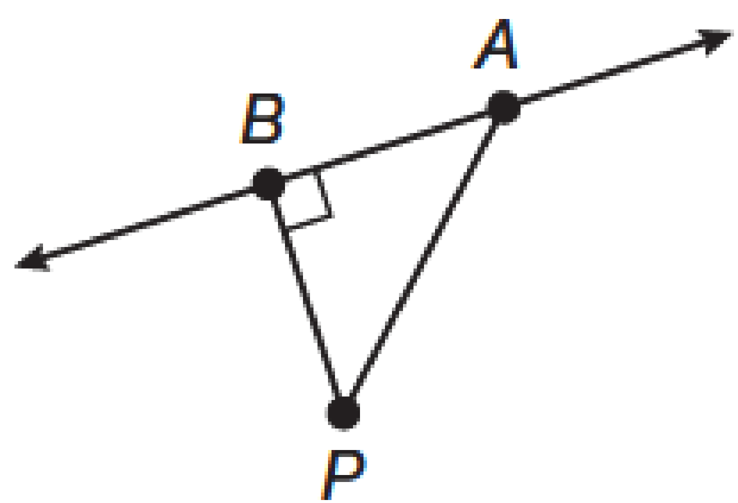
Lesson 42

Finding Distance from a Point to a Line

Given a line $\overleftrightarrow{AB}$ and a point *P*, what is the shortest distance between *P* and $\overleftrightarrow{AB}$? Notice that ΔABP is a right triangle, and $\overline{AP}$ is the hypotenuse. The hypotenuse is always the longest side of a right triangle, so $\overline{AP}$ must be longer than $\overline{PB}$.



Example 1 Choosing the Closest Point

Which point on the line $y = 0$ is closest to point D—$L(3.6, 0)$, $M(4, 0)$, or $N(4.25, 0)$?

SOLUTION

Use the distance formula:

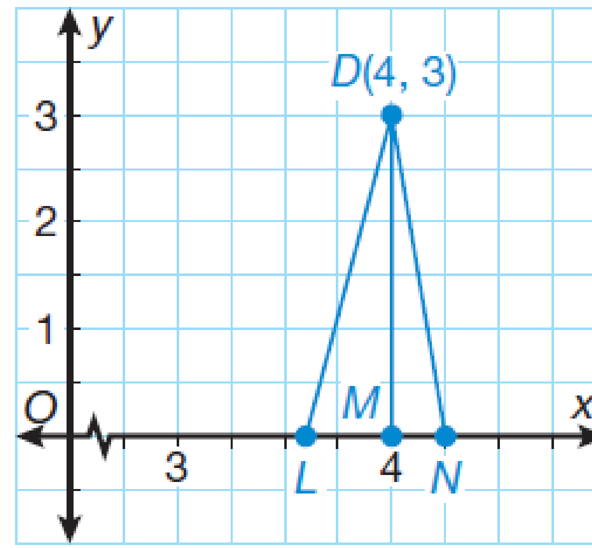
$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$DL = \sqrt{(3.6 - 4)^2 + (0 - 3)^2} \approx 3.03$$

$$DM = \sqrt{(4 - 4)^2 + (0 - 3)^2} = 3$$

$$DN = \sqrt{(4.25 - 4)^2 + (0 - 3)^2} \approx 3.01$$

M is the closest point to *D*.



Theorem 42-1 - Through a line and a point not on the line, there exists exactly one perpendicular line to the given line.

Theorem 42-2 - The perpendicular segment from a point to a line is the shortest segment from the point to the line.

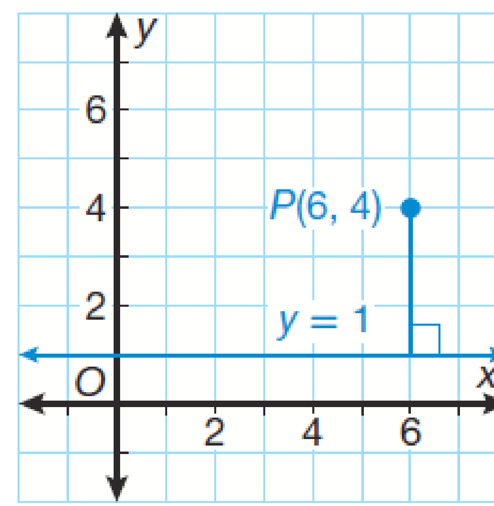
Theorem 42-1 indicates that there is only one such segment. The length of a perpendicular segment from a point to a line is referred to as the distance from a point to a line.

Example 2 Finding Distance to a Line

a. Find the distance from *P*(6, 4) to the line $y = 1$.

SOLUTION

The perpendicular distance is the distance between the *y*–coordinates of the point and the line. The distance between point *P* and the line is 3 units.

