Name: ______________________________

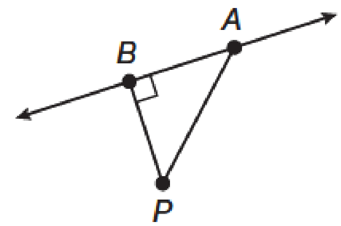
Date: ______________________________

Period: ____________________________

Geometry Lesson 42

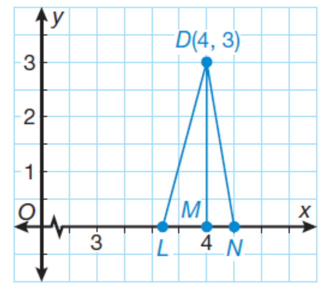
Objective: TSW find the distance from a point to a line.

Given a line $\overleftrightarrow{AB}$ and a point *P*, what is the shortest distance between *P* and $\overleftrightarrow{AB}$? Notice that ΔABP is a _______________ triangle, and $\overline{AP}$ is the hypotenuse. The hypotenuse is always the longest side of a right triangle, so $\overline{AP}$ must be _________________ than $\overline{PB}$.



Example 1 Choosing the Closest Point

Which point on the line *y* = 0 is closest to point *D*—*L*(3.6, 0), *M*(4, 0), or *N*(4.25, 0)?



Theorem 42-1 - Through a line and a point not on the line, there exists exactly one perpendicular line to the given line.

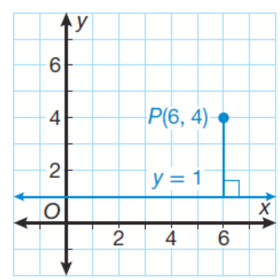
Theorem 42-2 - The perpendicular segment from a point to a line is the shortest segment from the point to the line.

Theorem 42-1 indicates that there is only one such segment. The length of a perpendicular segment from a point to a line is referred to as the distance from a point to a line.

Example 2 Finding Distance to a Line

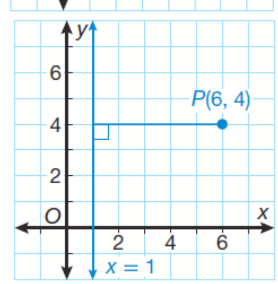
a. Find the distance from *P*(6, 4) to the line *y* = 1.

SOLUTION



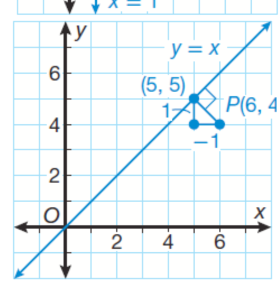
b. Find the distance from *P*(6, 4) to the line *x* = 1.

SOLUTION



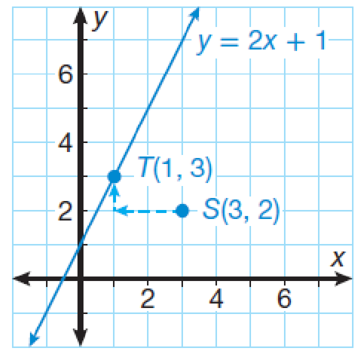
c. Find the distance from *P*(6, 4) to the line *y* = *x*.

SOLUTION



Example 3 Finding the Closest Point on a Line to a Point

Given the equation $y = 2x + 1$ and the point $S(3, 2)$, find the point on the line that is closest to S. Find the shortest distance from S to the line.



SOLUTION

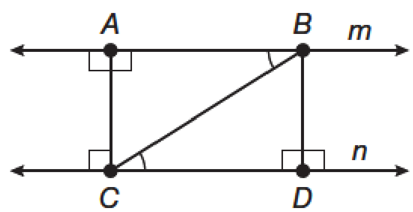
Theorem 42-3 - The perpendicular segment from a point to a plane is the shortest segment from the point to the plane.

Because parallel lines are always the same distance from one another, the distance from any point on a line to a line that is parallel is the same, regardless of which point you pick.

Theorem 42-4 - If two lines are parallel, then all points on one line are equidistant from the other line.

Example 4 Proving All Points on Parallel Lines are Equidistant

Prove that if two lines are parallel, then all the points on one line are equidistant from the other line.



Given: $m \parallel n$

Prove: $\overline{AC} \cong \overline{DB}$

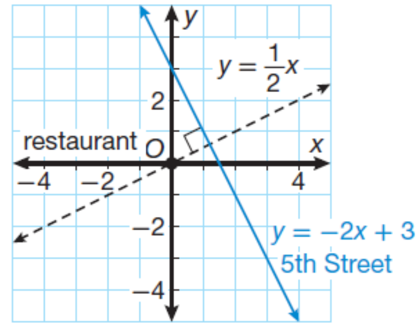
SOLUTION

Draw a diagram like the one above, with two parallel lines cut by a transversal. Draw two perpendicular segments from the endpoints of the transversal to the opposite parallel line, forming ΔABC and ΔDCB. To prove Theorem 42-4, we need to show that points A and C are the same distance apart as points B and D, or that $\overline{AC} \cong \overline{DB}$.

Statements	Reasons
1.	
2.	
3.	
4.	
5.	
6.	

Example 5 Application: Delivery Routes

A pizza restaurant only delivers to customers who are less than 3 miles away. The restaurant, located at the origin, receives a call from a customer who lives at the closest point to the restaurant on 5th Street, which can be represented by the line $y = -2x + 3$. If each unit on a coordinate plane represents 1 mile, does this customer live close enough for delivery?



SOLUTION

You Try!!!!!

a. Find the distance between the line $y = 6$ and (-3, -5).

b. Find the distance between the line $x = 9$ and (-3, -5).

d. Find the closest point on the line $y = 3x - 1$ to (5, 4).