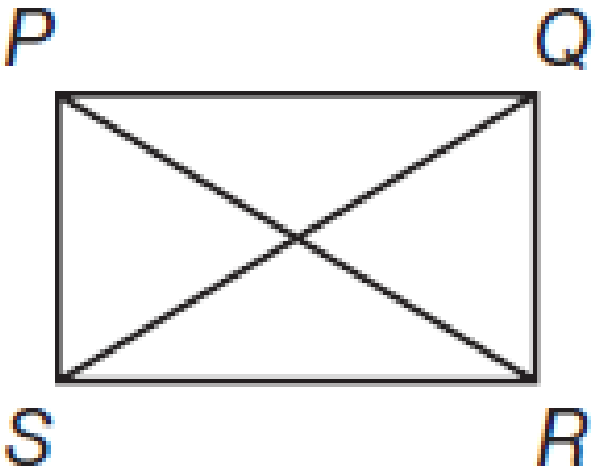
Lesson 52

Properties of Rectangles, Rhombuses, and Squares

The diagonals of parallelograms have special properties. Recall that a rhombus is a parallelogram with four congruent sides, a rectangle is a parallelogram with four right angles, and a square shares the properties of both a rectangle and a rhombus. One property of the diagonals of a parallelogram has already been introduced: they bisect each other. Three more are introduced in this lesson.

Properties of a Rectangle: Congruent Diagonals – The diagonals of a rectangle are congruent. $\overline{PR} \cong \overline{QS}$

If a quadrilateral is a parallelogram, it is a rectangle if and only if the above property is true.

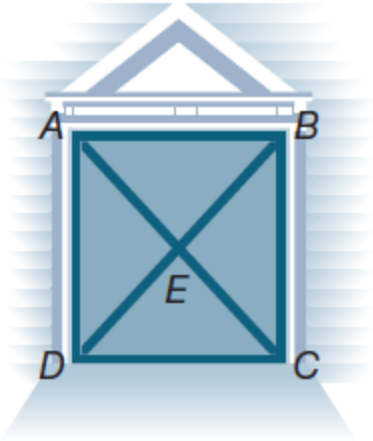


Example 1 Using Diagonals of a Rectangle

A rectangular barn door has diagonal braces. If AE is 6 feet, what is the length of $\overline{BD}$?

SOLUTION

$\overline{AC} \cong \overline{BD}$	Diag of a rectangle are congruent
$AE = EC$	Diag of a parallelogram bisect each other
$EC = 6$	Substitute.
$AC = 12$	Segment Addition Postulate
$BD = 12$	Def segment congruence



Exploration: Using Construction Techniques to Draw a Rhombus

In this exploration, you will use simple construction techniques to construct a quadrilateral, then classify it. You may wish to review Construction Lab 1 before this exploration.

1. Draw *JK*. Set your compass to *JK*. Place the compass point at *J* and draw an arc above $\overline{JK}$. Choose and label a point *L* on the arc. What is the relationship between *JK* and *JL*?

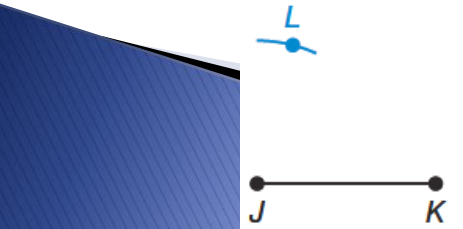
2. Place the compass point at *L* and draw an arc to the right of *L*.

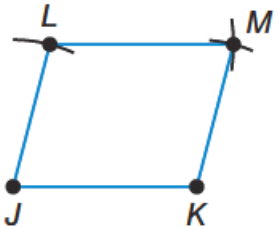
3. Place the compass point at *K* and draw an arc that intersects the arc you drew in step 2. Label the point of intersection *M*. How are *JK*, *KM*, *ML*, and *LJ* related?

