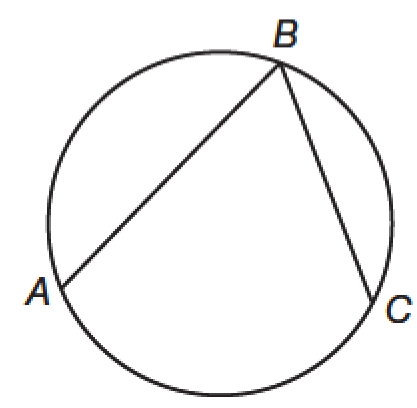
Lesson 47

Circles and Inscribed Angles

Lessons 23 and 26 introduce circles. This lesson also addresses circles and introduces inscribed angles of circles. Recall that a central angle is an angle with the center of a circle as its vertex. Another kind of angle found in circles is the inscribed angle.

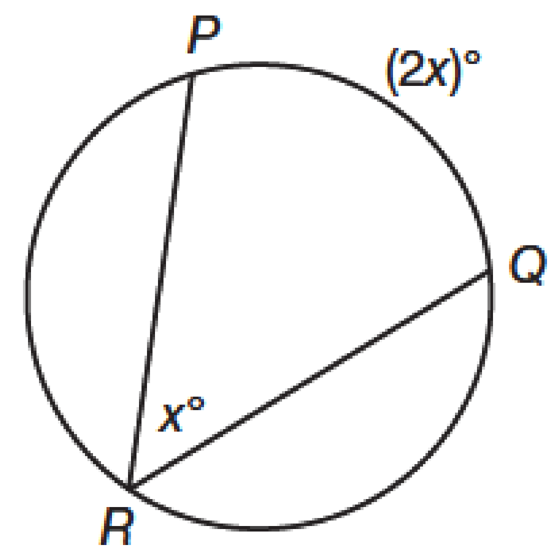
Inscribed Angle – An angle whose vertex is on a circle and whose sides contain chords of the circle. In the diagram, $\angle ABC$ is an inscribed angle.

Intercepted Arc – The arc formed by an inscribed angle. In the diagram, $\widehat{AC}$ is the intercepted arc of $\angle ABC$.

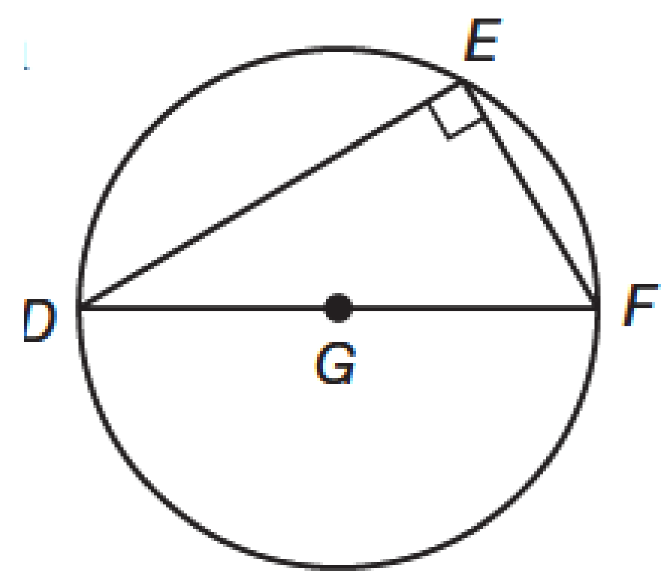


Theorem 47-1 - The measure of an inscribed angle is equal to half the measure of its intercepted arc.

$$m\angle PRQ = \left(\frac{1}{2}\right) m\widehat{PQ}$$



Theorem 47-2 - If an inscribed angle intercepts a semicircle, then it is a right angle. $\angle DEF$ intersects the semicircle, so $m\angle DEF = 90°$.



