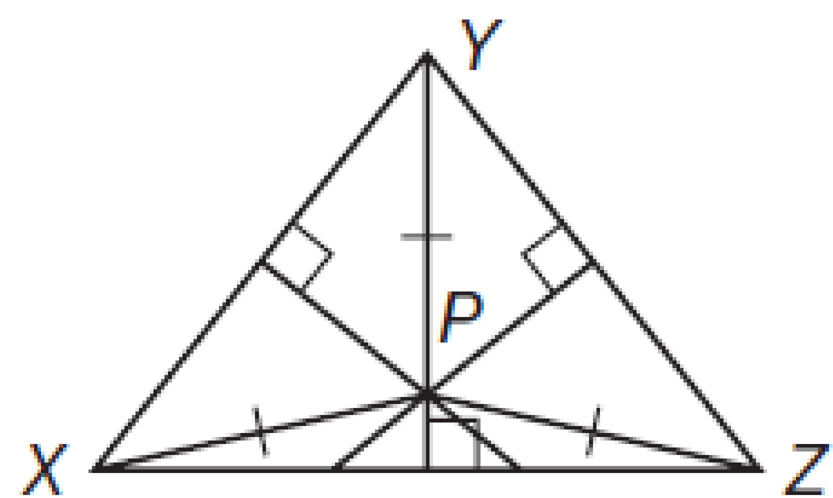
$$\frac{15}{8} = \frac{20}{BC}$$

$$15\text{BC} = 160$$

$$BC = 10.\overline{6}$$



Circumcenter of the Triangle – The point of concurrency when perpendicular bisectors are drawn for every side of a triangle. The circumcenter of a triangle is equidistant from every vertex in the triangle. In the diagram below, point P is the circumcenter of the triangle, so $PX = PY = PZ$.



The circumcenter is not always inside a triangle. A right triangle's circumcenter lies on the hypotenuse, and an obtuse triangle's circumcenter is outside the triangle.

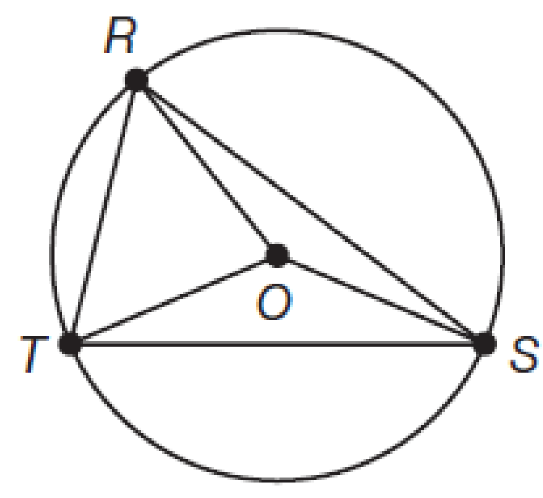
Math Language

The circumcenter, orthocenter, and centroid of a triangle will always be collinear. The line that all three points lie on is known as the **Euler Line**.

Circumscribed Circle – Any circle that contains all the vertices of a polygon.

The circumcenter lies at the center of the circle that contains the three vertices of the triangle.

