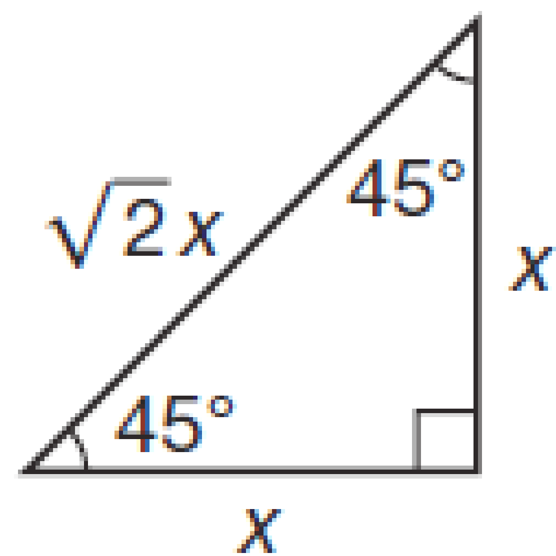
Lesson 53

45° – 45° – 90° Right Triangles

Some right triangles are used so frequently that it is helpful to remember some of their particular properties. These triangles are called special right triangles. The two most common special right triangles are the 30°-60°-90° triangle and the 45°-45°-90° triangle. Since the 45°-45°-90° triangle has two angles with equal measures, it is also an isosceles right triangle.

Properties of a 45°–45°–90° Triangle: Side Lengths – In a 45°–45°–90° right triangle, both legs are congruent and the length of the hypotenuse is the length of a leg multiplied by $\sqrt{2}$.

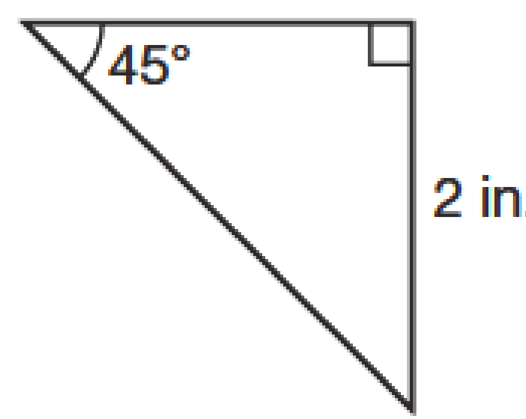


Example 1 Finding the Side Lengths in a 45°–45°–90° Triangle

a. Use the properties of a 45°–45°–90° right triangle to find the length of the hypotenuse of the triangle.

SOLUTION

The length of the hypotenuse is equal to the length of a leg times $\sqrt{2}$. Since the leg is 2 inches long, the hypotenuse has a length of $2\sqrt{2}$ inches.



Example 1 Finding the Side Lengths in a 45°–45°–90° Triangle

b. Use the properties of a 45°–45°–90° right triangle to find the length of a leg of the triangle.

SOLUTION

The length of the hypotenuse is equal to the length of the leg times $\sqrt{2}$. To find the length of a leg when given the hypotenuse, divide by $\sqrt{2}$ instead.

The length of a leg of the triangle is

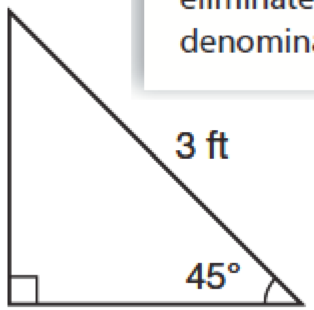
$$\frac{3}{\sqrt{2}}$$

$$\frac{3}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{3\sqrt{2}}{2}$$

$\frac{3\sqrt{2}}{2}$ feet.

Caution

When the denominator of a fraction has a square root in it, it must be rationalized. In this example, multiplying both the top and bottom of the fraction by $\sqrt{2}$ eliminates the root in the denominator.



Example 2 Finding the Perimeter of a 45°-45°-90° Triangle with Unknown Measures

Find the perimeter of the triangle.

