Lesson 49

Introduction to Solids

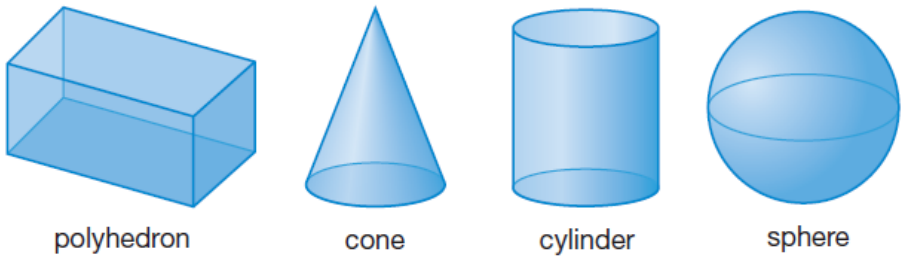
The figures discussed in previous lessons are two-dimensional figures. This lesson introduces three-dimensional figures called solids. Solids can have flat or curved surfaces.

Polyhedron – Any closed three-dimensional figure formed by four or more polygons that intersect only at their edges.

Cone – A three-dimensional figure with a circular base and a curved lateral surface that comes to a point.

Cylinder – A three-dimensional figure with two parallel circular bases and a curved lateral surface that connects the bases.

Sphere – The set of points in space that are a fixed distance from a given point, called the center of the sphere.



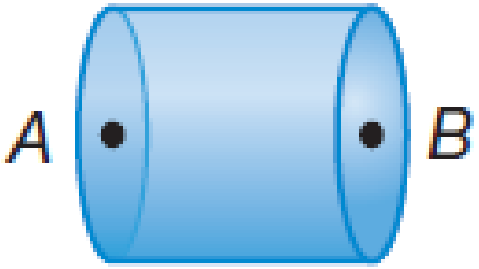
Math Reasoning

Write What are some common objects that are polyhedrons? Spheres?

Example 1 Classifying Solids

Classify each of the three-dimensional solids shown.

a.



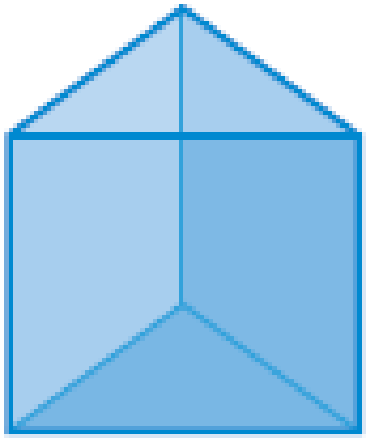
SOLUTION

The figure has two parallel circular bases, and a curved lateral surface. Therefore, the solid is a cylinder.

Example 1 Classifying Solids

Classify each of the three-dimensional solids shown.

b.



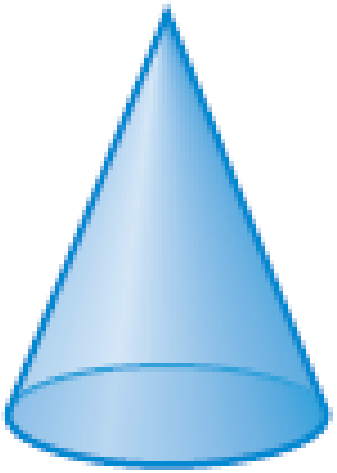
SOLUTION

The figure is made up of five polygons that meet at their edges. Therefore, the figure is a polyhedron.

Example 1 Classifying Solids

Classify each of the three-dimensional solids shown.

c.



SOLUTION

The figure has a circular base and a curved lateral surface that comes to a point. Therefore, the figure is a cone.

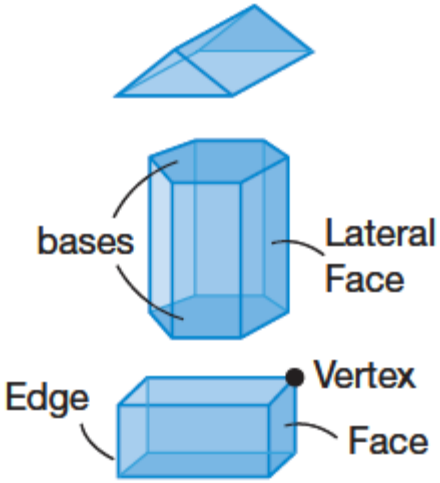
Face of the Polyhedron – Each flat surface of a polyhedron.

Edge – The segment that is the intersection of two faces of a solid.

Vertex – The point of intersection of three or more faces of the figure.

Prism – A polyhedron formed by two parallel congruent polygonal bases connected by lateral faces that are parallelograms.

