Lesson 46

Triangle Similarity: AA, SSS, SAS

Two triangles are similar if all their corresponding angles are congruent. Since the sum of any triangle's angles is 180°, only two angles are required to prove that two triangles are similar.

Postulate 21: Angle-Angle (AA) Triangle Similarity Postulate – If two angles of one triangle are congruent to two angles of another triangle, then the triangles are similar.

Example 1 Using the AA Similarity Postulate

Show that the two triangles are similar if $\overline{AB} \parallel \overline{DE}$. Then, find *DE*.

SOLUTION

Statements	Reasons
1. $\overline{AB} \parallel \overline{DE}$	1. Given
2. $m\angle ABC = m\angle DEC$	2. Corresponding Angles Postulate
3. $m\angle BAC = m\angle EDC$	3. Corresponding Angles Postulate
4. $\Delta ABC \sim \Delta DEC$	4. AA Similarity Postulate

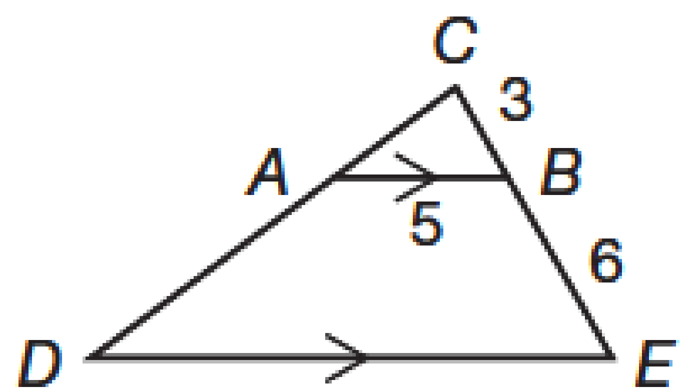
Since the two triangles are similar, the ratios of the lengths of corresponding sides are equal.

$$\frac{AB}{DE} = \frac{BC}{EC}$$

$$\frac{5}{DE} = \frac{3}{9}$$

$$3(DE) = 45$$

$$DE = 15$$



Exploration: Understanding AA Similarity

In this exploration, you will construct similar triangles and observe the properties of each.

1. Draw two different line segments on a sheet of paper. Make sure the segments each have a different length.
2. At each end of the first line segment, measure acute angles and draw the rays out to create a triangle.
3. On your second line segment, measure the same two angles with a protractor and draw a second triangle.
4. Measure the unknown angle of each of the triangles you have drawn.

What is the relationship between these two angles? What is the relationship between these two triangles?

5. Measure the side lengths of the larger triangle and one side of the small triangle. Now, use what you know about the triangles to predict the side lengths of the smaller triangle without using a ruler. Then, measure the lengths. Were your predictions correct?

It is not always necessary to know a triangle's angle measures to determine similarity. Another way to determine similarity is to verify that the lengths of all the corresponding sides of both triangles are related in the same ratio.

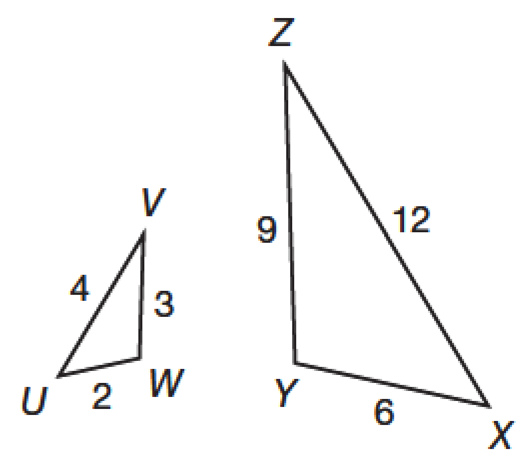
Theorem 46-1: SSS Similarity Theorem – If the lengths of the sides of a triangle are proportional to the lengths of the sides of another triangle, then the triangles are similar.

Example 2 Using the SSS Similarity Theorem

Given the two triangles with lengths as shown, show that they are similar triangles.

SOLUTION

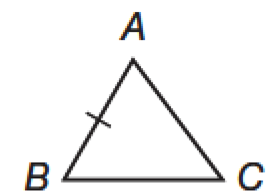
Statements	Reasons
1. $\frac{UW}{XY} = \frac{2}{6} = \frac{1}{3}$	1. Similarity ratio for $\overline{UW}:\overline{XY}$
2. $\frac{WV}{YZ} = \frac{3}{9} = \frac{1}{3}$	2. Similarity ratio for $\overline{WV}:\overline{XY}$
3. $\frac{VU}{ZX} = \frac{4}{12} = \frac{1}{3}$	3. Similarity ratio for $\overline{VU}:\overline{ZX}$
4. $\Delta UWV \sim \Delta XYZ$	4. SSS Similarity Theorem



Example 3 Proving the SSS Similarity Theorem

Prove Theorem 46-1: If the lengths of the sides of a triangle are proportional to the lengths of the sides of another triangle, then the triangles are similar.

