Lesson 41

Ratios, Proportions, and Similarity

A ratio is a comparison of two values by division. The ratio of two quantities, *a* and *b*, can be written in three ways: *a* to *b*, $a:b$, or $\frac{a}{b}$ (where $b \neq 0$). A statement that two ratios are equal is called a proportion.

Example 1 Writing Ratios and Proportions

Consider ΔMNO and ΔPQR.

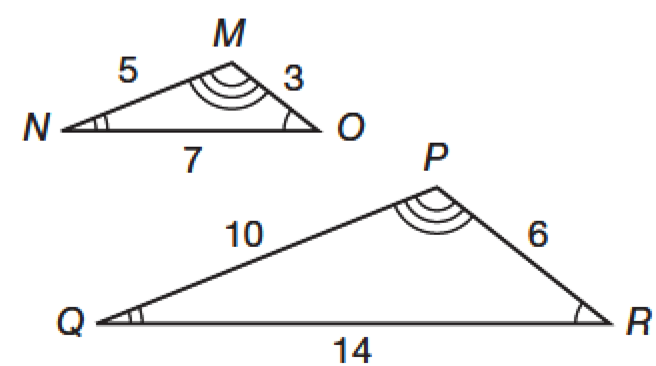
a. Write a ratio comparing the lengths of segments $\overline{MN}$ to $\overline{NO}$ to $\overline{OM}$.

SOLUTION

The quantity that is mentioned first in a ratio is written first.

$MN = 5$, $NO = 7$, $OM = 3$.

Therefore $MN{:}NO{:}OM = 5{:}7{:}3$.



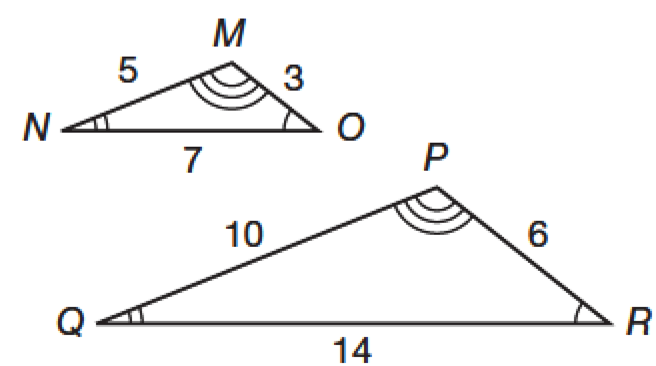
Example 1 Writing Ratios and Proportions

Consider ΔMNO and ΔPQR.

b. Write a ratio comparing *MN* to *PQ* in three ways.

SOLUTION

$MN = 5$ and $PQ = 10$. The ratio of *MN* to *PQ* can be written as 5 to 10, 5:10, or $\frac{5}{10}$. Ratios can be reduced just like fractions. Reducing this ratio results in $\frac{1}{2}$.



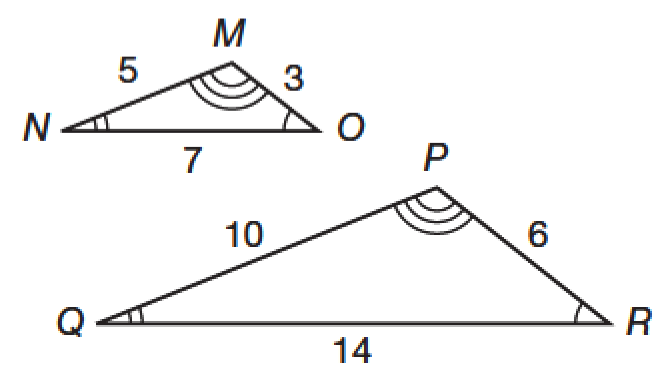
Example 1 Writing Ratios and Proportions

c. Write a proportion to show that $MN:PQ = NO$
$:QR$

SOLUTION

$MN:PQ = NO:QR$ can be written as $\frac{5}{10} = \frac{7}{14}$.

Notice that the proportion is true since both sides of the proportion reduce to $\frac{1}{2}$.



FYI: In the proportion $\frac{a}{b} = \frac{c}{d}$, a and d are the extremes, and b and c are the means. One way to solve a proportion to find a missing value is to use cross products. The cross product is the product of the means and the product of the extremes. In other words, if $\frac{a}{b} = \frac{c}{d}$, then $ad = bc$.

Example 2 Solving Proportions with Cross Products

Solve the proportion $\frac{3}{15} = \frac{x}{50}$ to find the value of *x*.

SOLUTION

Find the cross products to solve the proportion.