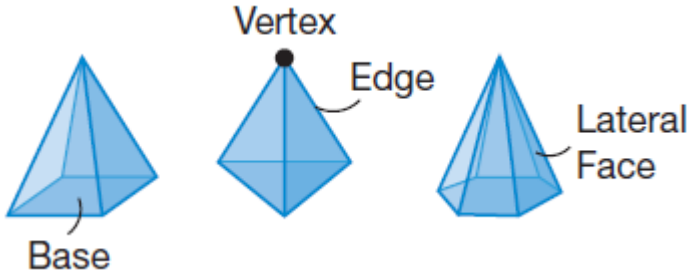
Base of a Prism – One of the two congruent parallel faces of the prism.



Lateral Face – Face of a prism that is not a base.

Pyramid – A polyhedron formed by a polygonal base and triangular lateral faces that meet at a common vertex. The faces of a pyramid all share a common vertex. The base is the side of the pyramid that does not share a single vertex with all of the other sides.

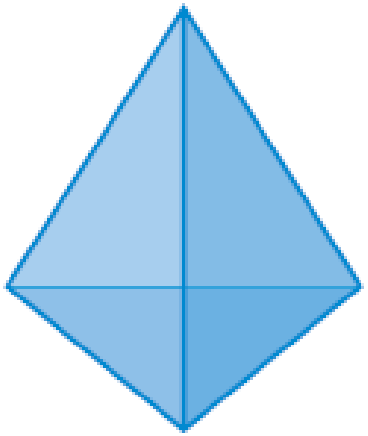
Prisms and pyramids are named by the shape of their bases. For example, a prism with a triangle for a base is called a triangular prism. A pyramid with a hexagon for a base would be called a hexagonal pyramid. A cube is the special name for a prism with six square faces.



Example 2 Classifying Polyhedra

Classify each polyhedron.

a.



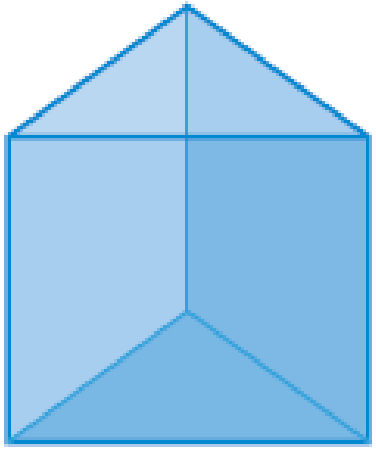
SOLUTION

The polyhedron has one base and the triangular faces meet at a common vertex. Therefore, the polyhedron is a pyramid. Since the base is a triangle, the polyhedron is a triangular pyramid.

Example 2 Classifying Polyhedra

Classify each polyhedron.

b.



SOLUTION

The polyhedron has two parallel bases and the lateral faces are parallelograms. Therefore, the polyhedron is a prism. Since the bases are triangles, the polyhedron is a triangular prism.

Regular Polyhedron – All of its faces are congruent, regular polygons.

Regular Pyramid – Its base is a regular polygon and its lateral faces are congruent isosceles triangles.

Regular Prism – Its base is regular and its faces are rectangles.

A cube is both a regular polyhedron and a regular rectangular prism.

A triangular prism with equilateral bases is a regular prism but is not a regular polyhedron, since its faces are not congruent to its bases.

Diagonal of a Polyhedron – A segment whose endpoints are the vertices of two different faces of a polyhedron.

Example 3 Describing Characteristics of Solids

Classify the polyhedron in the diagram, assuming all the angles of each pentagon are congruent. Is it a regular polyhedron? How many edges, vertices, and faces does it have? Name one diagonal segment of the polyhedron.

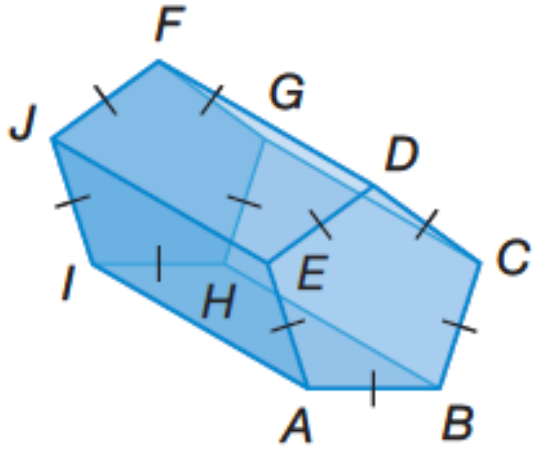
SOLUTION

The figure has two parallel pentagonal bases. Therefore, the polyhedron is a pentagonal prism.

The sides of the bases are all congruent, and it is given that the angles are congruent, so it is a regular prism.

Since the lateral faces are not congruent to the pentagonal bases, it is not a regular polyhedron.

It has 7 faces, 15 edges, and 10 vertices. One diagonal is the segment $\overline{BF}$.