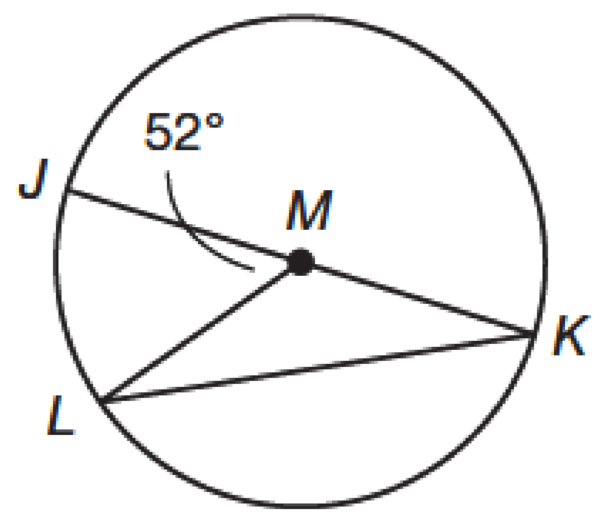
Example 1 Proving and Applying Inscribed Angle Theorems

Use $\odot M$ to answer each question.

a. Name the inscribed angle.

SOLUTION

The inscribed angle is $\angle JKL$.



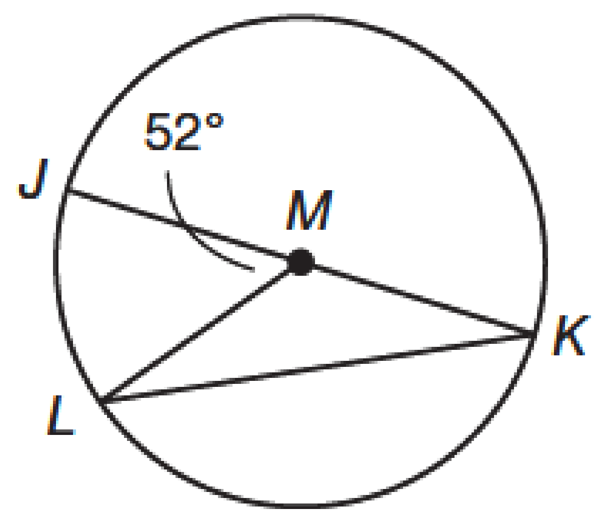
Example 1 Proving and Applying Inscribed Angle Theorems

Use $\odot M$ to answer each question.

b. Name the arc intercepted by $\angle JKL$.

SOLUTION

$\angle JKL$ intercepts the minor arc $\widehat{JL}$.



Example 1 Proving and Applying Inscribed Angle Theorems

Use $\odot M$ to answer each question.

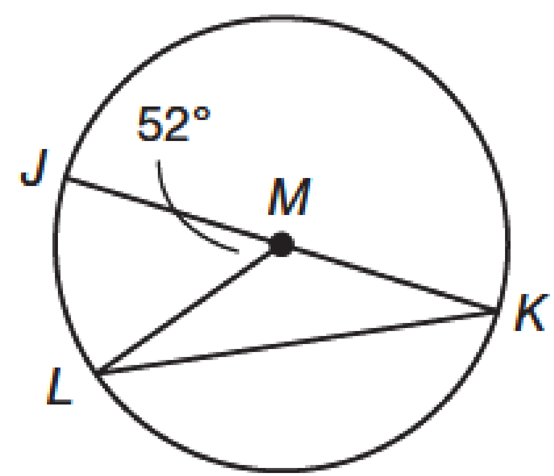
c. If $m\angle JML = 52°$, find $m\angle JKL$.