4. How do you know that the quadrilateral you have drawn is a rhombus?

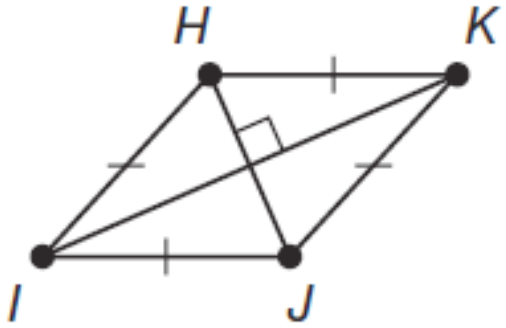
5. Draw the diagonals $\overline{JM}$ and $\overline{LK}$ and label their point of intersection *P*. Measure $\angle LPM$. What can you determine about the diagonals?

6. By measuring angles, determine the relationship between the diagonals and the angles of the rhombus.





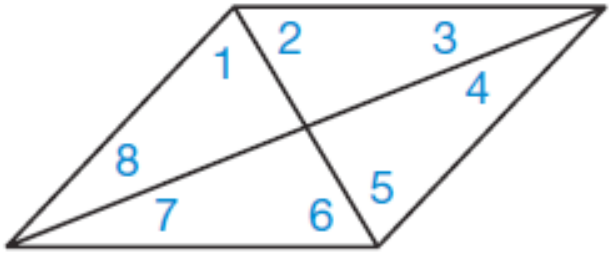
Properties of a Rhombus: Perpendicular Diagonals – The diagonals of a rhombus are perpendicular. $\overline{HJ} \perp \overline{IK}$



If a quadrilateral is a parallelogram, it is a rhombus if and only if the above property is true. Since a square is both a rhombus and a rectangle, its diagonals are both perpendicular and congruent.

Properties of a Rhombus: Diagonals as Angle Bisectors – Each diagonal of a rhombus bisects opposite angles. Because opposite angles of a rhombus are equal, when they are bisected by a diagonal, four congruent angles result.

$\angle 1 \cong \angle 2 \cong \angle 5 \cong \angle 6$, and $\angle 3 \cong \angle 4 \cong \angle 7 \cong \angle 8$.



If a quadrilateral is a parallelogram, it is a rhombus if and only if the above property is true.

Example 2 Using Properties of Diagonals of a Rhombus

BCDF is a rhombus. Find the measure of each angle.

a. m$\angle EBC$

SOLUTION

Since m$\angle BEC$ is 90°, then we know that m$\angle EBC$ + m$\angle ECB$ = 90°

$(3x + 12)° + (x + 10)° = 90°$ Substitute.

$4x + 22 = 90°$ Simplify.

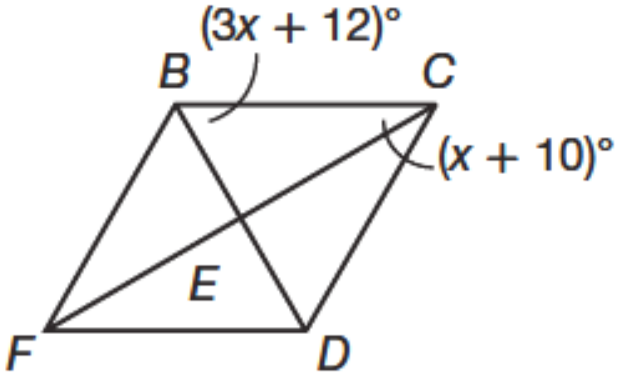
$x = 17$ Solve.

Now substitute the value of x to find the measure of $\angle EBC$.

m$\angle EBC$ = $3x + 12$

m$\angle EBC$ = $3(17) + 12$ Substitute for x.

m$\angle EBC$ = 63° Simplify.



Example 2 Using Properties of Diagonals of a Rhombus

BCDF is a rhombus. Find the measure of each angle.

b. $m\angle ECD$

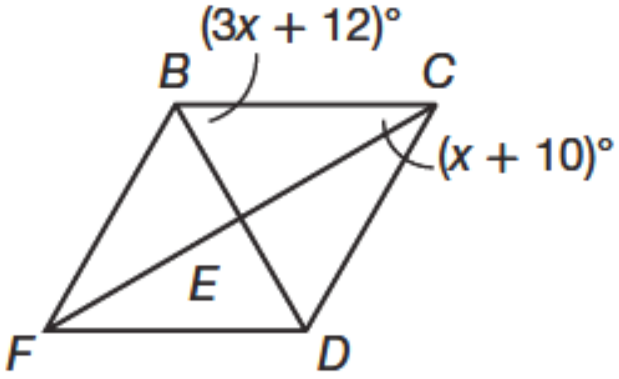
SOLUTION

Since the diagonals of a rhombus bisect the angles, $m\angle ECD = m\angle ECB$.