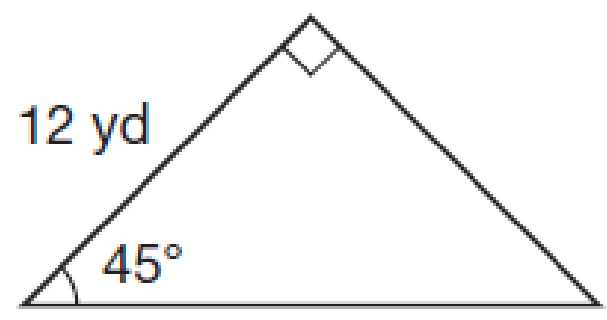
SOLUTION

The length of the hypotenuse is equal to the length of the leg times $\sqrt{2}$. Therefore, the hypotenuse is $12\sqrt{2}$ yards long.

The perimeter can be found by adding the lengths of the three sides together.

$$P = 12 + 12 + 12\sqrt{2}$$
$$P = 24 + 12\sqrt{2}$$
$$P \approx 40.97$$

Therefore, the perimeter is approximately 41 yards.



Though it is often faster to use the properties of 45°–45°–90° triangles to find unknown lengths, the Pythagorean Theorem can still be used to determine lengths in special right triangles.

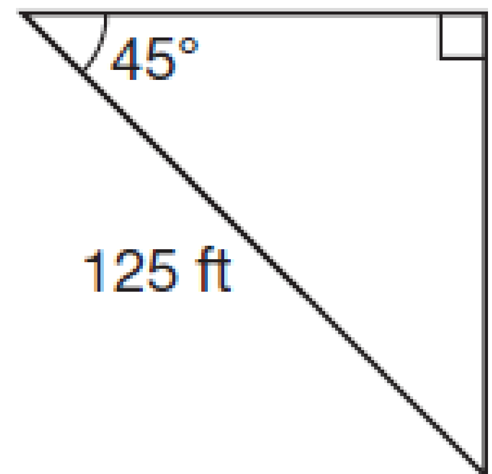
Example 3 Applying the Pythagorean Theorem with 45°–45°–90° Right Triangles

Find the length of the missing sides to the nearest foot, using the Pythagorean Theorem.

SOLUTION

Since the legs are congruent, let *x* represent the length of the legs of the triangle. Therefore,

$$a^2 + b^2 = c^2$$
$$x^2 + x^2 = 125^2$$
$$2x^2 = 15625$$
$$\frac{2x^2}{2} = \frac{15625}{2}$$
$$x^2 = 7812.5$$
$$x^2 \approx 88.4$$



Math Reasoning

Verify Use the properties of 45°-45°-90° triangles to find *x*. Is the result the same?

Therefore, the missing side length is approximately 88 feet.

Example 4 Application: Park Construction

A square park is to be fenced around the perimeter with a snow fence for an upcoming outdoor concert. There is a diagonal path that is 430 feet long through the park. How much snow fence is required? Use the 4-Step Problem-Solving Process.

SOLUTION

Understand: The fence is to be placed around the perimeter, so the perimeter must be found. The diagonal of the square is given. A diagram would be a helpful visual aid to understand this problem.

Plan: First, draw a diagram. Identify the lengths that need to be found and use the properties of 45°-45°-90° triangles to solve for them. Add the length of each side together to find the perimeter.

Solve: This involves finding the length of the legs of the 45°-45°-90° triangle created by two adjacent sides of the park and the diagonal path.

$$430 = x\sqrt{2}$$

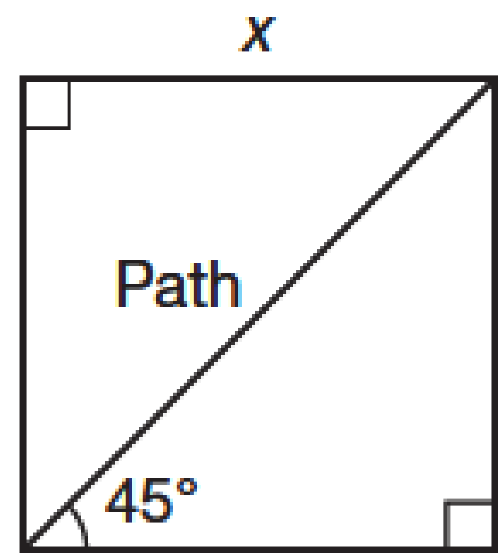
$$\frac{430}{\sqrt{2}} = \frac{x\sqrt{2}}{\sqrt{2}}$$

$$\frac{430}{\sqrt{2}} = x$$

$$\frac{430}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = x$$

$$\frac{430\sqrt{2}}{2} = x$$

$$215\sqrt{2} = x$$



Example 4 Application: Park Construction

Therefore, the length of the side of the square park is $215\sqrt{2}$ feet. To find the perimeter of the park, the formula for the perimeter of a square will be used.

