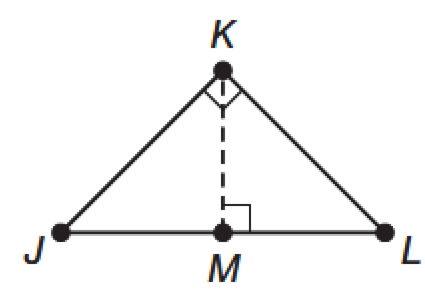
Lesson 50

Geometric Mean

When an altitude is drawn from the vertex of a right triangle's 90° angle to its hypotenuse, it splits the triangle into two right triangles that exhibit a useful relationship.

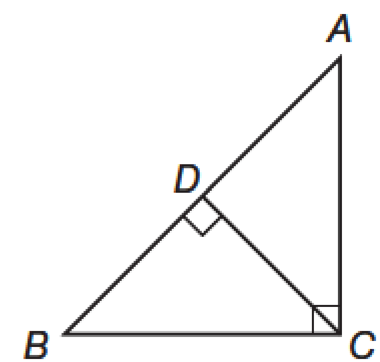
Theorem 50-1 – If the altitude is drawn to the hypotenuse of a right triangle, then the two triangles formed are similar to each other and to the original triangle.

In ΔJKL, for example, ΔJMK Is similar to ΔLMK, and both ΔJMK and ΔLMK are similar to ΔJKL.



Example 1 Proving Theorem 50-1

Given: $\overline{DC}$ is an altitude of ΔABC.
Prove: $\Delta ABC \sim \Delta CBD$, $\Delta ABC \sim \Delta ACD$, and $\Delta ACD \sim \Delta CBD$.
SOLUTION
In ΔABC, $\overline{CD} \perp \overline{AB}$ by the definition of an altitude.
All right angles are congruent, so $\angle BCA \cong \angle CDA$, $\angle BDC \cong \angle CDA$, and $\angle BCA \cong \angle BDC$.
By the Reflexive Property, $\angle B \cong \angle B$.
This is sufficient to show that $\Delta ABC \sim \Delta CBD$, by the AA Similarity Postulate.
Again, by the Reflexive Property $\angle A \cong \angle A$, so $\Delta ABC \sim \Delta ACD$, by the AA Similarity Postulate.
By the Transitive Property of Similarity, $\Delta ACD \sim \Delta CBD$ since both triangles are similar to ΔABC.



Example 2 Identifying Similar Right Triangles

Find *PS* and *PQ*.

SOLUTION

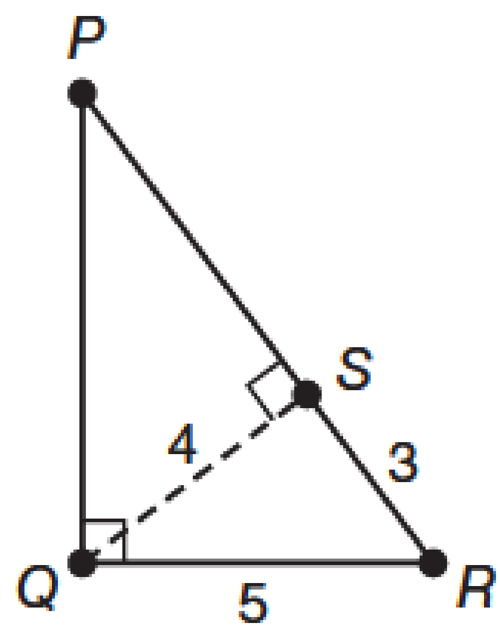
Since $\overline{QS}$ is a segment that is perpendicular to one side of the triangle with one endpoint on a vertex of the triangle, it is an altitude of ΔPQR. By Theorem 50-1, $\Delta PQR \sim \Delta PSQ \sim \Delta QSR$. Set up a proportion to solve for the missing sides.

$$\frac{SQ}{SR} = \frac{PQ}{QR} = \frac{PS}{QS}$$

$$\frac{4}{3} = \frac{PQ}{5} = \frac{PS}{4}$$

$$PQ = 6.\overline{6}$$

$$PS = 5.\overline{3}$$



Geometric Mean – When the means of a proportion are equal to one another. The geometric mean for positive numbers *a* and *b*, is the positive number *x* such that:

$$\frac{a}{x} = \frac{x}{b}$$

Math Reasoning

Write Take the cross product of the definition of the geometric mean and solve for *x*. What is another way to state the geometric mean of *a* and *b*, according to the formula you have found?

