Lesson 39

Inequalities in a Triangle

Theorem 39-1 - If one side of a triangle is longer than another side, then the angle opposite the first side is larger than the angle opposite the second side.

Theorem 39-2 - If one angle of a triangle is larger than another angle, then the side opposite the first angle is longer than the side opposite the second angle.

In other words, a triangle's largest side is always opposite its largest angle, and it smallest side is always opposite its smallest angle.

Math Reasoning

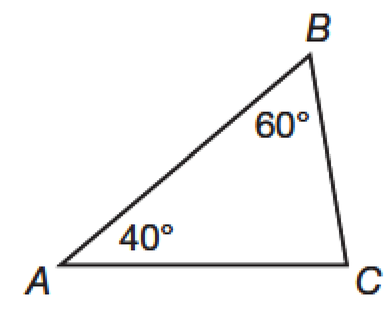
Predict Using Theorems 39-1 and 39-2, what can you say about the angle measures of an isosceles triangle?… of an equilateral triangle?

Example 1 Ordering Triangle Side Lengths and Angle Measures

a. Order the side lengths in ΔABC from least to greatest.

SOLUTION

The Triangle Angle Sum Theorem shows that the missing angle is 80°. Therefore, the side with the greatest length is $\overline{AB}$ because it is opposite the largest angle. The shortest side is $\overline{BC}$, as it is opposite the smallest angle. The final order of sides, from least to greatest length, is $\overline{BC}, \overline{AC}, \overline{AB}$.

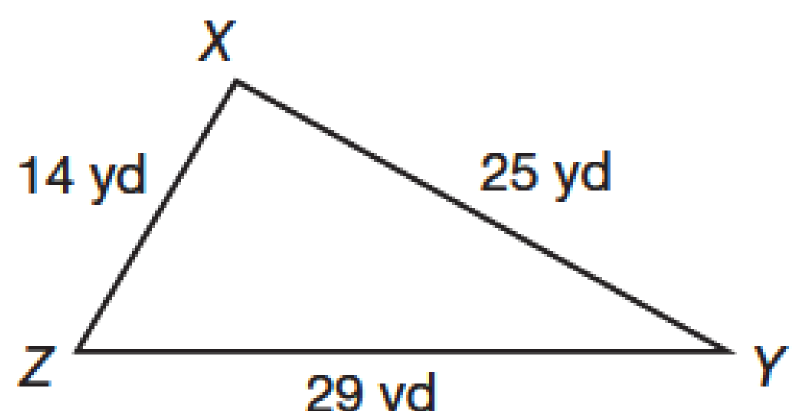


Example 1 Ordering Triangle Side Lengths and Angle Measures

b. Order the measures of the angles in ΔXYZ from least to greatest.

SOLUTION

The shortest side of the triangle is $\overline{XZ}$, therefore the measure of the opposite angle, $\angle Y$, is the least of the three angles. The longest side is $\overline{YZ}$, so it is opposite the angle with the greatest measure, $\angle X$. Therefore the order of angles is $\angle Y$, $\angle Z$, $\angle X$.



Recall from Lesson 18 that the measure of the exterior angle of a triangle is equal to the sum of the two remote interior angles. This result leads to the Exterior Angle Inequality Theorem.

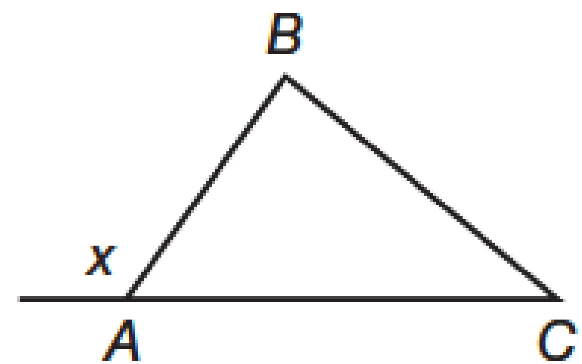
Theorem 39-3: Exterior Angle Inequality Theorem – The measure of an exterior angle is greater than the measure of either remote interior angle.

Example 2 Proving The External Angle Inequality Theorem

In the given triangle, the exterior angle is labeled as x. Prove that x is greater than the measures of $\angle B$ or $\angle C$.

SOLUTION

By the Exterior Angle Theorem, we know that m$\angle B$ + m$\angle C = x$. We also know, from the definition of an angle, that both $\angle B$ and $\angle C$ have a measure greater than 0°. Rearranging the Exterior Angle Theorem, m$\angle B = x$ – m$\angle C$. Since m$\angle C$ is greater than 0, m$\angle B$ must be less than x.