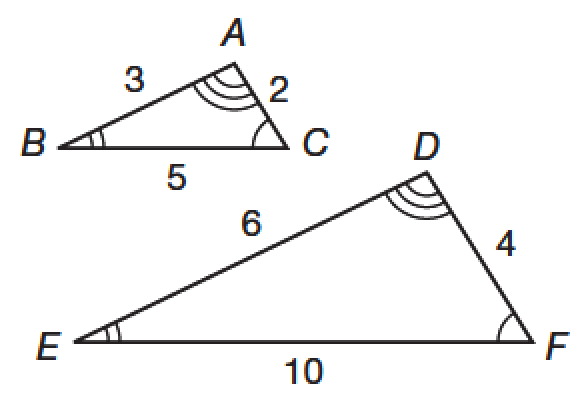
$$\frac{3}{15} = \frac{x}{50}$$

$$3 \cdot 50 = 15x$$

$$x = 10$$

Similar – Two figures that have the same shape, but not necessarily the same size.

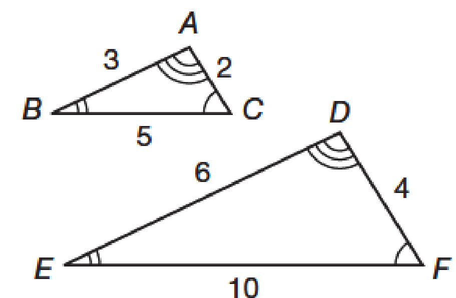
In the diagram, ΔABC is similar to ΔDEF. All congruent figures are also similar figures, but the converse is not always true.



In similar polygons, the corresponding angles are congruent and the corresponding sides are proportional. In the diagram, ΔABC and ΔDEF have congruent angles, and each pair of their corresponding sides has the same proportional relationship. A similarity ratio is the ratio of two corresponding linear measurements in a pair of similar figures. The following similarity ratios can be written for ΔABC and ΔDEF.

$$\frac{DE}{AB} = \frac{6}{3} = 2 \qquad \frac{EF}{BC} = \frac{10}{5} = 2 \qquad \frac{FD}{CA} = \frac{4}{2} = 2$$

Like congruence, similarity is a transitive relation. The Transitive Property of Similarity states that if $a \sim b$, and $b \sim c$, then $a \sim c$.



Example 3 Using Proportion to Find Missing Lengths

Find the unknown side lengths in the two similar triangles.

SOLUTION

The triangles are similar so corresponding sides are proportional:

$$\frac{VT}{YW} = \frac{TU}{WX} = \frac{UV}{XY}$$

$$\frac{5}{a} = \frac{7}{b} = \frac{4}{16}$$

Solve each proportion using a known ratio and a ratio with an unknown.

$\frac{5}{a} = \frac{4}{16}$

$4a = 5 \cdot 16$

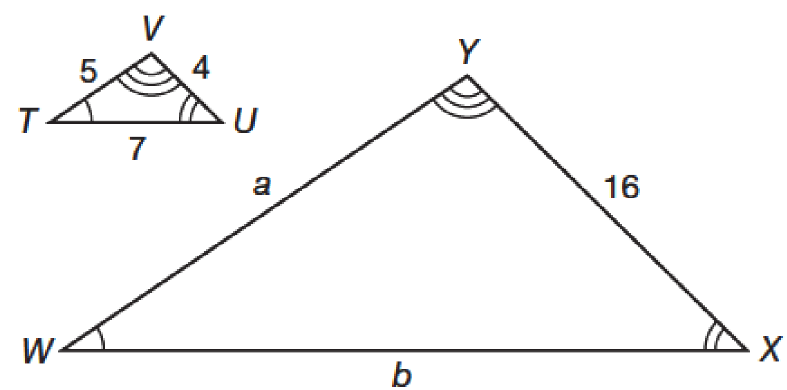
$a = 20$

$\frac{7}{b} = \frac{4}{16}$

$4b = 7 \cdot 16$

$b = 28$

Therefore, $WX = 28$ and $YW = 20$.



A similarity statement is a statement indicating that two polygons are similar by listing their vertices in order of correspondence. Much like writing a congruence statement, corresponding angles have to be named in the same order.

Example 4 Finding Missing Measures of Similar Polygons

In the diagram, $STUV \sim WXYZ$.