Example 3 Finding Geometric Mean

a. Find the geometric mean of 3 and 12.

SOLUTION

Using the definition of geometric mean, you can obtain the following algebraic expression, where *x* represents the geometric mean.

$$\frac{a}{x} = \frac{x}{b}$$

$$\frac{3}{x} = \frac{x}{12}$$

$$x \cdot x = 3 \cdot 12$$

$$x^2 = 36$$

$$x = \sqrt{36}$$

$$x = 6$$

Example 3 Finding Geometric Mean

b. Find the geometric mean of 2 and 9 to the nearest tenth.

SOLUTION

Using the definition of geometric mean, you can obtain the following algebraic expression, where *x* represents the geometric mean.

$$\frac{a}{x} = \frac{x}{b}$$

$$\frac{2}{x} = \frac{x}{9}$$

$$x \cdot x = 2 \cdot 9$$

$$x^2 = 18$$

$$x = \sqrt{18}$$

$$x \approx 4.2$$

$$x = 3\sqrt{2}$$

Math Reasoning

Formulate Write the answer to part *b* of Example 3 in simplified radical form.

Two corollaries to Theorem 50-1 use geometric means to relate the segments formed by the altitude of a right triangle to its hypotenuse.

Corollary 50-1-1 - If the altitude is drawn to the hypotenuse of a right triangle, then the length of the altitude is the geometric mean between the segments of the hypotenuse.

Corollary 50-1-2 - If the altitude is drawn to the hypotenuse of a right triangle, then the length of a leg is the geometric mean between the hypotenuse and the segment of the hypotenuse that is closer to that leg.

Example 4 Using Geometric Mean with Right Triangles

a. Given the triangle *STU*, find the missing value, *y*.

SOLUTION

Since *TV* is an altitude, by Corollary 50-1-1, *y* is the geometric mean of the segments of the hypotenuse, which are 3 and $\frac{4}{3}$. Using the definition of geometric mean, you can obtain the following algebraic expression.

$$\frac{a}{x} = \frac{x}{b}$$

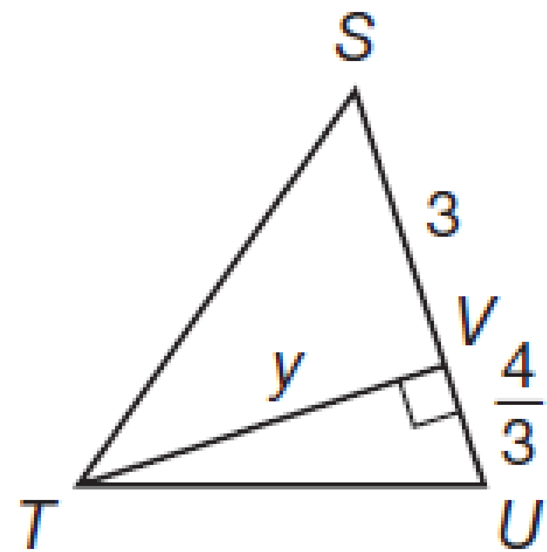
$$\frac{3}{y} = \frac{y}{\frac{4}{3}}$$

$$y \cdot y = 3 \cdot \frac{4}{3}$$

$$y^2 = 4$$

$$y = \sqrt{4}$$

$$y = 2$$



Example 4 Using Geometric Mean with Right Triangles

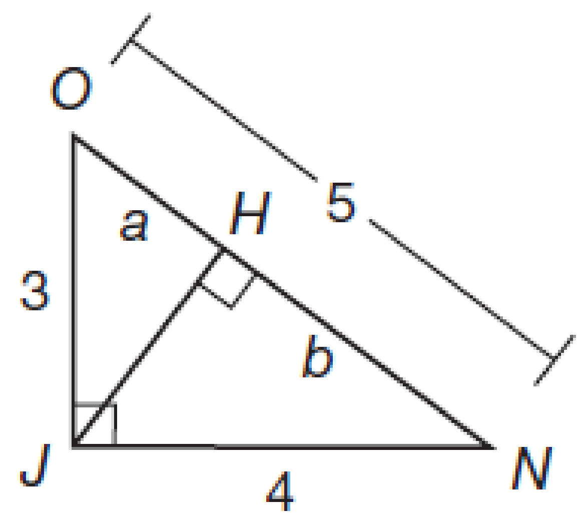
b. Given the triangle, find the missing values *a* and *b*.

