Lesson 44

Applying Symmetry

Recall from Lesson 41 that proportions can be used to find unknown measures in similar polygons. Any two regular polygons with the same number of sides are similar. Therefore, all regular polygons with the same number of sides are similar to each other.

Example 1 Using Similarity to Find Unknown Measures

The hexagons in the diagram are similar. Find the values of x and y.

SOLUTION

By looking at the corresponding segments with known lengths, a similarity ratio can be written: 20:8. Now, use a proportion to solve for x and y.

$$\frac{20}{8} = \frac{x}{6}$$

$$6 \cdot 20 = 8x$$

$$120 = 8x$$

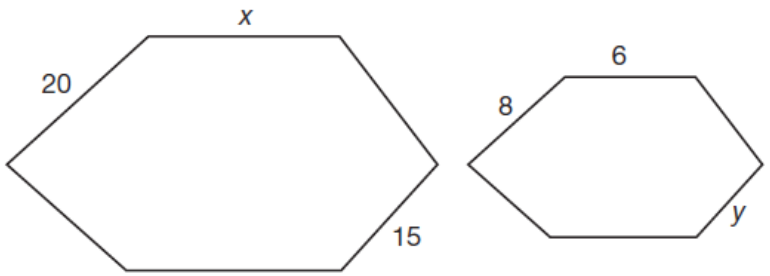
$$x = 15$$

$$\frac{20}{8} = \frac{15}{y}$$

$$20y = 15 \cdot 8$$

$$20y = 120$$

$$y = 6$$

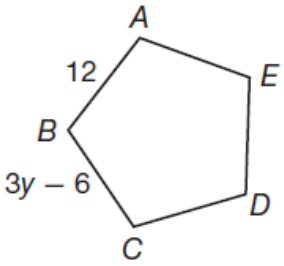


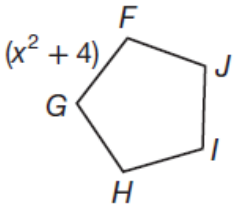
Example 2 Applying Similarity to Solve for Unknowns

a. Pentagons *ABCDE* and *FGHIJ* are regular pentagons, and are similar to each other. The similarity ratio of *ABCDE* to *FGHIJ* is 3:2. Find the values of *x* and *y*.

SOLUTION

Set up a proportion using the similarity ratio and the ratio of *AB* to *FG*, then find the cross product and solve for *x*.





$\frac{3}{2} = \frac{12}{x^2+4}$	Set up a proportion.
$3(x^2 + 4) = 12 \cdot 2$	Cross multiply.
$3x^2 + 12 = 24$	Distribute and simplify.
$x = \pm 2$	Solve.

Since *ABCDE* is a regular pentagon, its sides are congruent.

Therefore, $BC = AB$

$3y - 6 = 12$	Substitute.
$3y = 18$	Add 6 to each side.
$y = 6$	Divide each side by 3.

Example 2 Applying Similarity to Solve for Unknowns

b. What is the ratio of the perimeter of *ABCDE* to *FGHIJ*?

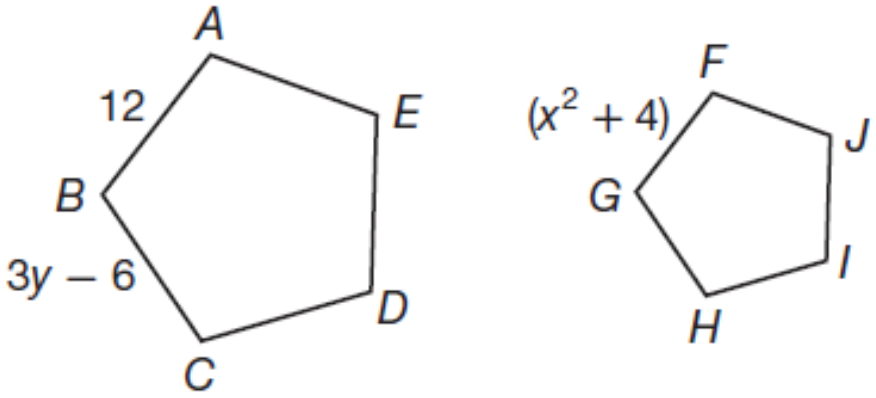
SOLUTION

Since both pentagons are regular, their sides are all congruent. *ABCDE* has five sides, each measuring 12 units, so its perimeter is 60.

Substituting $x = \pm 2$ to find the length of a side of *FGHIJ* shows that each side measures 8 units, so the perimeter of *FGHIJ* is 40.

Therefore, the ratio of the perimeters is 60:40, which reduces to 3:2.

As you can see from Example 2, the perimeters of two similar figures share the same similarity ratio as their sides.



Theorem 44-1 – If two polygons are similar, then the ratio of their perimeters is equal to the ratio of their corresponding sides.