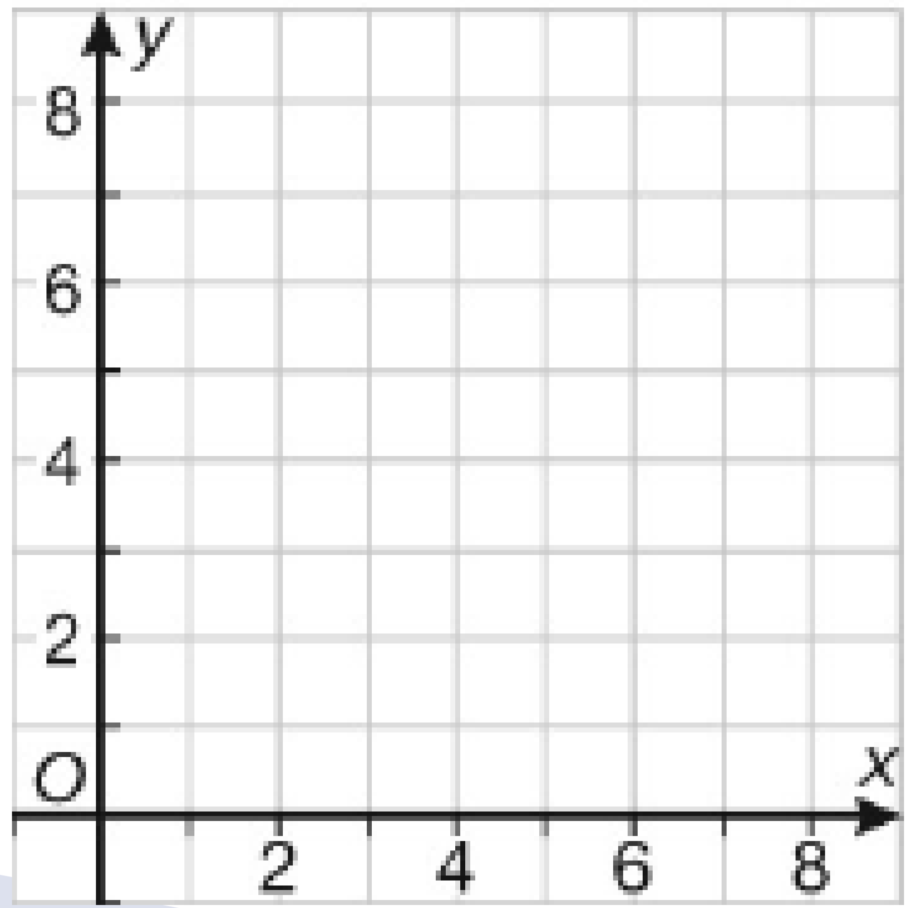
Inscribed Polygon – Any polygon with each vertex on a circle.



Example 3 Finding a Circumcenter in the Coordinate Plane

Find the circumcenter of a triangle with vertices at *A*(2, 2), *B*(8, 2), and *C*(4, 7).

SOLUTION



Example 3 Finding a Circumcenter in the Coordinate Plane

Find the circumcenter of a triangle with vertices at *A*(2, 2), *B*(8, 2), and *C*(4, 7).

SOLUTION

The perpendicular bisector of each segment needs to be found. The midpoint of $\overline{AB}$ is (5, 2) and the midpoint of $\overline{AC}$ is (3, 4.5). For each of these midpoints, a line needs to be found that is perpendicular to the segment on which it lies. Find the slope between the segment's two endpoints, and then use linear equations to find the line. $\overline{AB}$ lies on the line $y = 2$, and $\overline{AC}$ lies on $y = \frac{5}{2}x - 3$. Perpendicular lines can be found through the midpoints using the method learned in Lesson 37. The line perpendicular to $\overline{AB}$ through (5, 2) is $x = 5$. The line perpendicular to $\overline{AC}$ through (3, 4.5) is $y = -\frac{2}{5}x + 5.7$. To find the circumcenter, solve the system of equations.

$x = 5$

$y = -\frac{2}{5}x + 5.7$

$y = 3.7$

Finally, substitute this value of *y* into one of the equations above to find *x*. The coordinates of the orthocenter are $(5,3.1\overline{6})$.

Example 4 Application: City Planning

A gas company has three gas stations located at points $R(2, 8)$, $S(6, 2)$, and $T(2, 2)$, as shown. The storage facility is equidistant from the three gas stations. Find the location of the storage facility.