SOLUTION

Since *JH* is an altitude, there are two relationships that can be derived from Corollary 50–1–2.

$$\frac{a}{3} = \frac{3}{5}$$
$$5a = 3 \cdot 3$$
$$5a = 9$$
$$a = \frac{9}{5}$$
$$a = 1.8$$

$$\frac{b}{4} = \frac{4}{5}$$
$$5b = 4 \cdot 4$$
$$5b = 16$$
$$b = \frac{16}{5}$$
$$b = 3.2$$



Example 5 Real World Application

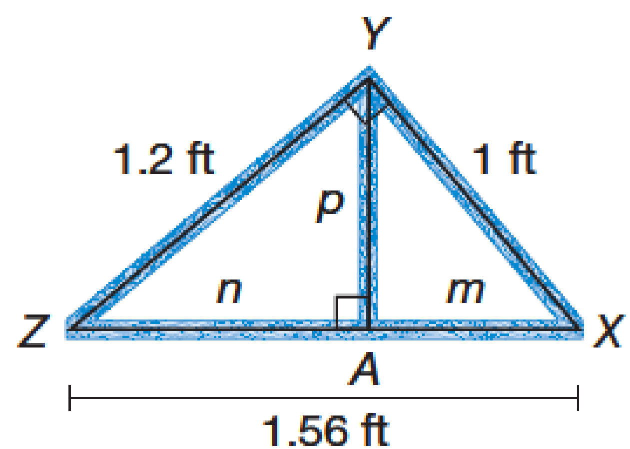
Jayden is building a truss for a shed, shown in the diagram. Jayden needs to find the lengths of the truss brace $\overline{AY}$, and the lengths of $\overline{XA}$ and $\overline{ZA}$.

SOLUTION

Since $\overline{AY}$ is an altitude to the triangle, then

$$\frac{n}{1.2} = \frac{1.2}{1.56}$$
$$1.56n = 1.2 \cdot 1.2$$
$$1.56n = 1.44$$
$$n = \frac{1.44}{1.56}$$
$$n \approx 0.92$$

$$\frac{m}{1} = \frac{1}{1.56}$$
$$1.56m = 1 \cdot 1$$
$$1.56m = 1$$
$$m = \frac{1}{1.56}$$
$$m \approx 0.64$$



Example 5 Real World Application

These are the lengths of $\overline{XA}$ and $\overline{ZA}$. To find the length of the truss brace $\overline{AY}$, apply Corollary 50-1-1.

$$\frac{m}{p} = \frac{p}{n}$$

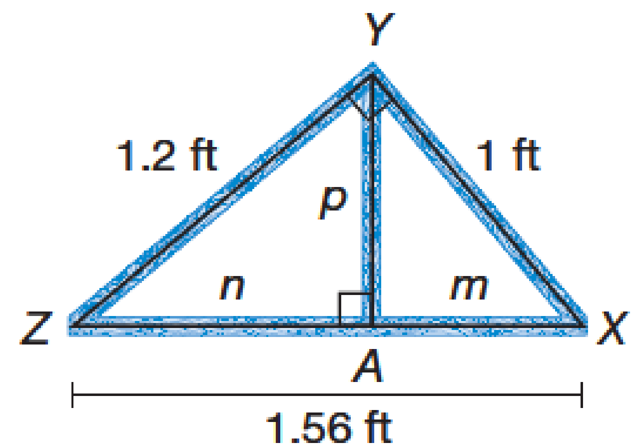
$$\frac{0.64}{p} \approx \frac{p}{0.92}$$

