Lesson 43

Chords, Secants, and Tangents

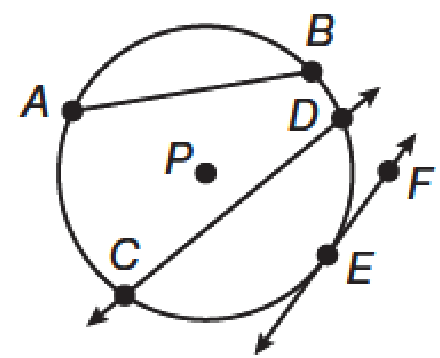
The diameter and radius of a circle are two special segments that can be used to find properties of a circle.

There are three more special segments common to every circle. They are chords, secants, and tangents.

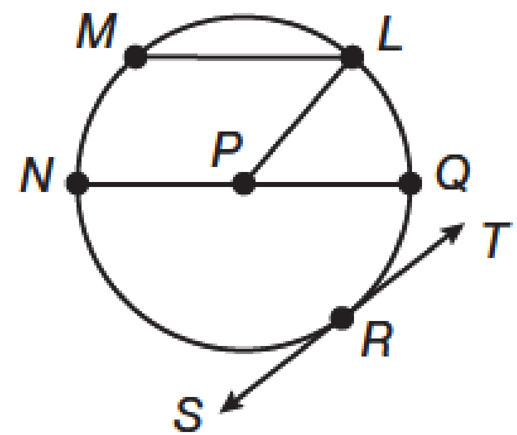
Chord – A line segment whose endpoints lie on a circle. In the diagram, $\overline{AB}$ and $\overline{CD}$ are chords of $\odot P$.

Secant of a Circle – A line that intersects a circle at two points. In the diagram, $\overleftrightarrow{CD}$ is a secant of $\odot P$.

Tangent of a Circle – A line in the same plane as the circle that intersects the circle at exactly one point, called a point of tangency. E is a point of tangency on $\odot P$, and $\overleftrightarrow{EF}$ is a tangent line.



Example 1 Identifying Lines and Segments that Intersect Circles



Use the figure at right to answer parts a, b, and c.
a. Identify three radii and a diameter.
SOLUTION
Three radii are $\overline{PL}$, $\overline{PQ}$, and $\overline{PN}$.
$\overline{PM}$ and $\overline{PR}$, are also radii.
A diameter is $\overline{NQ}$.

b. Identify two chords.
SOLUTION
One chord is $\overline{ML}$. Another chord is the diameter, $\overline{NQ}$.
Since any two endpoints on the circle can make a chord, you could also answer $\overline{MQ}$, $\overline{QR}$, $\overline{QL}$, $\overline{MR}$, $\overline{MN}$, $\overline{NR}$, $\overline{LN}$, and $\overline{LR}$ are also chords.

c. Name a tangent to the circle and identify the point of tangency.
SOLUTION
The tangent is $\overleftrightarrow{ST}$. The point of tangency is *R*.

These special segments can be used to find unknown lengths in circles with the help of the theorems presented in this lesson.

Theorem 43-1 – If a diameter is perpendicular to a chord, then it bisects the chord and its arcs.

Theorem 43-2 – If a diameter bisects a chord other than another diameter, then it is perpendicular to the chord.

Example 2 Finding Unknowns Using

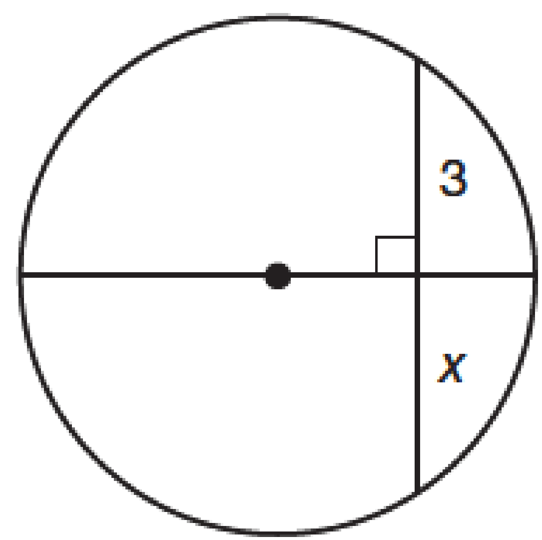
Chord–Diameter Relationships

a. Find the length of x.

SOLUTION

Since the diameter is perpendicular to the chord, the chord is bisected by the diameter by Theorem 43–1.

So, $x = 3$.