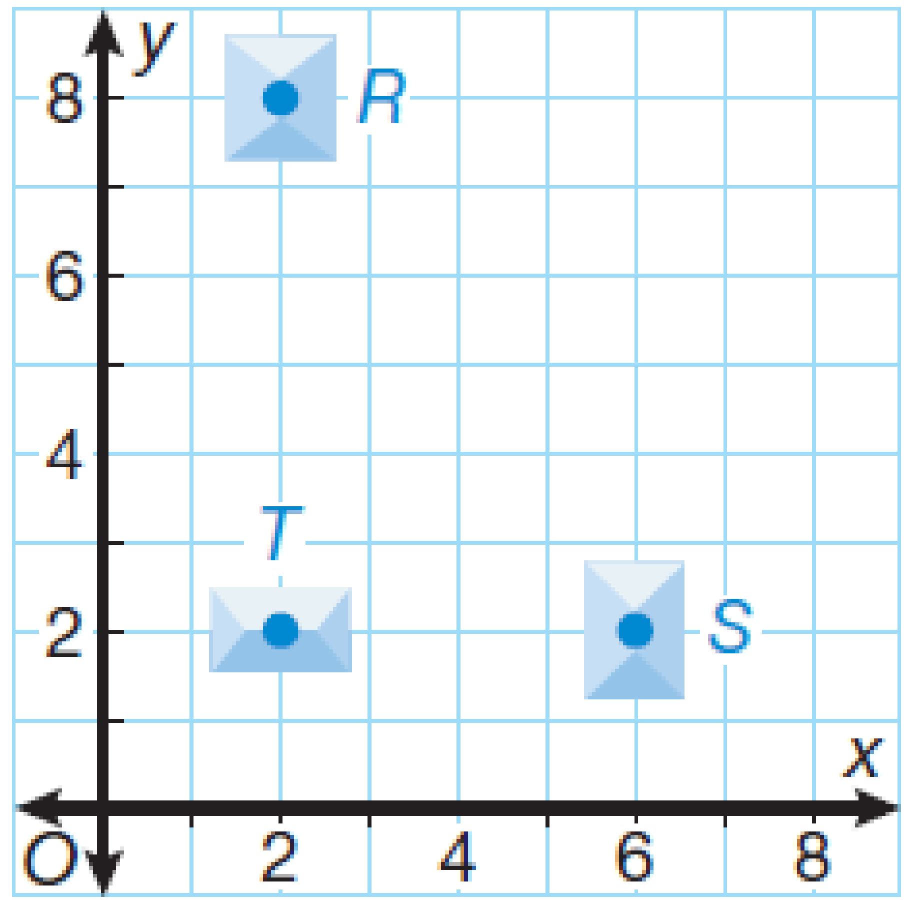
SOLUTION

Since the storage facility is equidistant from the gas stations, it is at the circumcenter of ΔRTS. By looking at the graph, you can see that the equation for $\overline{TS}$ is $y = 2$, and the equation for $\overline{RT}$ is $x = 2$. The midpoint of $\overline{TS}$ is (4, 2) and the midpoint of $\overline{RT}$ is (2, 5). The horizontal line $y = 5$ is perpendicular to $\overline{RT}$. The vertical line $x = 4$ is perpendicular to $\overline{TS}$. These two lines intersect on the hypotenuse at (4, 5), so the storage facility is located at (4, 5).

Example 4 Application: City Planning

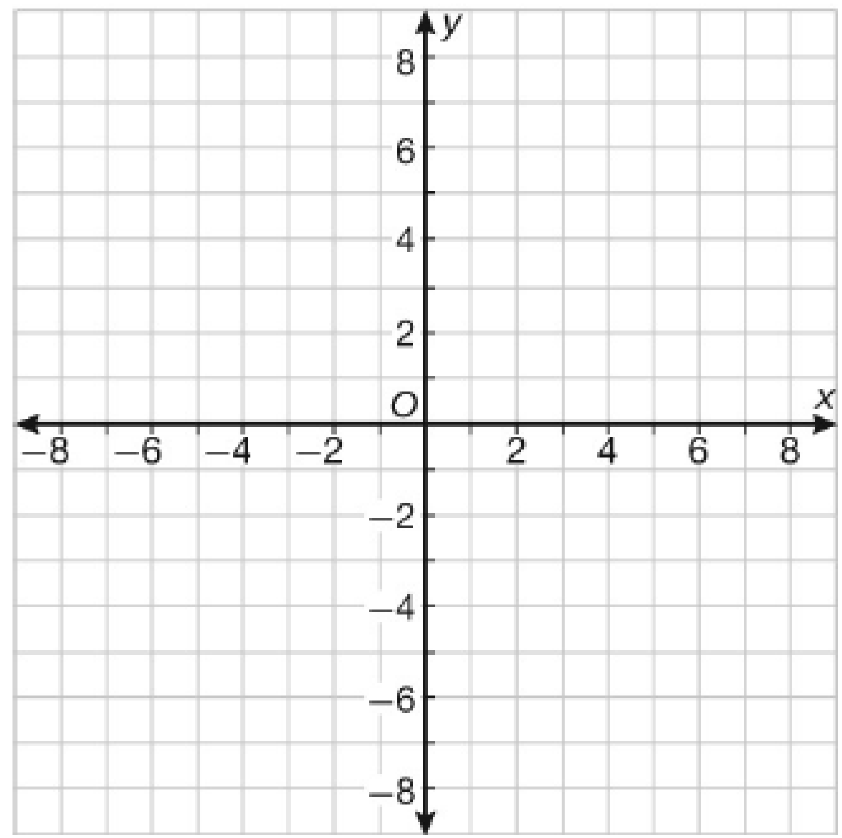
A gas company has three gas stations located at points $R(2, 8)$, $S(6, 2)$, and $T(2, 2)$, as shown.