$$p \cdot p \approx 0.64 \cdot 0.92$$

$$p^2 \approx 0.5888$$

$$p \approx \sqrt{0.5888}$$

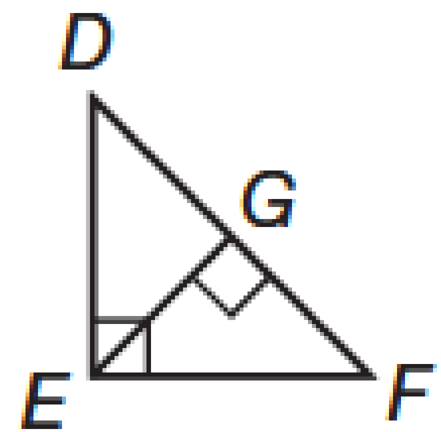
$$p \approx 0.77$$



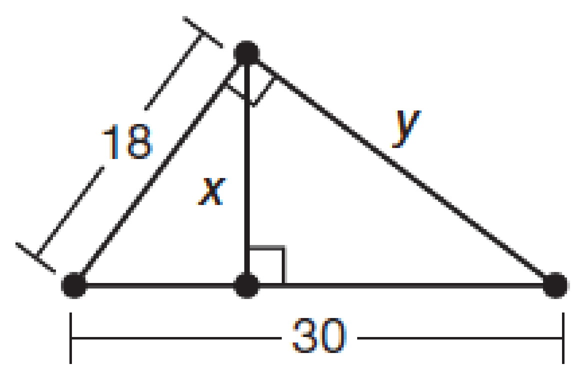
So, Jayden needs a brace that is 0.77 feet long, which will divide the truss into two pieces that are 0.64 feet long and 0.92 feet long, respectively.

You Try!!!!

a. Name the similar triangles.



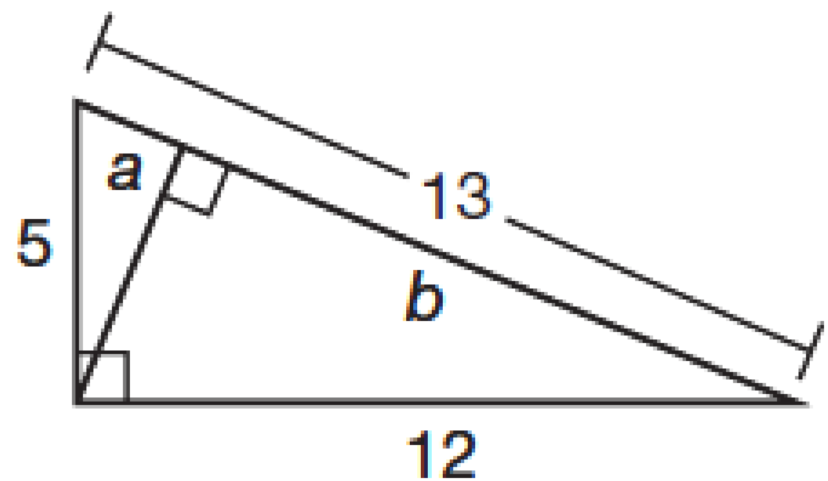
b. Find the values of *x* and *y*.



You Try!!!!

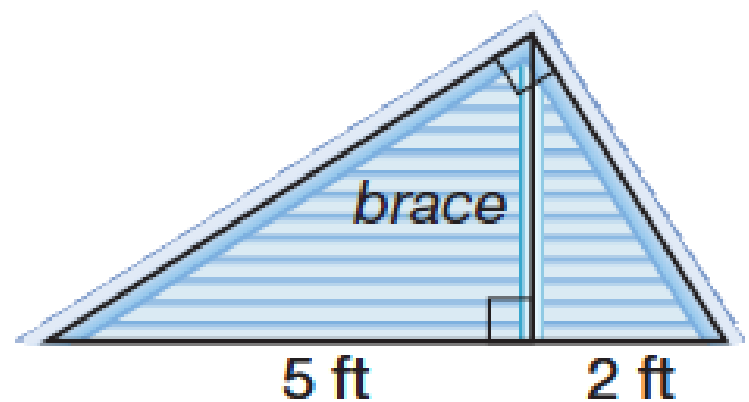
d.Find the geometric mean between 2 and 16 in simplified radical form.

f.Find the values of *a* and *b* to the nearest tenth.



You Try!!!!

g.To support an old roof, a brace must be installed at the altitude. Find the length of the brace to the nearest tenth of a foot.



Assignment

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Lesson Practice (Ask Mr. Heintz)

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Practice 1–30 (Do the starred ones first)