

Geometry Lesson 41

Objective: TSW use ratios and proportions in similar shapes.

A ratio is a comparison of two values by division. The ratio of two quantities, a and b , can be written in three ways: a to b , $a:b$, or $\frac{a}{b}$ (where $b \neq 0$). A statement that two ratios are equal is called a proportion.

Example 1 Writing Ratios and Proportions

Consider $\triangle MNO$ and $\triangle PQR$.

a. Write a ratio comparing the lengths of segments \overline{MN} to \overline{NO} to \overline{OM} .

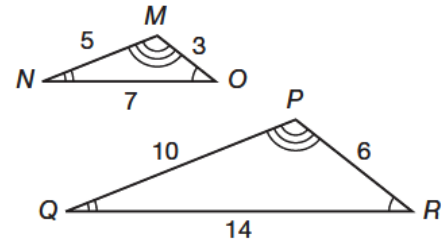
SOLUTION

b. Write a ratio comparing MN to PQ in three ways.

SOLUTION

c. Write a proportion to show that $MN:PQ = NO:QR$

SOLUTION



Math Reasoning

Write Explain the difference between a ratio and a proportion.

FYI: In the proportion $\frac{a}{b} = \frac{c}{d}$, a and d are the extremes, and b and c are the means. One way to solve a proportion to find a missing value is to use cross products. The cross product is the product of the means and the product of the extremes. In other words, if $\frac{a}{b} = \frac{c}{d}$, then $ad = bc$.

Example 2 Solving Proportions with Cross Products

Solve the proportion $\frac{3}{15} = \frac{x}{50}$ to find the value of x .

SOLUTION

Reading Math

The symbol \sim shows that two figures are similar, and should be read "is similar to."

Similar - Two figures that have the same shape, but not necessarily the same size.

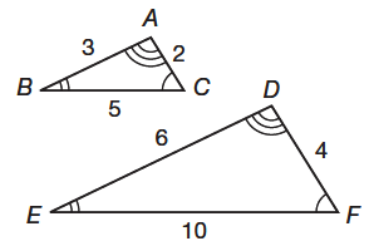
In the diagram, $\triangle ABC$ is similar to $\triangle DEF$. All congruent figures are also similar figures, but the converse is not always true.

In similar polygons, the corresponding angles are congruent and the corresponding sides are proportional. In the diagram, $\triangle ABC$ and $\triangle DEF$ have congruent angles, and each pair of their corresponding sides has the same proportional relationship. A similarity ratio is the ratio of two corresponding linear measurements in a pair of similar figures. The following similarity ratios can be written for $\triangle ABC$ and $\triangle DEF$.

$$\frac{DE}{AB} = \dots =$$

$$\frac{EF}{BC} = \dots =$$

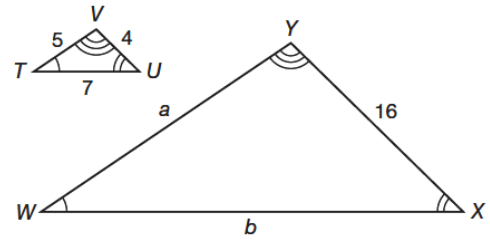
$$\frac{FD}{CA} = \dots =$$



Like congruence, similarity is a transitive relation. The Transitive Property of Similarity states that if $a \sim b$, and $b \sim c$, then $a \sim c$.

Example 3 Using Proportion to Find Missing Lengths
Find the unknown side lengths in the two similar triangles.

SOLUTION



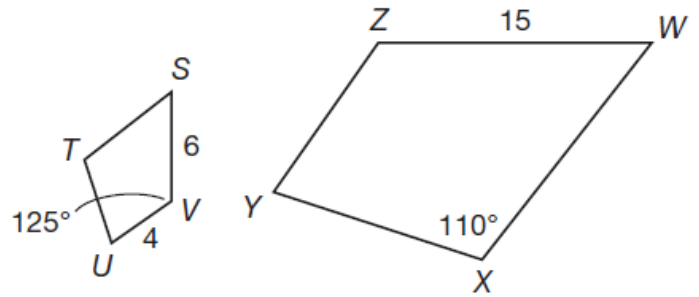
A similarity statement is a statement indicating that two polygons are similar by listing their vertices in order of correspondence. Much like writing a congruence statement, corresponding angles have to be named in the same order.

Example 4 Finding Missing Measures of Similar Polygons

In the diagram, $STUV \sim WXYZ$.

a. What are the measures of $\angle T$ and $\angle Z$?

SOLUTION



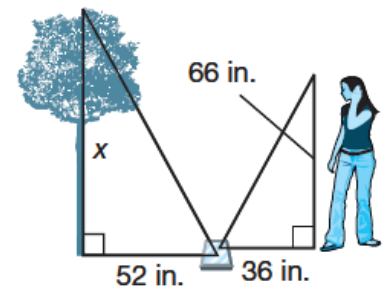
b. What is the length of \overline{YZ} ?

SOLUTION

Example 5 Application: Optics

Siobhan is using a mirror and similar triangles to determine the height of a small tree. She places the mirror at a distance where she can see the top of the tree in the mirror. According to the measures in Siobhan's triangles, what is the height of the tree to the nearest inch?

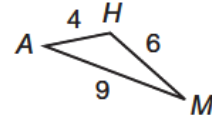
SOLUTION



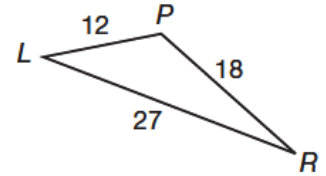
You Try!!!!

Use the two similar triangles to answer a through a through c.

a. Write a ratio comparing the lengths of segments \overline{HA} to \overline{AM} to \overline{MH} .



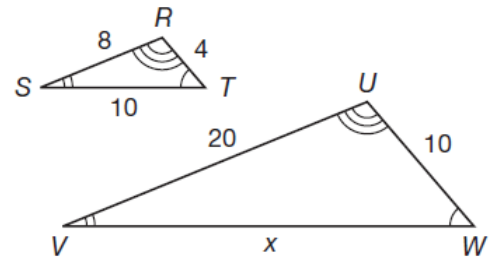
b. Write a ratio comparing AM to LR in three ways, in simplest form.



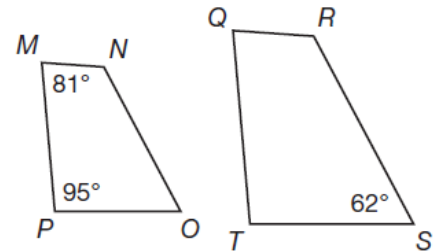
c. Write a proportion to show that $HM:PR = AM:LR$.

d. What is the value of x in the proportion $\frac{8}{7} = \frac{x}{21}$?

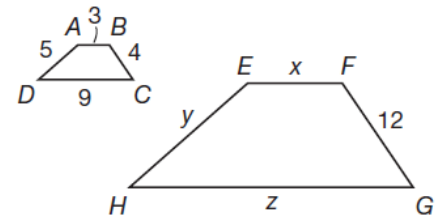
e. If $\triangle RST \sim \triangle UVW$, find the missing length in $\triangle UVW$.



f. If the polygons $MNOP$ and $QRST$ are similar, what are the measures of $\angle O$ and $\angle R$?



g. If the polygons $ABCD$ and $EFGH$ are similar, what are the values of x , y , and z ?



h. Cree uses a 21-foot ladder and a 12-foot ladder while painting the exterior of a house. Each ladder forms the same angle with the ground. If the longer ladder reaches 18 feet up the wall, how high does the other ladder reach, to the nearest foot?