$m\angle ECD = x + 10$

$m\angle ECD = 17 + 10$

$m\angle ECD = 27°$



Example 3 Using Properties of Parallelograms

UVWX is a parallelogram. Decide what type of parallelogram it is by using the properties of rectangles and rhombuses.

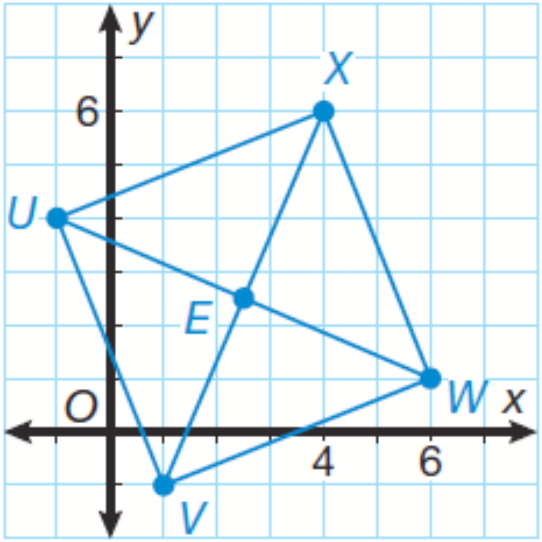
a. Determine whether the diagonals are congruent and classify the parallelogram.

SOLUTION

$UW = \sqrt{(-1-6)^2 + (4-1)^2} = \sqrt{58}$

$VX = \sqrt{(1-4)^2 + (-1-6)^2} = \sqrt{58}$

Since $UW = VX$, then the diagonals are congruent. By the Congruent Diagonals Property of a Rectangle, the shape must be a rectangle.



Example 3 Using Properties of Parallelograms

UVWX is a parallelogram. Decide what type of parallelogram it is by using the properties of rectangles and rhombuses.

b. Determine whether the diagonals are perpendicular and classify the parallelogram.

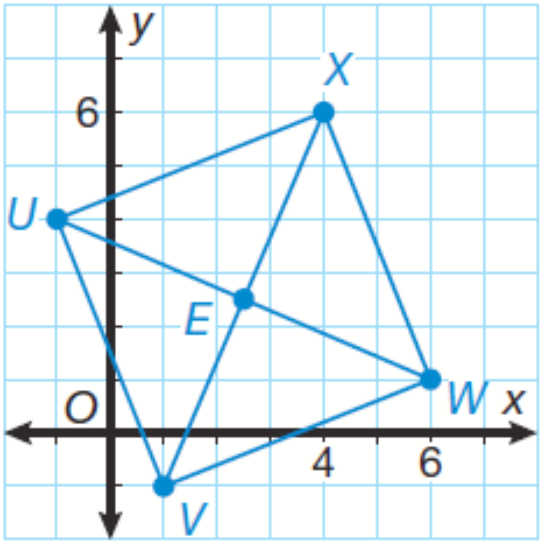
SOLUTION

$$slope\ of\ \overline{UW} = \frac{4-1}{-1-6} = -\frac{3}{7}$$

$$slope\ of\ \overline{VX} = \frac{-1-6}{1-4} = \frac{7}{3}$$

Since $-\frac{3}{7} \cdot \frac{7}{3} = -1$ $\overline{UW}$ is perpendicular to $\overline{VX}$.

This implies that the parallelogram is a rhombus. Since the shape is both a rectangle and a rhombus, it is also a square.