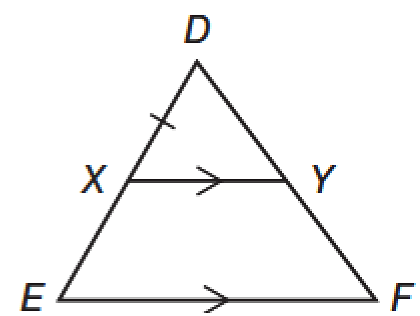
Given: $\frac{AB}{DE} = \frac{BC}{EF} = \frac{CA}{FD}$, $\overline{DX} \cong \overline{AB}$, $\overline{XY} \parallel \overline{EF}$



Prove: $\Delta ABC \sim \Delta DEF$

SOLUTION

Statements	Reasons
1. $\overline{DX} \cong \overline{AB}$, $\overline{XY} \parallel \overline{EF}$	1. Given; Parallel Postulate
2. $\angle DXY \cong \angle DEF$	2. Corresponding angles are congruent
3. $\angle D \cong \angle D$	3. Reflexive Property of Congruence
4. $\Delta DXY \sim \Delta DEF$	4. AA Similarity Postulate
5. $\frac{AB}{DE} = \frac{BC}{EF} = \frac{CA}{FD}$	5. Given
6. $\frac{DX}{DE} = \frac{XY}{EF} = \frac{YD}{FD}$	6. Similarity ratio from $\Delta DXY \sim \Delta DEF$
7. $DX = AB$	7. Definition of congruent segments
8. $\frac{AB}{DE} = \frac{XY}{EF} = \frac{YD}{FD}$	8. Substitute AB for DX in step 6
9. $\frac{BC}{EF} = \frac{XY}{EF}, \frac{CA}{FD} = \frac{YD}{FD}$	9. Substitute
10. $BC = XY$, $CA = YD$	10. Simplify
11. $\overline{BC} \cong \overline{XY}, \overline{CA} \cong \overline{TD}$	11. Definition of Congruent Segments
12. $\Delta ABC \cong \Delta \mathrm{DXY}$	12. SSS Triangle Congruence Postulate
13. $\Delta ABC \sim \Delta DEF$	13. Transitive Property of Similarity



One final way to prove triangle similarity is the SAS Similarity Theorem. You will notice that it is similar to one of the congruence postulates you have learned about.

Theorem 46-2: SAS Similarity Theorem – If two sides of one triangle are proportional to two sides of another triangle and the included angles are congruent, then the triangles are similar.

Caution

To apply SAS similarity, one angle pair has to be congruent, but the side pairs only have to be proportional. Do not confuse this with SAS congruence, where the two pairs of sides must be congruent.

Example 4 Proving Similarity

a. Prove that $\Delta EXY \sim \Delta EDF$.

SOLUTION

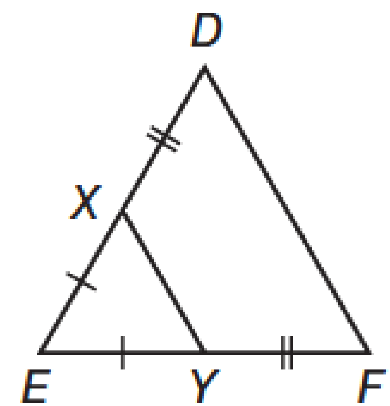
By the Reflexive Property, $\angle XEY \cong \angle DEF$.

It is given in the diagram, $\overline{EX} \cong \overline{EY}$ and $\overline{XD} \cong \overline{YF}$.

The ratio of *EX* to *ED* can be given by $\frac{EX}{EX+XD}$.

By substituting the congruent segments, it can be rewritten as $\frac{EY}{EY+YF}$, which is also the ratio of *EY* to *EF*. So the triangles have two proportional sides and one congruent angle.

By the SAS Similarity Theorem, they are similar triangles.



Example 4 Proving Similarity

b. If $EX = 6$, $ED = 11$, and $XY = 7$, find DF.