It is not true that any three line segments can make a triangle. Only line segments of certain lengths can form the three sides needed for a triangle. The requirements are given in the Triangle Inequality Theorem.

Theorem 39-4: Triangle Inequality Theorem - The sum of the lengths of any two sides of a triangle must be greater than the length of the third side.

For example, a triangle could not have side lengths of 3, 5, and 9 because the sum of 3 and 5 is less than 9.

Example 3 Applying the Triangle Inequality Theorem

a. Decide if each set of side lengths could form a valid triangle: (3, 4, 5), (5, 11, 6), and (1, 9, 5).

SOLUTION

For the first set, no combination of two sides can be found that will sum to less than the length of the other side, so since $3 + 4 = 7$, and $7 > 5$, these side lengths could represent a triangle.

For the second set, the sum of the two short sides, 5 and 6, equals exactly the length of the third side, 11. Since the sum is not greater than the third side, but only equal to it, this set cannot represent a triangle.

For the third set, the sum of the two short sides, 1 and 5, sum to less than the third side, 9, so this set also cannot represent a triangle.

Example 3 Applying the Triangle Inequality Theorem

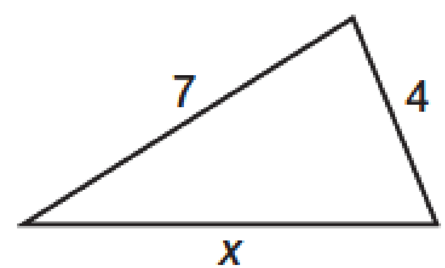
b. Find the range of values for x in the given triangle.

SOLUTION

Using the Triangle Inequality Theorem, the following three statements are true:

$x + 4 > 7$	$x + 7 > 4$	$7 + 4 > x$
$x > 3$	$x > -3$	$11 > x$

The second inequality is invalid, since a side length cannot be negative. Combining the two valid statements, we obtain the solution $3 < x < 11$.



Example 4 Application: Planning a Trip

Simone took a flight from Atlanta to London (a distance of 4281 miles), then flew from London to New York City (a distance of 3470 miles), and then took a flight back to Atlanta. Assuming that all three trips are straight lines, determine the range of distances (from least to greatest) she could have traveled altogether.

SOLUTION

Let x represent the distance Simone traveled back to Atlanta from New York City. Using the Triangle Inequality Theorem, the following three statements are true:

$$x + 3470 > 4281 \qquad x + 4281 > 3470 \qquad 3470 + 4281 > x$$

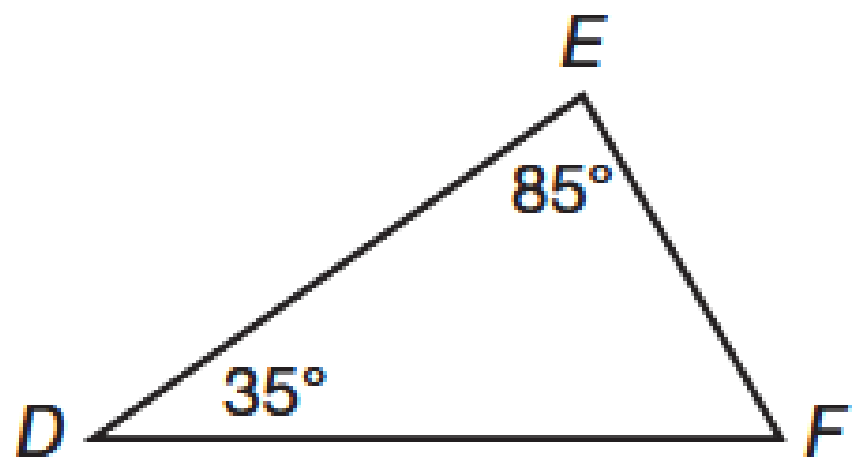
$$x > 811 \qquad x > -811 \qquad 7751 > x$$

Combining the two valid statements, $811 < x < 7751$.

Therefore, the shortest distance Simone could have traveled is $3470 + 4281 + 811 = 8562$ miles. The farthest she could have traveled is $3470 + 4281 + 7751 = 15{,}502$ miles.