A unique relationship exists among the number of faces, vertices, and edges of any polyhedron.

Euler's Formula – For any polyhedron with V vertices, E edges and F faces,

$$V - E + F = 2.$$

Example 4 Using Euler's Formula

How many faces does a polyhedron with 12 vertices and 18 edges have?

SOLUTION

Substitute $V = 12$ and $E = 18$ and solve for F.

$$V - E + F = 2$$

$$12 - 18 + F = 2$$

$$F = 8$$

The polyhedron has 8 faces.

Example 5 Application: Diamond Cutting

Diamonds are cut to change them from a rough stone into a gemstone. The figure below shows two steps in cutting a particular diamond. If each of the other vertices is cut in the next steps, what is the number of faces, vertices, and edges of the diamond in Step 4? Verify your answer.

SOLUTION

At the start the diamond has 4 faces, 4 vertices, and 6 edges.

After cutting in Step 1, the diamond has 5 faces, 6 vertices, and 9 edges.

After Step 2, the diamond has 6 faces, 8 vertices, and 12 edges.

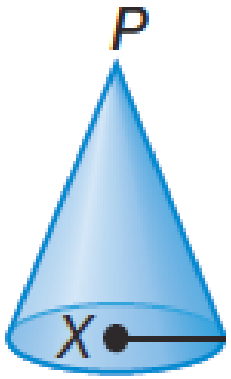
Since this pattern continues, after Step 3, the diamond will have 7 faces, 10 vertices, and 15 edges.

After Step 4, the diamond will have 8 faces, 12 vertices, and 18 edges.

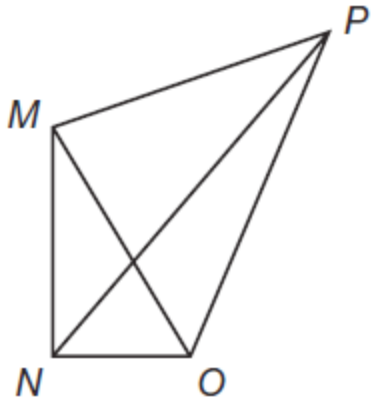
Euler's Formula can verify the relationship among the faces, vertices, and edges: $12 - 18 + 8 = 2$.

You Try!!!!!!

a.Classify the solid. Name its vertices, edges, and bases.

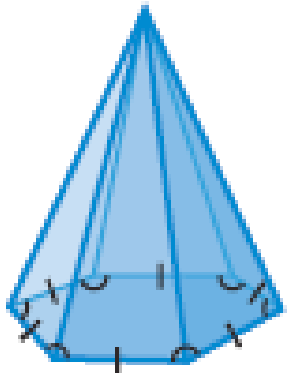


b.Classify the solid. How many vertices, edges, and bases does it have?

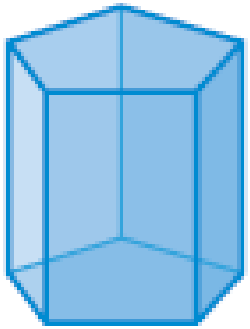


You Try!!!!!!

c.Classify the polyhedron. Determine whether it is a regular polyhedron.



d.Classify the polyhedron. Determine whether it is a regular polyhedron.

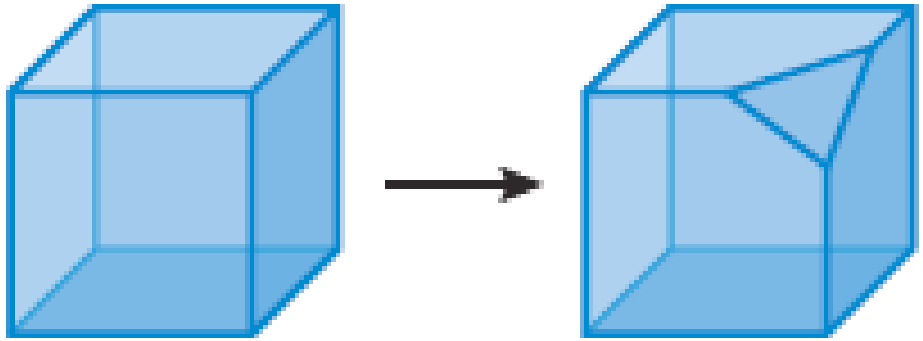


You Try!!!!!!

e.How many edges does a polyhedron with 14 vertices and 9 faces have?

You Try!!!!!!

f.Gemstones A gemstone is cut in the shape of a cube. Each vertex of the cube is then cut so that there is a triangular facet at each vertex. What is the number of faces, vertices and edges when the first four vertices of the cube are removed? Verify the results with Euler's Formula.



Assignment

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Lesson Practice (Ask Mr. Heintz)

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Practice 1-30 (Do the starred ones first)