SOLUTION

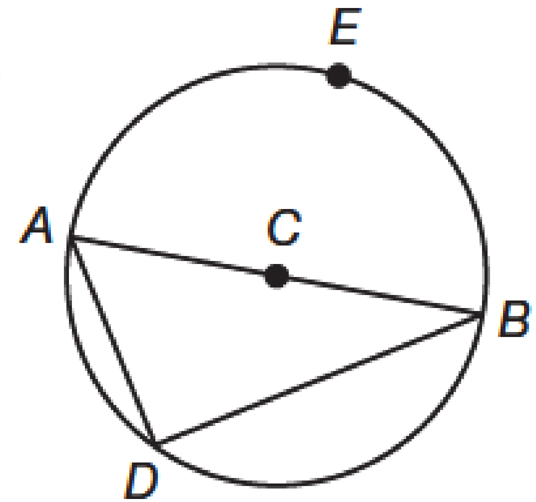
$\angle JML$ is a central angle, so $m\angle JML = m\,\widehat{JL}$. By Theorem 47-1, the measure of inscribed angle $\angle JKL$ is half the measure of $\widehat{JL}$.

$$m\angle JKL = \frac{1}{2}(52°)$$

$$m\angle JKL = 26°$$



Example 1 Proving and Applying Inscribed Angle Theorems



d. Prove Theorem 47–2.

Given: $\overline{AB}$ is a diameter of $\odot C$

Prove: $m\angle ADB = 90°$

SOLUTION

Statements	Reasons
1. $\overline{AB}$ is a diameter	1. Given
2. $\angle ACB = 180°$	2. Protractor Postulate
3. $m\widehat{AEB} = 180°$	3. Definition of the measure of an arc
4. $m\angle ADB = 90°$	4. Theorem 47–1

Example 2 Finding Angle Measures in Inscribed Triangles

Find the measure of $\angle 1$, $\angle 2$, and $\angle 3$.

SOLUTION

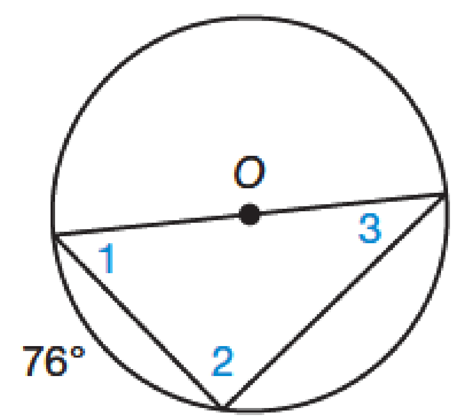
The arc intercepted by $\angle 3$ measures 76°.

$m\angle 3 = \frac{1}{2}(76°)$	Theorem 47–1
$m\angle 3 = 38°$	Simplify.

Because $\angle 2$ is an inscribed angle that intercepts a semicircle, it measures 90°, by Theorem 47–2.

You can use the Triangle Angle Sum Theorem (Theorem 18–1) to find $m\angle 1$.

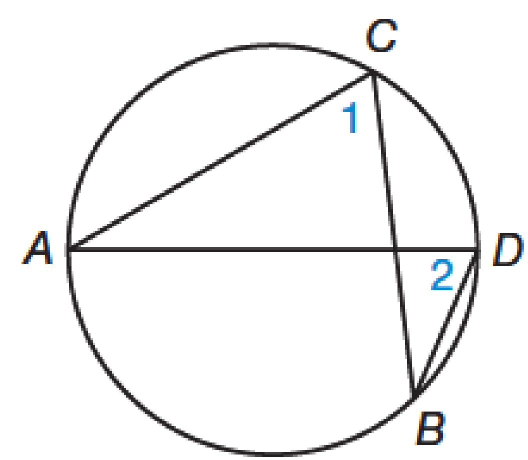
$m\angle 1 + m\angle 2 + m\angle 3 = 180°$	Triangle Angle Sum Th.
$m\angle 1 + 90° + 38° = 180°$	Substitute.
$m\angle 1 = 52°$	Solve.



More than one inscribed angle can intercept the same arc. Since both of these inscribed angles measure one-half what the arc does, they have the same measure, and are congruent.

Theorem 47-3 - If two inscribed angles intercept the same arc, then they are congruent.

$$\angle 1 \cong \angle 2$$



Example 3 Finding Measures of Arcs and Inscribed Angles

a. Find the measures of $\angle FGH$ and of $\widehat{GJ}$.

