

## Geometry Lesson 39

Objective: TSW find inequalities in a triangle.

**Theorem 39-1 - If one side of a triangle is longer than another side, then the angle opposite the first side is larger than the angle opposite the second side.**

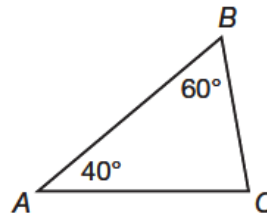
**Theorem 39-2 - If one angle of a triangle is larger than another angle, then the side opposite the first angle is longer than the side opposite the second angle.**

In other words, a triangle's largest side is always opposite its largest angle, and its smallest side is always opposite its smallest angle.

Example 1 Ordering Triangle Side Lengths and Angle Measures

a. Order the side lengths in  $\triangle ABC$  from least to greatest.

SOLUTION

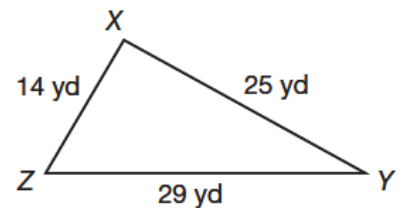


### Math Reasoning

**Predict** Using Theorems 39-1 and 39-2, what can you say about the angle measures of an isosceles triangle?... of an equilateral triangle?

b. Order the measures of the angles in  $\triangle XYZ$  from least to greatest.

SOLUTION



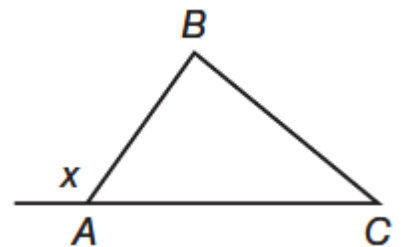
Recall from Lesson 18 that the measure of the exterior angle of a triangle is equal to the sum of the two remote interior angles. This result leads to the Exterior Angle Inequality Theorem.

**Theorem 39-3: Exterior Angle Inequality Theorem - The measure of an exterior angle is greater than the measure of either remote interior angle.**

Example 2 Proving The External Angle Inequality Theorem

In the given triangle, the exterior angle is labeled as  $x$ . Prove that  $x$  is greater than the measures of  $\angle B$  or  $\angle C$ .

SOLUTION



It is not true that any three line segments can make a triangle. Only line segments of certain lengths can form the three sides needed for a triangle. The requirements are given in the Triangle Inequality Theorem.

**Theorem 39-4: Triangle Inequality Theorem - The sum of the lengths of any two sides of a triangle must be greater than the length of the third side.**

For example, a triangle could not have side lengths of 3, 5, and 9 because the sum of 3 and 5 is less than 9.

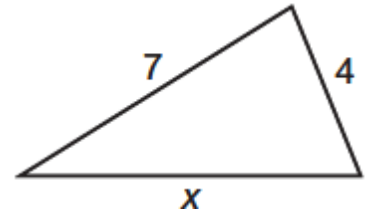
Example 3 Applying the Triangle Inequality Theorem

- a. Decide if each set of side lengths could form a valid triangle: (3, 4, 5), (5, 11, 6), and (1, 9, 5).

SOLUTION

- b. Find the range of values for  $x$  in the given triangle.

SOLUTION



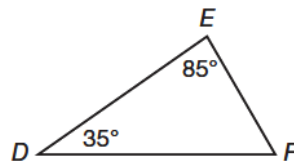
Example 4 Application: Planning a Trip

Simone took a flight from Atlanta to London (a distance of 4281 miles), then flew from London to New York City (a distance of 3470 miles), and then took a flight back to Atlanta. Assuming that all three trips are straight lines, determine the range of distances (from least to greatest) she could have traveled altogether.

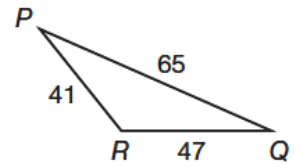
SOLUTION

You Try!!!!!!

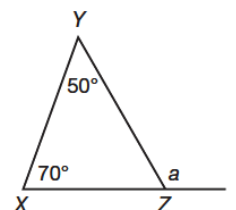
- a. Order the side lengths in  $\triangle DEF$  from least to greatest.



- b. Order the measures of the angles in  $\triangle PQR$  from least to greatest.



- c. Show that in the triangle, the measure of the exterior angle at vertex  $Z$  is greater than the angle measure at vertex  $X$  or at vertex  $Y$ .



- d. Find the range of values for  $x$  in the given triangle.