SOLUTION

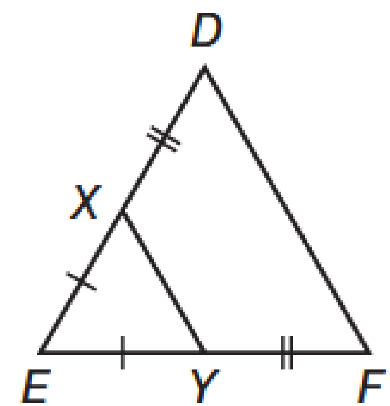
Use the similarity ratio given by $EX : ED$ and a proportion.

$$\frac{EX}{ED} = \frac{XY}{DF}$$

$$\frac{6}{11} = \frac{7}{DF}$$

$$11 \cdot 7 = 6 \cdot DF$$

$$DF = \frac{77}{6} = 12\frac{5}{6}$$



Example 5 Application: Land Surveying

A surveyor needs to find the distance across a lake. The surveyor makes some measurements as shown.

Find the distance across the lake.

SOLUTION

First, determine if the triangles are similar.

The angles at *C* are congruent since they are vertical angles.

AB and *DE* are parallel lines, so $\angle ABC$ and $\angle EDC$ are congruent.

Therefore, by the AA Similarity Postulate, $\Delta ABC \sim \Delta EDC$.

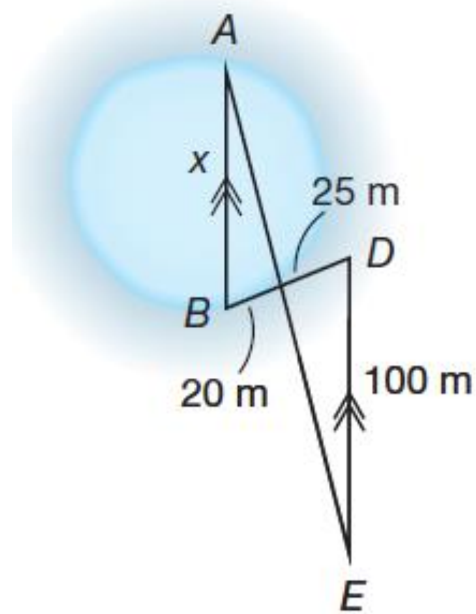
Now, find the missing value using a proportion.

$$\frac{x}{100} = \frac{20}{25}$$

$$25X = 2000$$

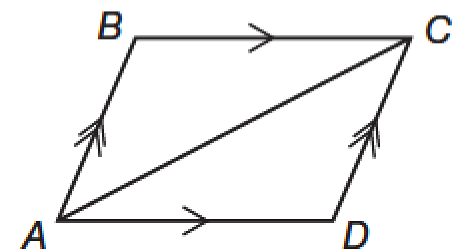
$$x = 80$$

The distance across the lake is 80 meters.

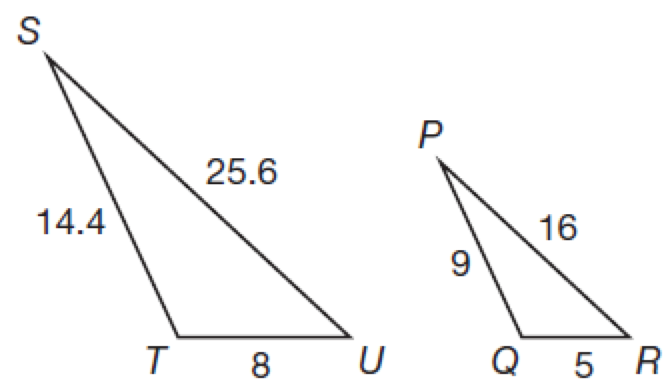


You Try!!!!!

a. Given the two triangles shown, prove they are similar using the AA Similarity Postulate.

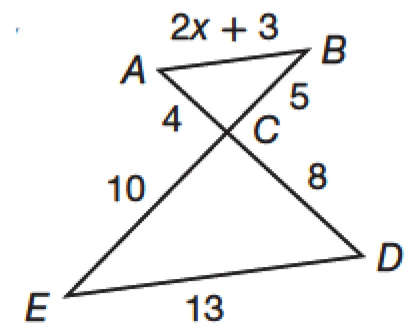


b. Given the two triangles shown, use SSS similarity to prove that they are similar.



You Try!!!!!

c.Given the two triangles shown, use SAS similarity to prove that they are similar. Find the value of x.



d.Laura wants to find out how tall a tree is. She notices that the tree makes a shadow on the ground. The top of the shadow of the treehouse is 25 feet away from the base of the tree. Laura is 5 feet 8 inches tall and she casts a shadow that is 6 feet 2 inches long. How tall is the tree, to the nearest foot?

Assignment

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Lesson Practice (Ask Mr. Heintz)

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Practice 1–30 (Do the starred ones first)