Name: _____

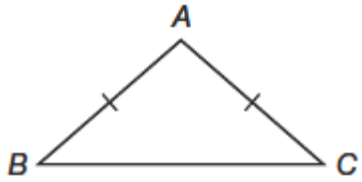
Date: _____

Period: _____

Geometry Lesson 51

Objective: TSW use properties of isosceles and equilateral triangles.

In an isosceles triangle, the sides and the angles of the triangle are classified by their position in relation to the triangle's congruent sides.

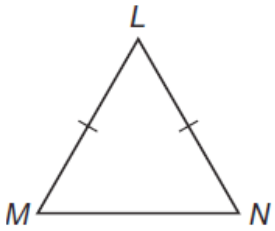


_____ of an Isosceles Triangle - One of the two congruent sides of the triangle. In the diagram, $\overline{AB}$ and $\overline{AC}$ are the legs.

_____ Angle of an Isosceles Triangle - The angle formed by the legs of the triangle. The vertex angle is $\angle A$.

_____ of an Isosceles Triangle - The side opposite the vertex angle. The base of ΔABC is $\overline{BC}$.

Base _____ of an Isosceles Triangle - One of the two angles that have the base of the triangle as a side. In ΔABC, $\angle B$ and $\angle C$ are base angles.



Theorem 51-1: Isosceles Triangle Theorem - If a triangle is isosceles, then its base angles are congruent. ΔLMN is isosceles. Therefore, $\angle M \cong \angle N$.

<u>Corollary 51-1-1 - If a triangle is equilateral, then it is equiangular.</u>

Example 1 Proving the Isosceles Triangle Theorem

Prove the Isosceles Triangle Theorem.

Given: ΔABC is an isosceles triangle with $\overline{AB} \cong \overline{AC}$.

D is the midpoint of $\overline{BC}$.

Prove: $\angle B \cong \angle C$

SOLUTION

	Statements	Reasons
1.		
2.		
3.		
4.		
5.		
6.		
7.		

Theorem 51-2: Converse of the Isosceles Triangle Theorem - If two angles of a triangle are congruent, then the sides opposite those angles are also congruent.

<u>Corollary 51-2-1 - If a triangle is equiangular, then it is equilateral.</u>

Example 2 Using the Isosceles Triangle Theorem and Its Converse

a. Triangle *DEF* is isosceles, and its vertex angle is at *E*. If m∠*D* = 36°, determine m∠*E* and m∠*F*.

SOLUTION

b. The perimeter of ΔGHJ is 12 inches, and $\angle G \cong \angle H$. If *GH* = 5 inches, find *GJ*.

SOLUTION

Example 3 Using Relationships in Equilateral Triangles

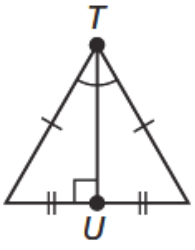
A triangle is equiangular and has a perimeter of 22.5 centimeters. Determine the length of each side.

SOLUTION

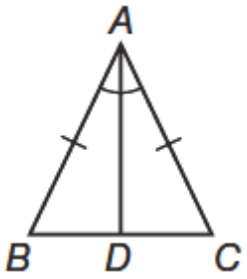
Theorem 51-3 - If a line bisects the vertex angle of an isosceles triangle, then it is the perpendicular bisector of the base.

Theorem 51-4 - If a line is the perpendicular bisector of the base of an isosceles triangle, then it bisects the vertex angle.

The diagram illustrates both of these theorems. The altitude $\overline{TU}$ bisects the vertex angle and is a perpendicular bisector of the base of the triangle.



Example 4 Proving Theorems 51-3 and 51-4



a. Prove Theorem 51-3.

Given: ΔABC is isosceles, $\overline{AD}$ bisects $\angle A$

Prove: $\overline{AD}$ is the perpendicular bisector of $\overline{BC}$

SOLUTION

Statements	Reasons
1.	
2.	
3.	
4.	
5.	
6.	
7.	
8.	
9.	
10.	
11.	

b. Write a paragraph proof of Theorem 51-4.

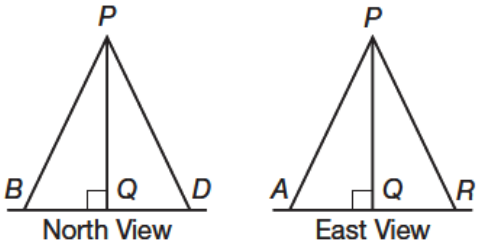
Given: ΔABC is isosceles, $\overline{AD}$ is the perpendicular bisector of $\overline{BC}$

Prove: $\overline{AD}$ bisects $\angle A$

SOLUTION

Example 5 Application: Infrastructure

This figure shows the north and east view of a telephone pole that is secured by four cables of equal length.



a. Explain why the base angles, $\angle PAQ$ and $\angle PRQ$, are congruent.

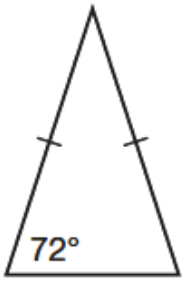
SOLUTION

b. Prove that these angles are also congruent to the base angles $\angle B$ and $\angle D$.

SOLUTION

You Try!!!!

a.For the isosceles triangle shown, determine the missing angle measures.



b. The perimeter of ΔXYZ is 15.2 centimeters, and $\angle X \cong \angle Z$. If XY = 6.3 centimeters, determine XZ.

c. If the vertex angle of an isosceles triangle measures 20°, what are the measures of each of its base angles?

d. A triangle is equiangular and its perimeter is 7 feet. Determine the length of each side.

e.Engineering: This diagram shows the side-view profile of a bridge. Determine the angle that each half of the bridge makes with the horizontal.