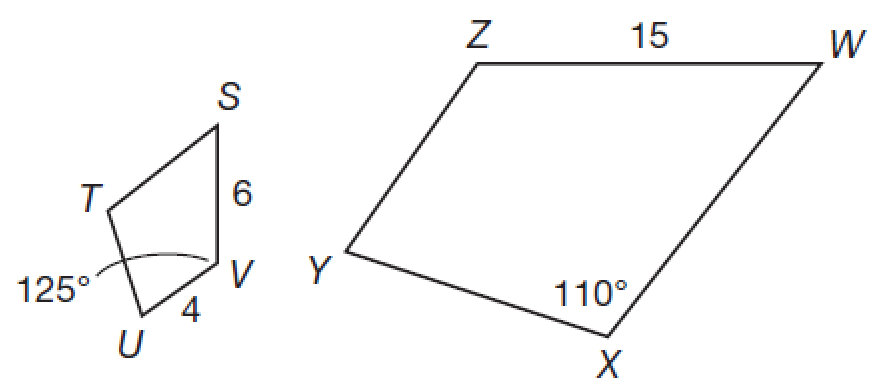
a. What are the measures of $\angle T$ amd $\angle Z$?

SOLUTION

The quadrilaterals are named in order of corresponding vertices, and corresponding angles of similar figures are congruent. Therefore,

$\angle S \cong \angle W, \angle T \cong \angle X, \angle U \cong \angle Y,\ and\ \angle V \cong \angle Z$.

m$\angle T =$ m$\angle X = 110°$, and m$\angle Z =$ m$\angle V = 125°$.



Example 4 Finding Missing Measures of Similar Polygons

b. What is the length of $\overline{YZ}$?

SOLUTION

In the quadrilaterals, $\overline{SV}$ and $\overline{WZ}$ are corresponding sides and $\overline{UV}$ and $\overline{YZ}$ are corresponding sides. Therefore,

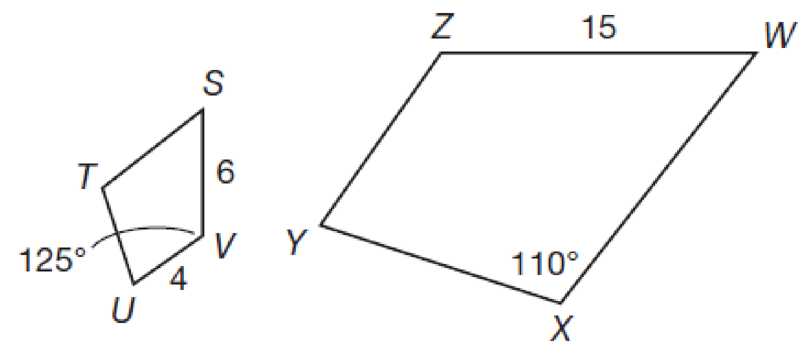
$$\frac{SV}{WZ} = \frac{UV}{YZ}$$

$$\frac{6}{15} = \frac{4}{YZ}$$

$$6 \cdot YZ = 15 \cdot 4$$

$$YZ = 10$$

The length of $\overline{YZ}$ is 10 units.



Example 5 Application: Optics

Siobhan is using a mirror and similar triangles to determine the height of a small tree. She places the mirror at a distance where she can see the top of the tree in the mirror. According to the measures in Siobhan's triangles, what is the height of the tree to the nearest inch?

SOLUTION

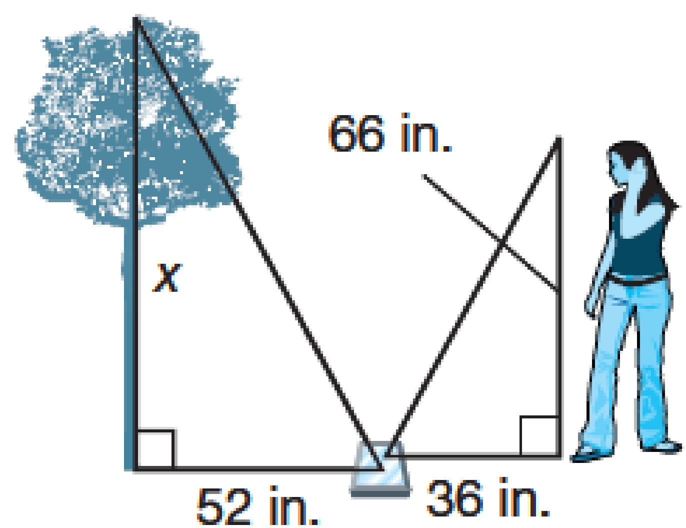
The two triangles are similar, so corresponding sides are proportional.

$$\frac{x}{66} = \frac{52}{36}$$

$$36x = 66 \cdot 52$$

$$x \approx 95$$

The tree is about 95 inches tall.