$$P = 4l$$

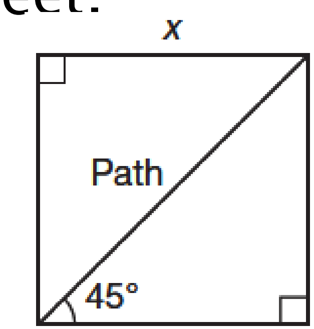
$$P = 4\left(215\sqrt{2}\right)$$

$$P = 860\sqrt{2}$$

$$P \approx 1216.22$$

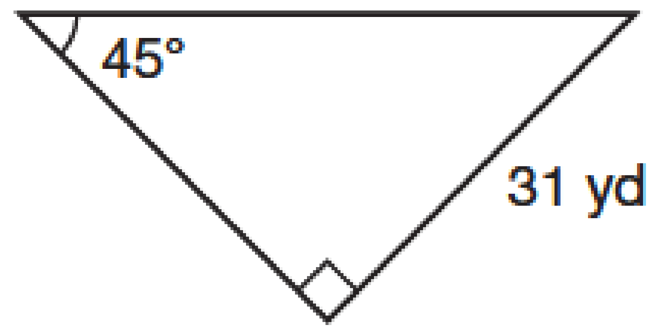
Therefore, the perimeter of the park, and thus the amount of fencing needed, is approximately 1216 feet.

Check: Here, since the diagonal is longer than a side, each side must be less than 430 feet. So the perimeter must be less than 4×430, or 1720 feet. The answer of 1216 feet seems to make sense because it is less than 1720 feet.

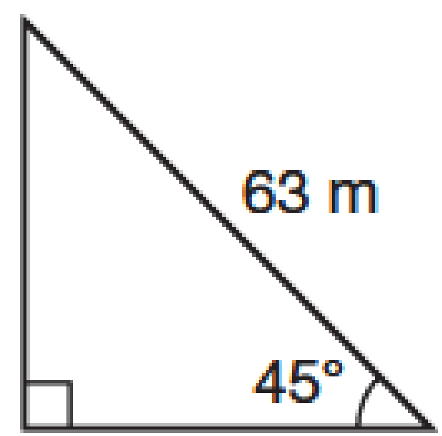


You Try!!!!!

a. Find the length of this triangle's hypotenuse.

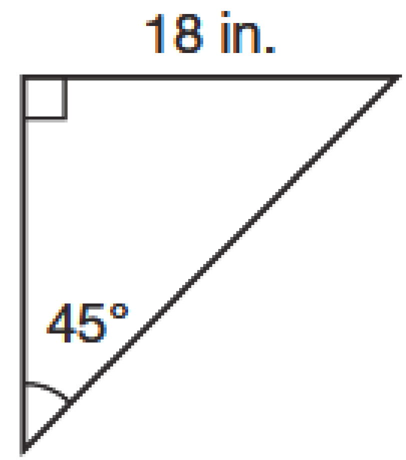


b. Find the length of this triangle's missing side.

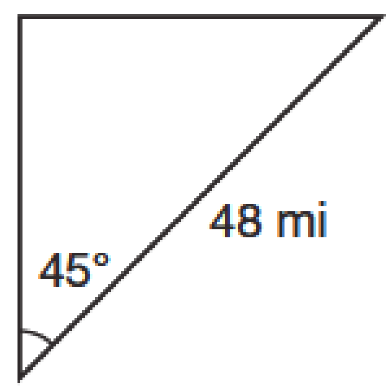


You Try!!!!!

c. Find the perimeter of the triangle, to the nearest tenth of an inch.



d. Find the length of the missing sides to the nearest mile.



You Try!!!!!

e. A square building has a diagonal length of 150 feet. What would be the square footage of one floor of the building?

Assignment

Page 351

Lesson Practice (Ask Mr. Heintz)

Page 352

Practice 1–30 (Do the starred ones first)