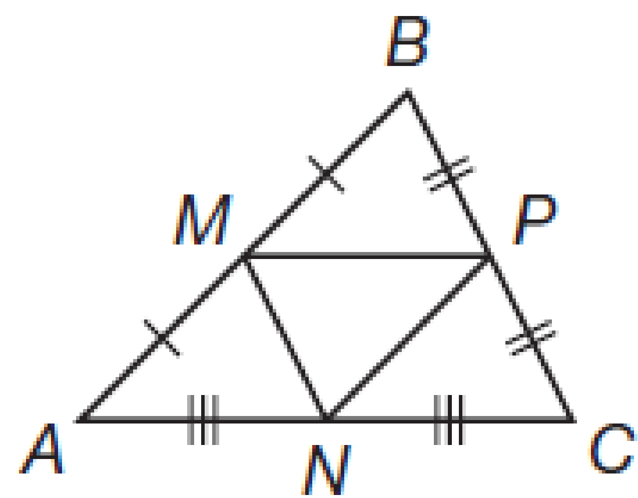
Lesson 55

Triangle Midsegment Theorem

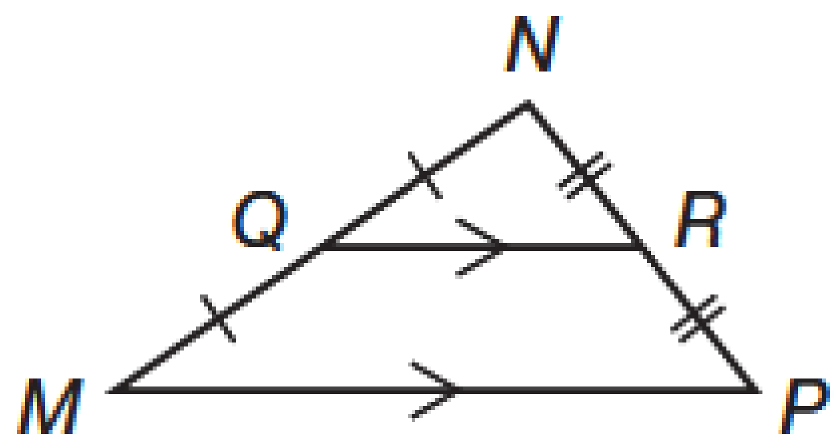
Midsegment of a Triangle – A segment that joins the midpoints of two sides of the triangle. Every triangle has three midsegments.

The midsegment is always half the length of the side that does not have a midsegment endpoint on it.



Theorem 55–1: Triangle Midsegment Theorem – The segment joining the midpoints of two sides of a triangle is parallel to, and half the length of, the third side.

$\overline{RQ} \parallel \overline{PM}$ and $RQ = \frac{1}{2}PM$



Example 1 Using the Triangle Midsegment Theorem

In the diagram, $\overline{DE}$ is a midsegment of ΔABC. Find the values of x and y.

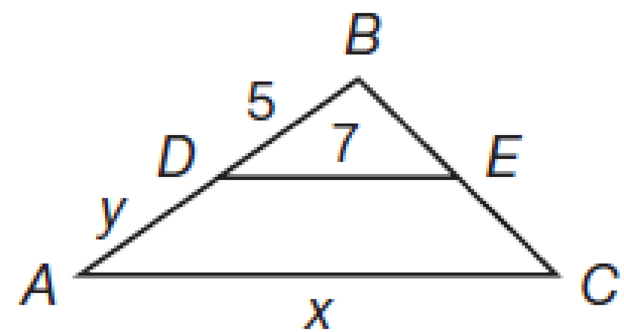
SOLUTION

From the Triangle Midsegment Theorem,

$DE = \frac{1}{2}AC$, so $AC = 2DE$.

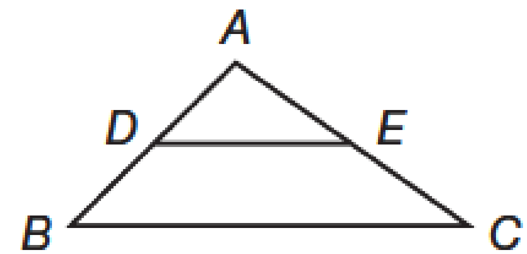
$AC = 2(7)$

Therefore, $x = 14$.



From the definition of a midsegment, $AD = DB$. So, $y = 5$.

Example 2 Proving the Triangle Midsegment Theorem



Given: D is the midpoint of $\overline{AB}$ and E is the midpoint of $\overline{AC}$.