Example 3 Proving the Relationship Between Perimeters of Similar Figures

Given $\Delta PQR \sim \Delta STU$, prove that the ratio of their perimeters is 1:2 if the ratio of their corresponding sides is 1:2.

SOLUTION Use a 2-column proof.

Statements	Reasons
1. $\Delta PQR \sim \Delta STU$	1. Given
2. $\frac{PQ}{ST} = \frac{QR}{TU} = \frac{RP}{US} = \frac{1}{2}$	2. Given
3. $2PQ = ST$	3. Cross multiply
4. $2QR = TU$	4. Cross multiply
5. $2RP = US$	5. Cross multiply
6. perimeter of $\Delta STU = ST + TU + US$	6. Definition of perimeter
7. perimeter of $\Delta STU = 2PQ + 2QR + 2RP$	7. Substitution Property of Equality
8. perimeter of $\Delta STU = 2(PQ + QR + RP)$	8. Simplify
9. perimeter of $\Delta PQR = PQ + QR + RP$	9. Definition of perimeter
10. perimeter of$\Delta STU = 2(perimeter\ of\ \Delta PQR$	10. Substitution Property of Equality

Therefore, the ratio of the perimeter of ΔPQR to the perimeter of ΔSTU is 1:2.

Example 4 Applying Similarity to Solve a Perimeter Problem

Figures *HIJK* and *LMNO* are similar polygons. Their corresponding sides have a ratio of 2:5. If the perimeter of figure *HIJK* is 27 inches, what is the perimeter of figure *LMNO*?

SOLUTION

Because *HIJK* and *LMNO* are similar polygons, the ratio of their perimeters is equal to the ratio of their corresponding sides.

Therefore, the ratio of *HIJK*'s perimeter to *LMNO*'s perimeter is 2:5.

Set up a proportion using this ratio to solve for the perimeter of *LMNO*.

$$\frac{27}{x} = \frac{2}{5}$$

$$2x = 5 \cdot 27$$

$$x = 67.5$$

The figure *LMNO* has a perimeter of 67.5 inches.

Example 5 Application: Map Scales

Foxx plans to jog 5000 meters a day in training for a race. The park where Foxx jogs is in the shape of a regular pentagon. The side length of the park is 5 centimeters long on a map with the scale $\frac{1\ cm}{50\ m}$. How many times does Foxx need to jog along the perimeter of the park to complete his daily training ?

SOLUTION

First, find the perimeter of the park on the map.

Perimeter of a regular pentagon $= 5s$

$P = 5(5\text{ cm})$

$P = 25\text{ cm}$

The park on the map and the actual park are similar polygons. Therefore, the ratio of their perimeters is the same as the ratio of their corresponding sides.

So, the ratio of the park's perimeter on the map to the perimeter of the actual park is $\frac{1\ cm}{50\ m}$.

$$\frac{perimeter\ on\ the\ map}{perimeter\ of\ the\ actual\ park} = \frac{1\ cm}{50\ m}$$

$$\frac{25\ cm}{perimeter\ of\ the\ actual\ park} = \frac{1\ cm}{50\ m}$$

$$perimeter\ of\ the\ actual\ park \cdot 1\ cm = 50\ m \cdot 25\ cm$$

$$perimeter\ of\ the\ actual\ park = \frac{50\ m \cdot 25\ cm}{1\ cm}$$

$$perimeter\ of\ the\ actual\ park = 1250\ m$$

Since 1250 is $\frac{1}{4}$ of 5000, Foxx needs to jog along the perimeter of the park 4 times to complete his daily training.

You Try!!!!!

b. In ΔABC , $AB = x^2 - 7$, $BC = y + 4$, and $CA = 5$. In $\Delta DEF, DE = 6, EF = 12, and\ FD = 15$. $\Delta ABC \sim \Delta DEF$. Find the values of *x* and *y*. Then find the ratio of the perimeters of the two triangles.

c. Figures *ABCD* and *EFGH* are similar. The ratio of their corresponding sides is 3:5. If the perimeter of *EFGH* is 45 inches, what is the perimeter of figure *ABCD*?

You Try!!!!!

d. Pentagons *ABCDE* and *FGHIJ* are similar figures. The perimeter of *ABCDE* is 32 centimeters. The similarity ratio of *ABCDE* to *FGHIJ* is 2:9. What is the perimeter of *FGHIJ*?

e. Jana and her brother Jacob are designing their own tree house with two separate doors, one that is proportional to Jana's height and one that is proportional to Jacob's height. Jacob is 3 feet tall and Jana is 4 feet tall, so Jana decides that her door should be 5 feet tall by 2 feet wide. How tall should Jacob's door be, and what will its perimeter be?

Assignment

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Lesson Practice (Ask Mr. Heintz)

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Practice 1-30 (Do the starred ones first)