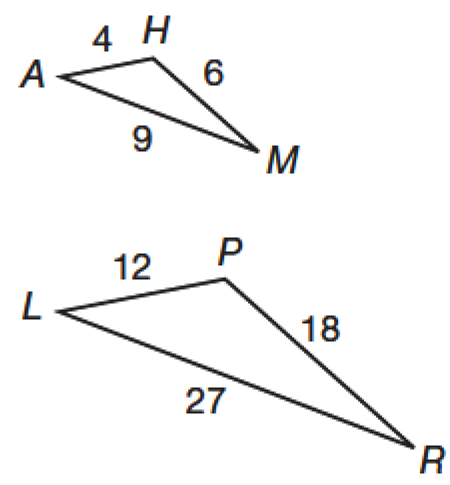
You Try!!!!!

Use the two similar triangles to answer *a* through a through *c*.

a. Write a ratio comparing the lengths of segments $\overline{HA}$ to $\overline{AM}$ to $\overline{MH}$.

b. Write a ratio comparing *AM* to *LR* in three ways, in simplest form.

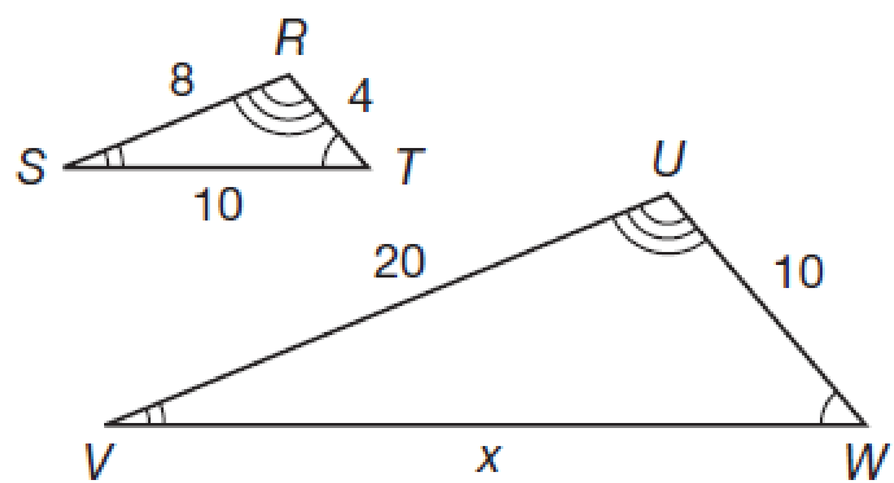
c. Write a proportion to show that $HM{:}PR = AM{:}LR$.



You Try!!!!!

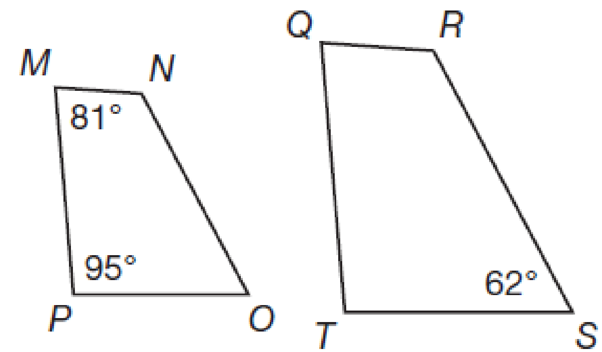
d. What is the value of x in the proportion $\frac{8}{7} = \frac{x}{21}$?

e. If $\Delta RST \sim \Delta UVW$, find the missing length in ΔUVW.

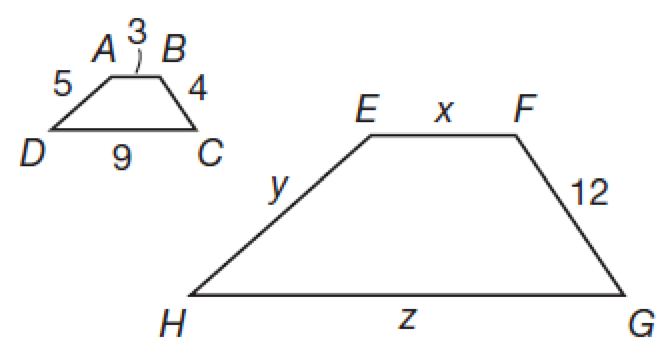


You Try!!!!!

f. If the polygons *MNOP* and *QRST* are similar, what are the measures of $\angle O$ and $\angle R$?



g. If the polygons *ABCD* and *EFGH* are similar, what are the values of *x*, *y*, and *z*?



You Try!!!!!

Cree uses a 21-foot ladder and a 12-foot ladder while painting the exterior of a house. Each ladder forms the same angle with the ground. If the longer ladder reaches 18 feet up the wall, how high does the other ladder reach, to the nearest foot?

Assignment

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Lesson Practice (Ask Mr. Heintz)

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Practice 1–30 (Do the starred ones first)