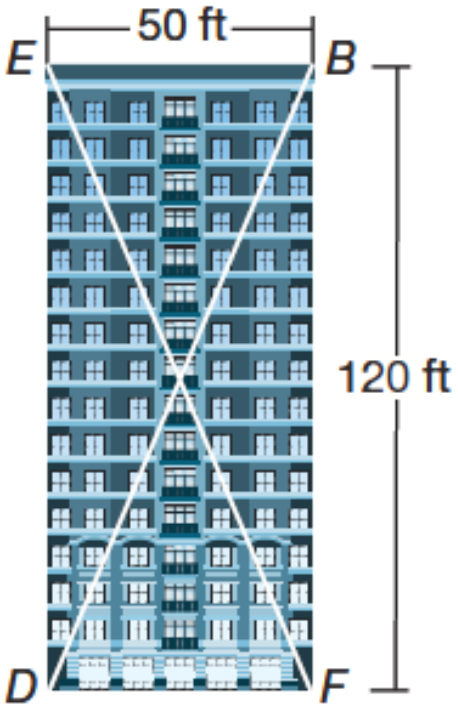


Example 4 Application: Architecture

A rectangular building is designed with steel support braces placed diagonally in the interior. Determine the length of the steel brace that will be used for diagonal $\overline{BD}$.

SOLUTION

$a^2 + b^2 = c^2$	Pythagorean Theorem
$50^2 + 120^2 = c^2$	Substitute
$c = 130$ ft	Solve
$EF = 130$ ft	Substitute
$\overline{EF} \cong \overline{BD}$	Diag of a rectangle are congruent
$BD = 130$ ft	Substitute



You Try!!!!!

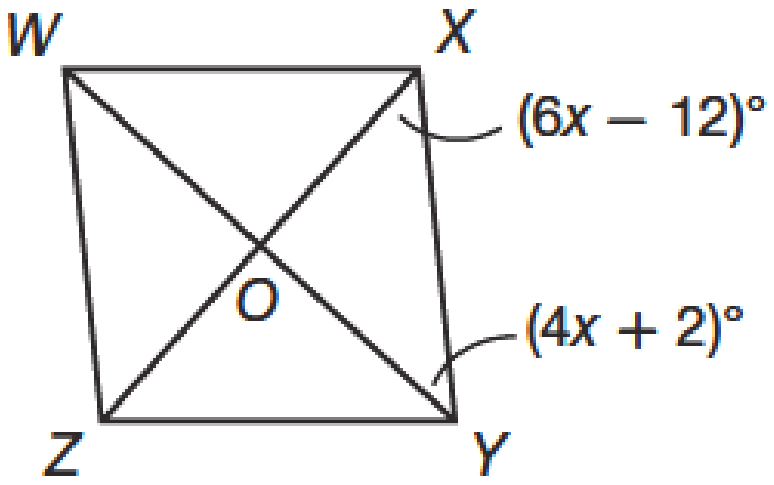
a.In rectangle *MNOP*, $MO = 5.4$ inches. What is the length of *NP*?

You Try!!!!!

WXYZ is a rhombus. Using the diagram, answer the questions that follow.

b.Find m$\angle OXY$.

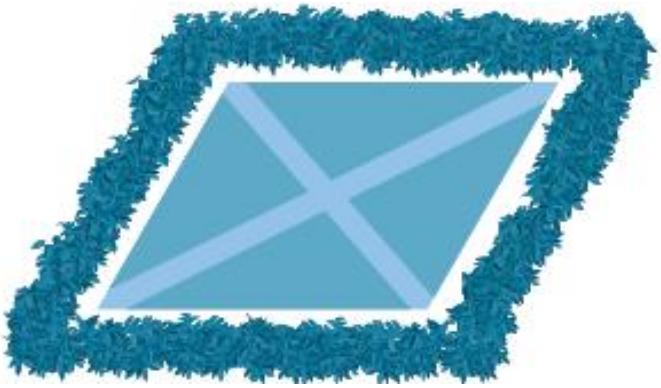
c.Find m$\angle OYZ$.



You Try!!!!!

d.Quadrilateral *RSTU* has a center point, *V*. If $\overline{RT} \cong \overline{SU}$, and $\overline{RT} \perp \overline{SU}$, classify the quadrilateral.

e.Architecture A building is made with a rhombus-shaped courtyard. If the longer diagonal walkway is 50 feet and the shorter one is 40 feet, what is the perimeter of the courtyard to the nearest foot?



Assignment

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Lesson Practice (Ask Mr. Heintz)

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Practice 1-30 (Do the starred ones first)