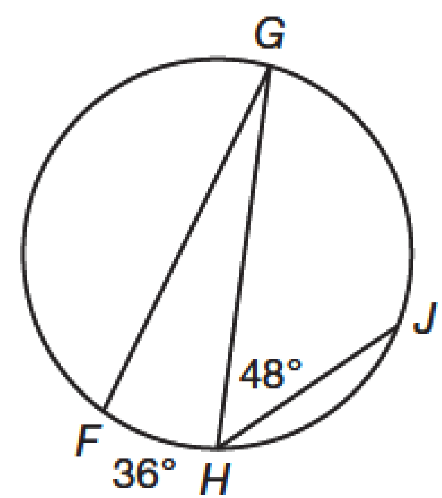
SOLUTION

$\angle FGH$ is an inscribed angle with intercepted arc $\widehat{FH}$.
Use Theorem 47-1.

$m\angle FGH = \left(\frac{1}{2}\right) m\widehat{FH}$	Theorem 47-1
$m\angle FGH = \left(\frac{1}{2}\right) 36°$	Substitute.
$m\angle FGH = 18°$	Solve.

$\widehat{GJ}$ is the intercepted arc of $\angle GHJ$. Use Theorem 47-1.

$m\angle GHJ = \left(\frac{1}{2}\right) m\widehat{GJ}$	Theorem 47-1
$48° = \left(\frac{1}{2}\right) m\widehat{GJ}$	Substitute.
$m\widehat{GJ} = 96°$	Solve.



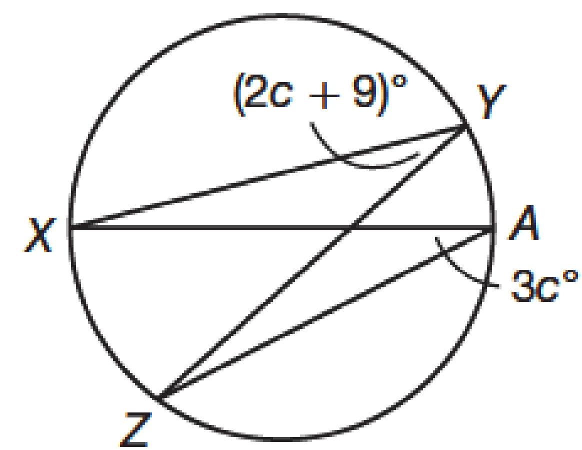
Example 3 Finding Measures of Arcs and Inscribed Angles

b. Find the measure of $\angle XYZ$.

SOLUTION

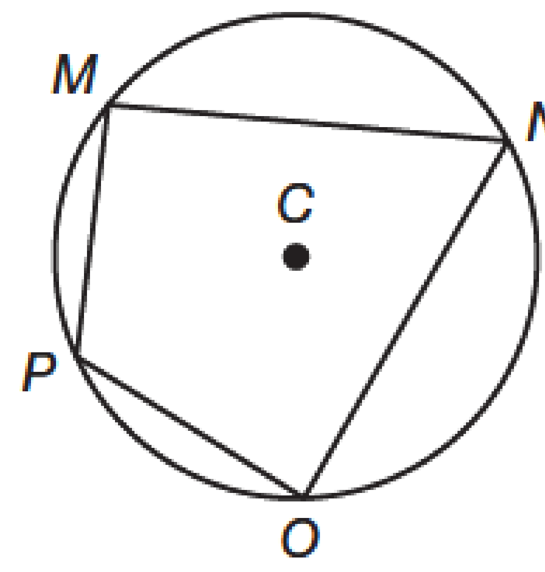
$\angle XYZ \cong \angle XAZ$	Theorem 47-3
$2c + 9 = 3c$	Substitute.
$c = 9$	Solve.

Substituting $c = 9$ into the expression for $\angle XYZ$ yields $m\angle XYZ = 27^\circ$.