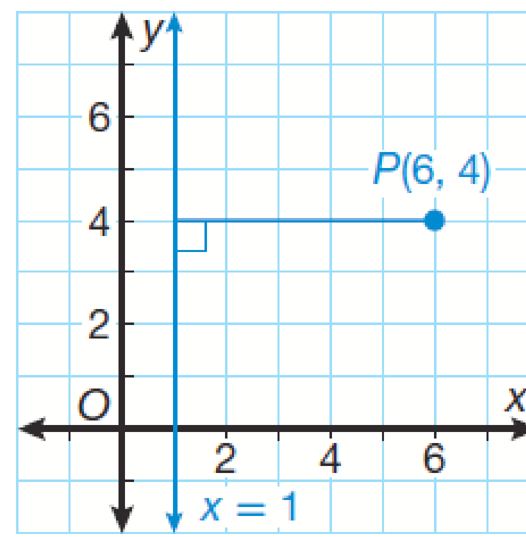


Example 2 Finding Distance to a Line

b. Find the distance from $P(6, 4)$ to the line $x = 1$.

SOLUTION

Point P is 5 units to the right of the line $x = 1$. The perpendicular distance is the difference between the x-coordinates. The distance is 5 units.



Example 2 Finding Distance to a Line

c. Find the distance from *P*(6, 4) to the line $y = x$.

SOLUTION

Find the line perpendicular to the line $y = x$ that includes point *P*(6, 4). The slope of the perpendicular line is the negative reciprocal of the slope of the line $y = x$, which is −1. Start at point *P*(6, 4). Use the slope to find other points on the perpendicular line. Draw the line. Notice that the lines intersect at (5, 5). Find the distance between (6, 4) and (5, 5) using the distance formula.

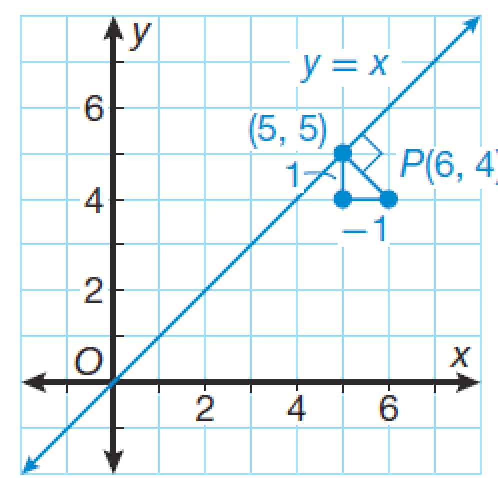
$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ Distance formula

$d = \sqrt{(6-5)^2 + (4-5)^2}$ Substitute

$d = \sqrt{2}$ Simplify

$d \approx 1.414$

So, the distance from the point to the line is about 1.41 units.



Example 3 Finding the Closest Point on a Line to a Point

Given the equation $y = 2x + 1$ and the point $S(3, 2)$, find the point on the line that is closest to S. Find the shortest distance from S to the line.

SOLUTION

Draw the line and the point on a coordinate grid.

Next, find the slope of the line. In this case, the slope is 2.

Find the slope of a line perpendicular to the given line. The negative reciprocal of 2 is $-\frac{1}{2}$.

Use the slope to find more points on the perpendicular line. Draw the line.

The lines intersect at (1, 3).

Use the distance formula to find the distance between (3, 2) and (1, 3).

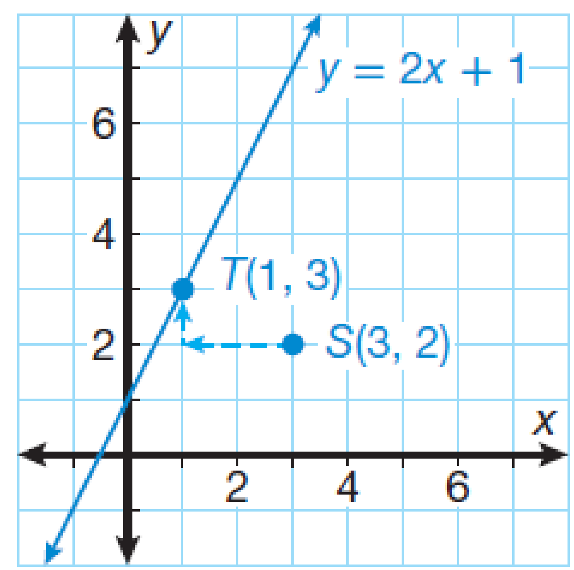
$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ Distance formula

$d = \sqrt{(1 - 3)^2 + (3 - 2)^2}$ Substitute

$d = \sqrt{5}$ Simplify

$d \approx 2.24$

So, the distance from the point to the line is about 2.24 units.

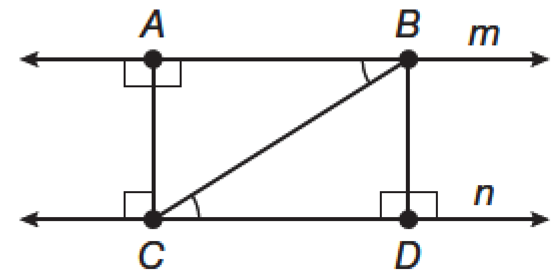


Theorem 42-3 – The perpendicular segment from a point to a plane is the shortest segment from the point to the plane.

Because parallel lines are always the same distance from one another, the distance from any point on a line to a line that is parallel is the same, regardless of which point you pick.