SOLUTION



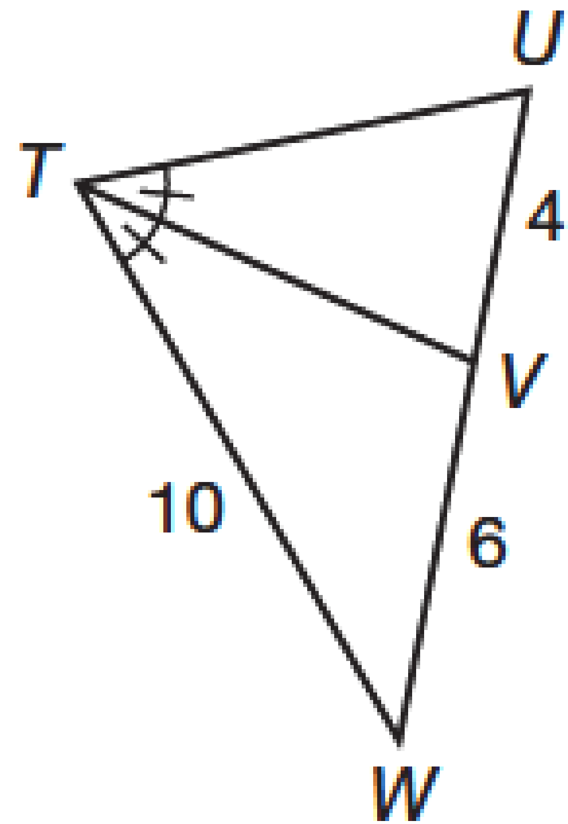
You Try!!!!!

a. Use a compass and a straightedge to find the incenter of a triangle whose vertices are at (–3, 1), (3, –2), and (2, 4) in a coordinate plane.



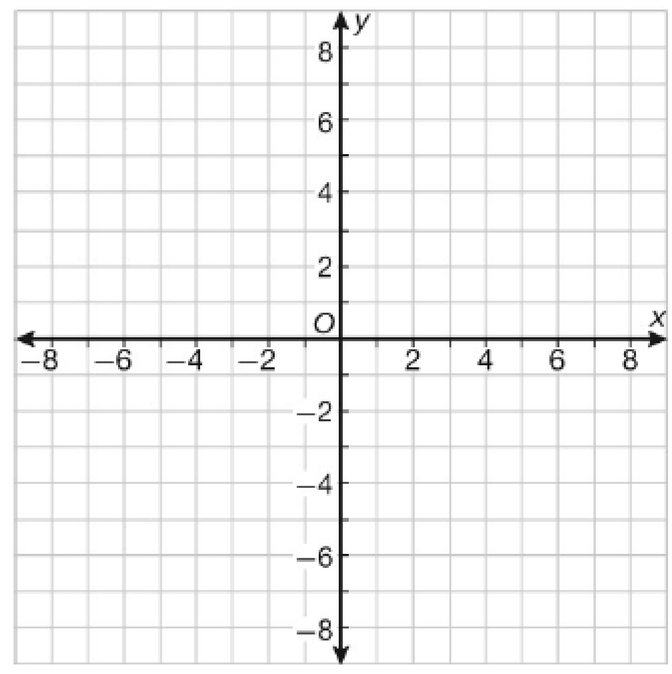
You Try!!!!!

Using the diagram at the right, find the length of $\overline{TU}$ *TU* if *UV* = 4, *TW* = 10, and *WV* = 6.



You Try!!!!!

d. A restaurant owner wants to place his new restaurant equidistant from three nearby grocery stores that will supply him. They are located at $A(0, 0)$, $B(4, 0)$ and $C(0, 6)$. Where should he place his restaurant?



Assignment

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Lesson Practice (Ask Mr. Heintz)

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Practice 1–30 (Do the starred ones first)