Example 2 Finding Unknowns Using

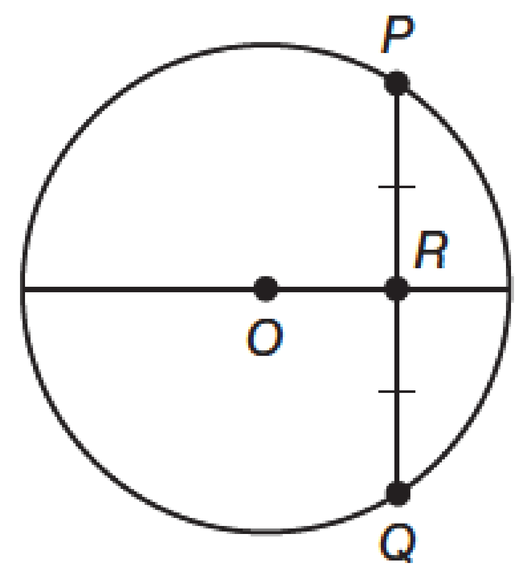
Chord–Diameter Relationships

b. Find m$\angle ORP$.

SOLUTION

Since the diameter bisects the chord, the chord must be perpendicular to the diameter by Theorem 43–2.

Thus, m$\angle ORP = 90^\circ$



Example 2 Finding Unknowns Using

Chord-Diameter Relationships

c. The circle shown has a diameter of 14 inches. Chord $\overline{AB}$ is 8 inches long. How far is $\overline{AB}$ from the center of the circle, to the nearest hundredth of an inch?

SOLUTION

Construct $\overline{AP}$ and $\overline{CP}$ so C is the midpoint of $\overline{AB}$. Since C is the midpoint of $\overline{AB}$, $\overline{AC}$ is 4 inches long. AP is a radius of the circle, so it is 7 inches long. Since ΔPAC is a right triangle, you can apply the Pythagorean Theorem to find the length of $\overline{CP}$.

$a^2 + b^2 = c^2$

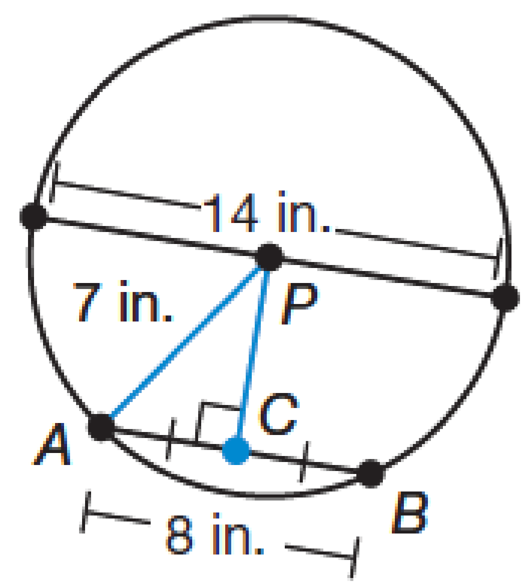
$x^2 + 4^2 = 7^2$

$x^2 + 16 = 49$

$x^2 = 33$

$x \approx 5.74$

The chord is about 5.74 inches from the center of the circle.

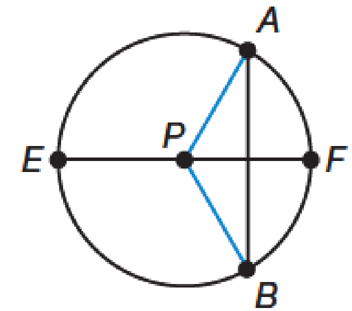


In fact, any segment that is a perpendicular bisector of a chord is also a diameter of the circle. This leads to Theorem 43-3.

Theorem 43-3 The perpendicular bisector of a chord contains the center of the circle.

Every diameter passes through the center of the circle, so another way of stating Theorem 43-3 is that the perpendicular bisector of a chord is also a diameter or a line containing the diameter.

Example 3 Proving the Perpendicular Bisector of a Chord Contains the Center



Given: $\overline{EF}$ is the perpendicular bisector of $\overline{AB}$.
Prove: $\overline{EF}$ passes through the center of $\odot P$.
SOLUTION

Statements	Reasons
1. $\overline{EF}$ is the $\perp$ bisector of $\overline{AB}$.	1. Given
2. $\overline{PA}$ and $\overline{PB}$ are congruent.	2. Definition of a radius
3. Point P lies on the perpendicular bisector of $\overline{AB}$.	3. If a point is equidistant from the endpoints of a segment, then the point lies on the perpendicular bisector of the segment (Theorem 6-6).
4. $\overline{EF}$ passes through the center of $\odot P$.	4. $\overline{EF}$ is the perpendicular bisector of $\overline{AB}$.

Therefore, the center of the circle lies on the perpendicular bisector of a chord.

One final property of chords is that all chords that lie the same distance from the center of the circle must be the same length, as stated in Theorem 43-4.