Theorem 42-4 – If two lines are parallel, then all points on one line are equidistant from the other line.

Example 4 Proving All Points on Parallel Lines are Equidistant



Prove that if two lines are parallel, then all the points on one line are equidistant from the other line.

Given: $m \parallel n$

Prove: $\overline{AC} \cong \overline{DB}$

SOLUTION

Draw a diagram like the one above, with two parallel lines cut by a transversal. Draw two perpendicular segments from the endpoints of the transversal to the opposite parallel line, forming ΔABC and ΔDCB. To prove Theorem 42-4, we need to show that points *A* and *C* are the same distance apart as points *B* and *D*, or that $\overline{AC} \cong \overline{DB}$.

Statements	Reasons
1. $m \parallel n$	1. Given
2. $\angle CAB \cong \angle BDC$	2. All right angles are congruent
3. $\angle ABC \cong \angle DCB$	3. Alternate Interior Angles
4. $\overline{BC} \cong \overline{CB}$	4. Reflexive Property
5. $\Delta ABC \cong \Delta DCB$	5. AAS Congruence Theorem
6. $\overline{AC} \cong \overline{DB}$	6. CPCTC

Therefore, all points on one line are equidistant from the other line.

Example 5 Application: Delivery Routes

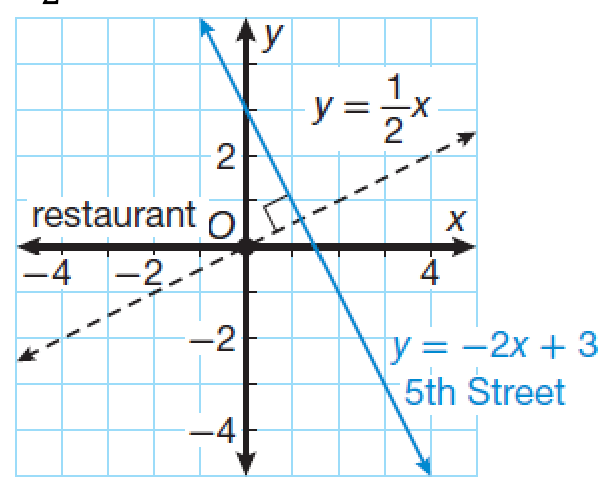
A pizza restaurant only delivers to customers who are less than 3 miles away. The restaurant, located at the origin, receives a call from a customer who lives at the closest point to the restaurant on 5th Street, which can be represented by the line $y = -2x + 3$. If each unit on a coordinate plane represents 1 mile, does this customer live close enough for delivery?

SOLUTION

Draw the restaurant at the origin of a coordinate plane. Graph the line representing 5th Street, as shown in the diagram. To find the shortest distance the customer could be from the restaurant, find a line perpendicular to $y = -2x + 3$ through the origin.

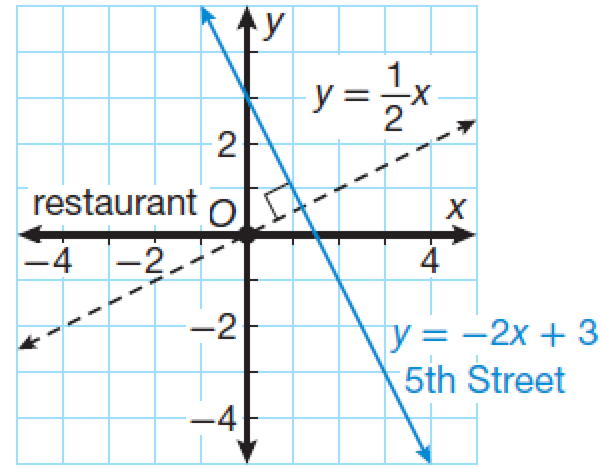
The opposite reciprocal of 5th Street's slope is $\frac{1}{2}$. Use the point–slope formula to find an equation for the line.

$y - y_1 = m(x - x_1)$	Point–slope form
$y - 0 = \frac{1}{2}(x - 0)$	Substitute.
$y = \frac{1}{2}x$	Simplify.



Example 5 Application: Delivery Routes

To find the intersection of these lines, graph them or solve them as a system of equations, as shown below.



$$y = \frac{1}{2}x \qquad y = -2x + 3$$

$$\frac{1}{2}x = -2x + 3$$

$$x = -4x + 6$$

$$5x = 6$$

$$x = 1.2$$

Substituting this value into either equation will reveal that the *y*–coordinate for this point is 0.6. Use the distance formula to find the distance between (1.2, 0.6) and the origin.

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ Distance formula

$d = \sqrt{(1.2 - 0)^2 + (0.6 - 0)^2}$ Substitute

$d = \sqrt{1.8}$ Simplify

$d \approx 1.34\, mi.$

The distance between these points is less than 3 miles, so the customer is within delivery distance.

You Try!!!!!

Find the distance between the line $y = 6$ and $(-3, -5)$.

Find the distance between the line $x = 9$ and $(-3, -5)$.

Find the closest point on the line $y = 3x - 1$ to $(5, 4)$.

Assignment

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Lesson Practice (Ask Mr. Heintz)

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Practice 1–30 (Do the starred ones first)