Theorem 47–4 – If a quadrilateral is inscribed in a circle, then it has supplementary opposite angles.

$$\angle M + \angle O = 180°$$
$$\angle P + \angle N = 180°$$



Example 4 Finding Angle Measures in Inscribed Quadrilaterals

Find the measure of $\angle U$.

SOLUTION

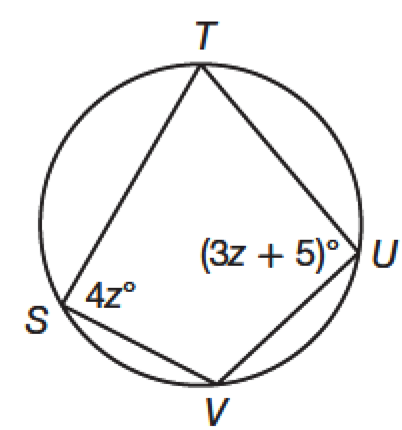
By Theorem 47-4, $\angle S$ is supplementary to $\angle U$.

$m\angle S + m\angle U = 180°$	Theorem 47-4
$4z + 3z + 5 = 180°$	Substitute.
$z = 25$	Solve for *z.*

Next, find the measure of $\angle U$.

$m\angle U = 3z + 5$	Given
$m\angle U = 3\,(25) + 5$	Substitute.
$m\angle U = 80$	Simplify.

The measure of $\angle U$ is 80°.



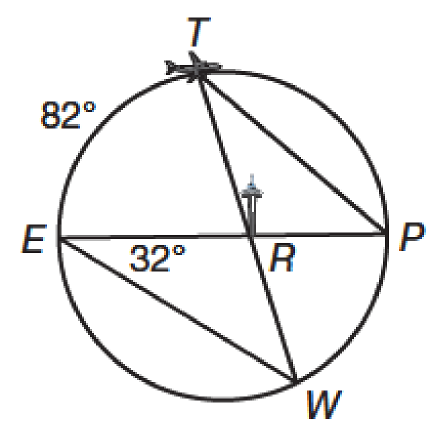
Example 5 Application: Air Traffic Control

A circular radar screen in an air traffic control tower shows aircraft flight paths. The control tower is labeled *R*. One aircraft must fly from point *T* to the control tower, and then to its destination at point *P*. Find $m\angle TRP$.

SOLUTION

$\angle WEP \cong \angle WTP$	Theorem 47-3
$m\angle WEP = m\angle WTP$	Definition of Congruence
$m\angle WEP = 32°$	Given
$m\angle WTP = 32°$	Transitive PoE
$m\angle TPE = \left(\frac{1}{2}\right)(82) = 41°$	Theorem 47-1
$m\angle WTP + m\angle TPE + m\angle TRP = 180°$	Triangle Angle Sum Theorem
$32° + 41° + m\angle TRP = 180°$	Substitution PoE
$m\angle TRP = 107°$	Solve

The measure of $\angle TRP$ is 107°.

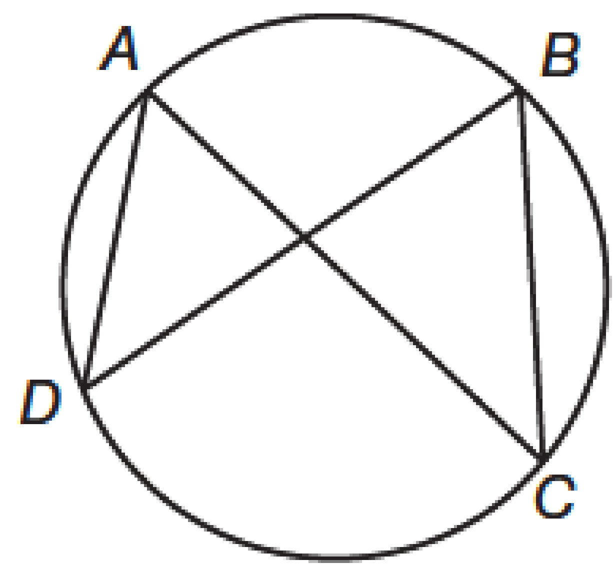


You Try!!!!!

a. Prove Theorem 47-3.

