Lesson 48

Indirect Proofs

Direct reasoning – The process of reasoning that begins with a true hypothesis and builds a logical argument to show that a conclusion is true.

Indirect Proof – A proof in which the statement to be proved is assumed to be false and a contradiction is shown.

Proof by Contradiction – Another name for an indirect proof.

Follow these three steps to write an indirect proof.

1. Assume the conclusion is false.
2. Show that the assumption you made is contradicted by a theorem, a postulate, a definition, or the given information.
3. State that the assumption must be false, so the conclusion must be true.

Example 1 Writing an Indirect Proof

Prove Theorem 4-1: If two lines intersect, then they intersect at exactly one point.

SOLUTION

Since this is an indirect proof, we start by assuming that the statement is not true. In other words, it must be possible for two lines to intersect at more than one point.

Assume that the lines *m* and *n* intersect at both points, *A* and *B*. Now we must show that this contradicts another theorem or postulate.

It contradicts Postulate 5, which states that through any 2 points, there exists exactly one line.

Since it is not possible for two lines to pass through both point *A* and point *B*, the assumption we have made is contradicted, and Theorem 4-1 must be correct.

In an indirect proof, it is often helpful to draw a diagram, just as you would for a 2-column, paragraph, or flowchart proof. A diagram is helpful in determining what assumptions should be made to prove the statement, and in finding the postulate, theorem, or definition that contradicts the assumption.

Example 2 Writing an Indirect Proof

Use the diagram to prove Theorem 47-2: If an inscribed angle intercepts a semicircle, then it is a right angle.

Given: $\overline{AC}$ is a diameter of $\odot M$.

Prove: $m\angle ABC \neq 90°$

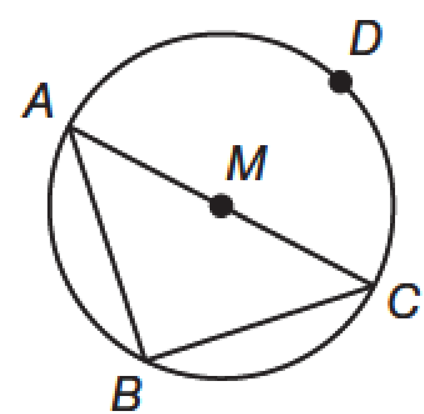
SOLUTION

Assume that $m\angle ABC \neq 90°$. By Theorem 47-1, this implies that $\widehat{ADC} \neq 180°$, since the arc is twice the measure of the inscribed angle.

It is given that $\overline{AC}$ is a diameter. Since $\overline{AC}$ goes through the center of the circle, $\angle AMC$ is the central angle that intercepts $\widehat{ADC}$.

Since $\angle AMC$ is a straight angle, its measure is 180°. An arc's measure is equal to the measure of its central angle, so the measure of $\widehat{ADC}$ must be 180°.

This contradicts our assumption.



Example 3 Writing an Indirect Proof

Use the diagram to prove Theorem 42-1: The perpendicular segment from a point to a line is the shortest segment from the point to the line.

Given: $\overline{AB} \perp \overleftrightarrow{BC}$

Prove: $\overline{AB}$ is the shortest segment from *A* to $\overleftrightarrow{BC}$.

SOLUTION

First, assume that there is another segment from *A* to $\overleftrightarrow{BC}$ that is shorter than $\overline{AB}$. In the diagram, this segment is shown as $\overline{AC}$. Our assumption is that $AC < AB$.

ΔABC is a right triangle, so by the Pythagorean Theorem, it must be true that $AB^2 + BC^2 = AC^2$.

Since $AB > AC$, and both *AB* and *AC* are greater than 0, by squaring both sides of the inequality we know that $AB^2 > AC^2$.

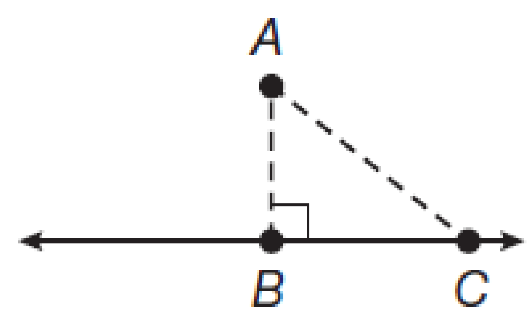
Using the Subtraction Property of Equality, subtract AB^2 from both sides. Then $BC^2 = AC^2 - AB^2$.

Since $AB^2 > AC^2$, $AC^2 - AB^2 < 0$.

Substituting shows that $BC^2 < 0$.

However, the length of *BC* must be greater than 0, so this contradicts the definition of a line segment.

Therefore, *AC* is not less than *AB*, and the theorem is true.



You Try!!!!!!

a. State the assumption you would make to start an indirect proof to show that $m\angle X = m\angle Y$.

b. State the assumption you would make to start an indirect proof to show that $\overleftrightarrow{AB} \perp \overleftrightarrow{CB}$.

You Try!!!!!!

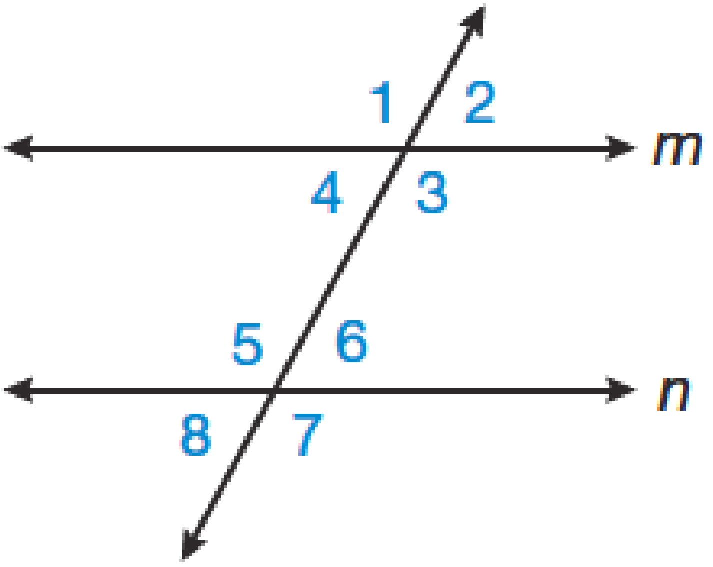
c. An isosceles triangle has at least two congruent sides. To prove this statement indirectly, assume an isosceles triangle does not have at least two congruent sides. What case needs to be explored to find a contradiction?

You Try!!!!!!

d. Use an indirect proof to prove that a triangle can have at most one right angle.

You Try!!!!!!

e. Use an indirect proof to show that $\angle 4 \cong \angle 6$, if $m \parallel n$.



Assignment

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Lesson Practice (Ask Mr. Heintz)

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Practice 1–30 (Do the starred ones first)