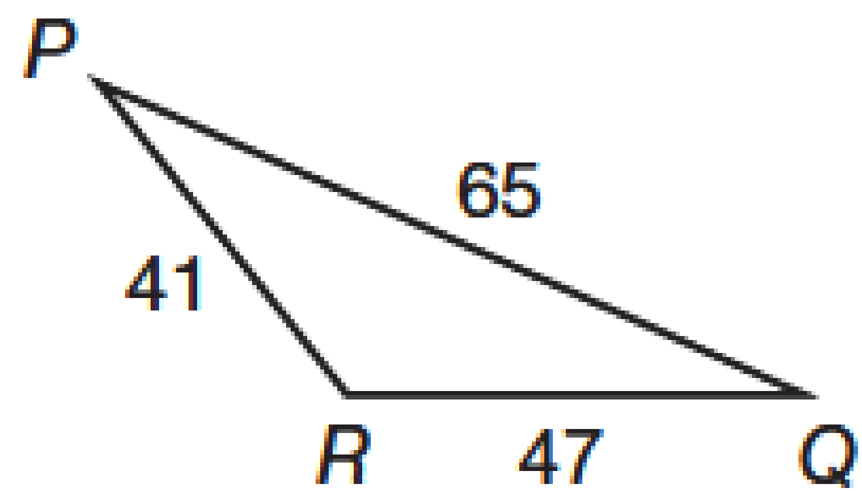
You Try!!!!!!

a. Order the side lengths in ΔDEF from least to greatest.



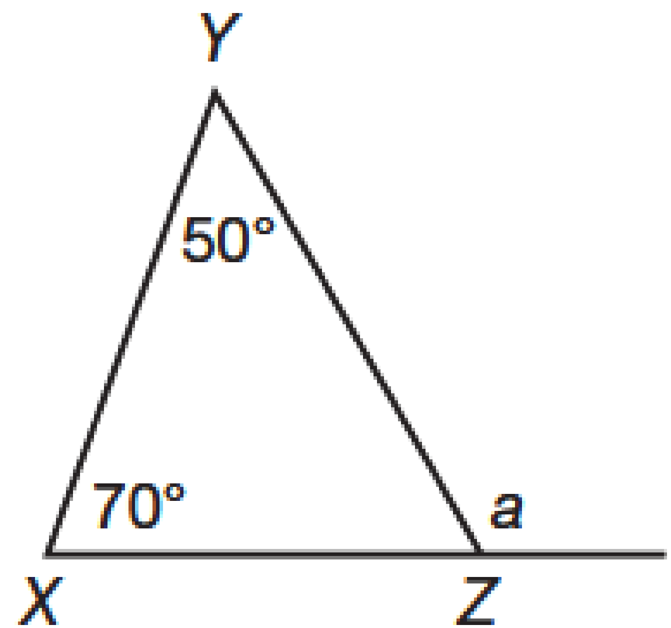
You Try!!!!!!

b. Order the measures of the angles in ΔPQR from least to greatest.



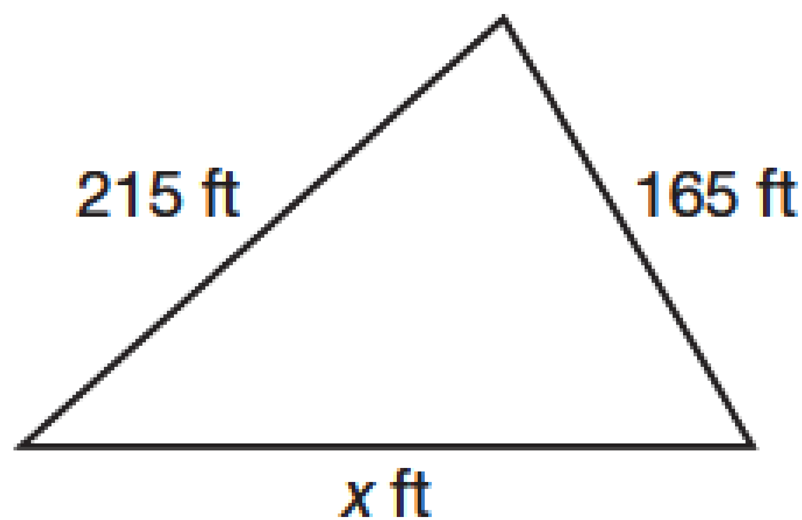
You Try!!!!!!

c. Show that in the triangle, the measure of the exterior angle at vertex *Z* is greater than the angle measure at vertex *X* or at vertex *Y*.



You Try!!!!!!

d. Find the range of values for *x* in the given triangle.



Assignment

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Lesson Practice (Ask Mr. Heintz)

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Practice 1–30 (Do the starred ones first)