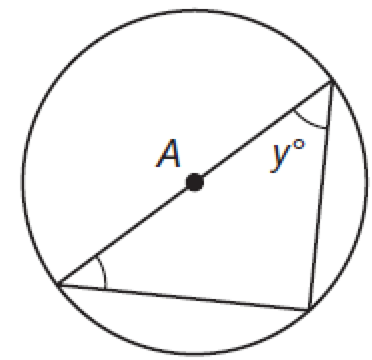
Given: Inscribed angles $\angle ADB$ and $\angle ACB$

Prove: $\angle ADB \cong \angle ACB$

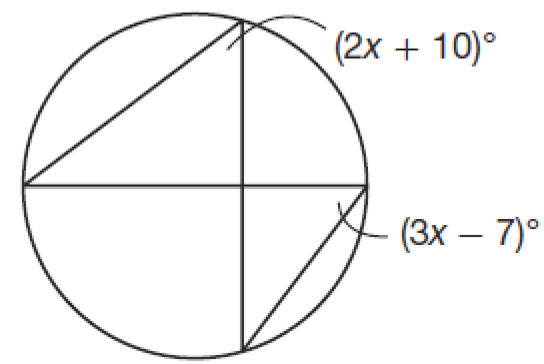


You Try!!!!!

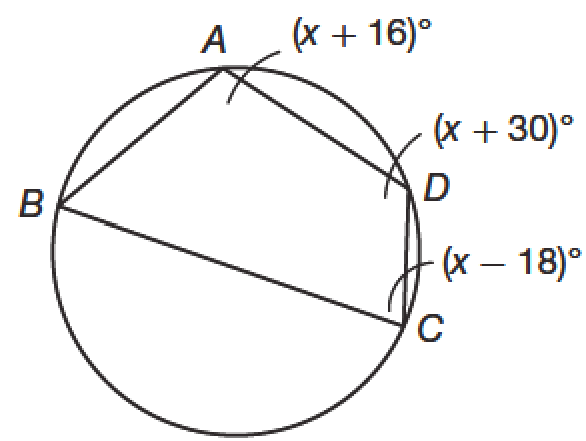
b. Find the value of y in the triangle inscribed in $\odot A$.



c. Find the value of x.



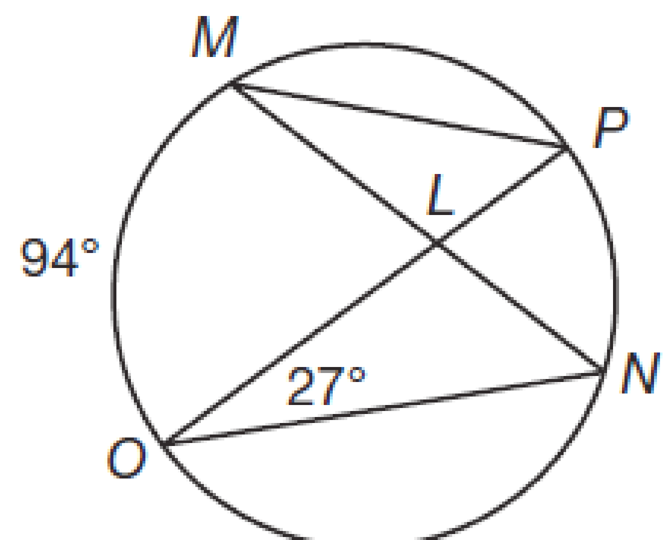
d. Find the measure of $\angle A$.



You Try!!!!!

e.Air Traffic Control A radar screen in an air traffic control tower shows flight paths. The control tower is labeled *L*.

Points *M*, *L*, and *P* mark the flight path of a commercial jet. Find the measure of $\angle MLP$.



Assignment

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Lesson Practice (Ask Mr. Heintz)

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Practice 1–30 (Do the starred ones first)