Theorem 43-4 – In a circle or congruent circles:

- **Chords equidistant from the center are congruent.**
- **Congruent chords are equidistant from the center of the circle.**

Example 4 Applying Properties of Congruent Chords

Find *CD*, if $AP = 5$ units and $PE = 3$ units.

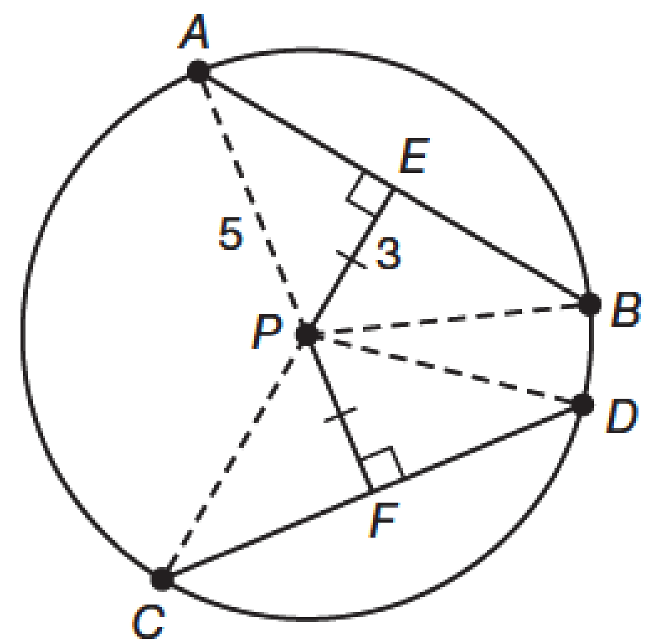
SOLUTION

$\overline{AB}$ and $\overline{CD}$ are equidistant from the center of the circle, so they are congruent by Theorem 43-4.

Use the Pythagorean Theorem to find the length of $\overline{AE}$.

$$a^2 + b^2 = c^2$$
$$3^2 + b^2 = 5^2$$
$$b = 4$$

By Theorem 43-1, *E* is the midpoint of $\overline{AB}$, so $AB = 8$. Since $\overline{AB} \cong \overline{CD}$, $CD = 8$ units.



Example 5 Application: Plumbing

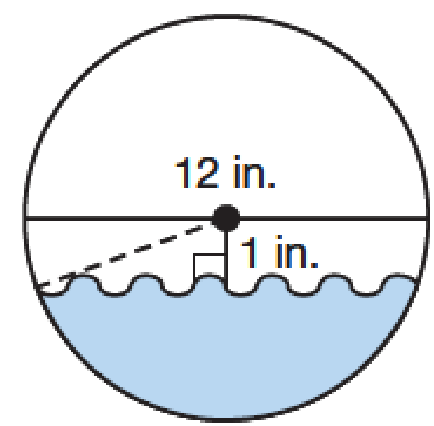
Two identical circular pipes have diameters of 12 inches. Water is flowing 1 inch below the center of both pipes. What can be concluded about the width of the water surface in both pipes? Calculate the width.

SOLUTION

Taking a cross section of the pipe, the width of the water surface is a chord of the circle. Since it is given that the water surface is one inch below the center in both pipes, we can conclude that the width of the water surface is equal in both pipes.

The radius that has been drawn into the diagram forms a right triangle with the surface of the water and the distance between the water and the center of the pipe. The radius of the pipe is 6 inches. Let x represent half the width of the water surface. Use the Pythagorean Theorem to solve for x.

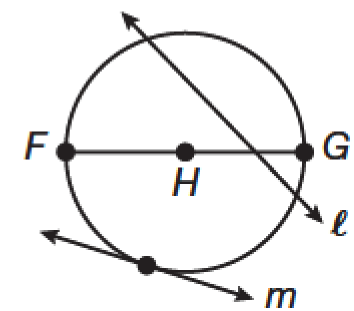
$$a^2 + b^2 = c^2$$
$$1^2 + x^2 = 6^2$$
$$x^2 = 35$$
$$x \approx 5.92$$



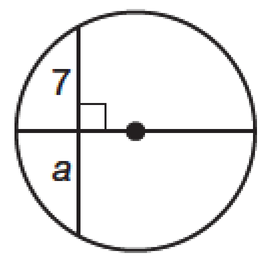
The width of the water surface is two times x, or approximately 11.84 inches.

You Try!!!!!!

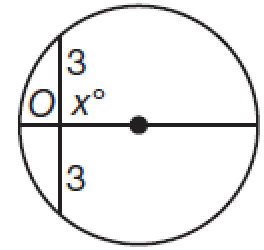
a. Identify each line or segment that intersects the circle.



b. Determine the value of *a*.



c. Determine the value of *x*.



Assignment

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Lesson Practice (Ask Mr. Heintz)

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Practice 1–30 (Do the starred ones first)