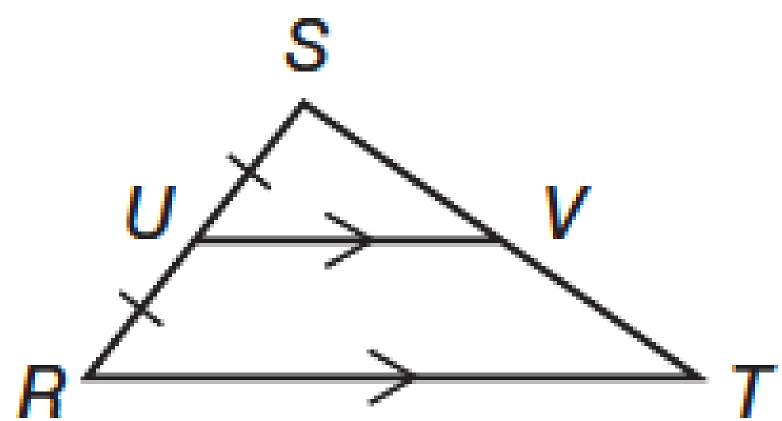
Prove: $\overline{DE} \parallel \overline{BC}$ and $DE = \frac{1}{2}BC$

SOLUTION

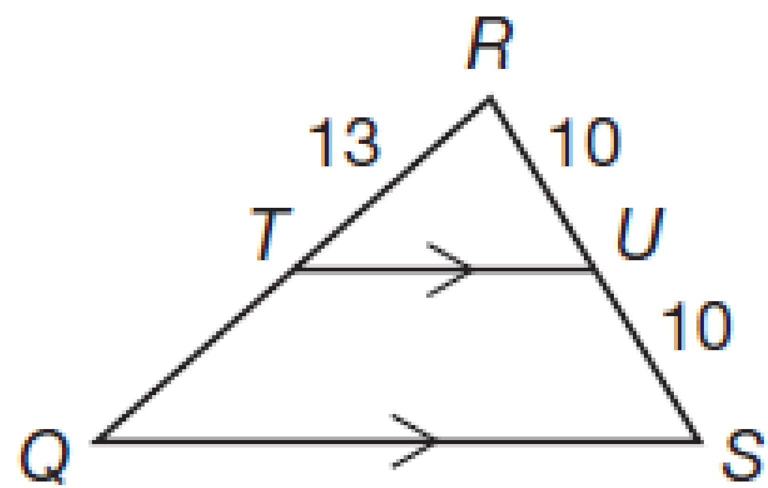
Statements	Reasons
1. D is the midpoint of $\overline{AB}$; E is the midpoint of $\overline{AC}$	1. Given
2. $AD = DB$; $AE = EC$	2. Definition of midpoint
3. $AD + DB = AB$; $AE + EC = AC$	*3*. Segment Addition Postulate
4. $AD + AD = AB$; $AE + AE = AC$	4. Substitute
5. $AD = \frac{1}{2}AB$	5. Solve
6. $\angle A \cong \angle A$	6. Reflexive Property of Congruence
7. $\Delta ABC \sim \Delta ADE$	7. SAS Triangle Similarity Theorem
8. $\angle AED \cong \angle ACB$	8. Definition of similar polygons
9. $\overline{DE} \parallel \overline{BC}$	9. If corresponding angles are congruent, lines cut by a transversal are parallel
10. $DE = \frac{1}{2}BC$ and step 5	10. Definition of similar polygons

Theorem 55-2 – If a line is parallel to one side of a triangle and it contains the midpoint of another side, then it passes through the midpoint of the third side.

Since $\overline{UV} \parallel \overline{RT}$ and $\overline{RU} \cong \overline{US}$, $\overline{SV} \cong \overline{VT}$.



The measure of $\overline{QT}$ can be determined using Theorem 55-2. Since $RU = US$ in triangle *QRS*, then *U* is the midpoint of $\overline{RS}$. By Theorem 55-2, since $\overline{TU} \parallel \overline{QS}$ and *U* is the midpoint of $\overline{RS}$, then *T* is the midpoint of $\overline{QR}$. Since *T* is the midpoint of $\overline{QR}$, then $QT = TR$. The measure of $\overline{QT}$ is 13 units.



Example 3 Identifying Midpoints of Sides of a Triangle

Triangle *MNP* has vertices *M*(−2, 4), *N*(6, 2), and *P*(2, −1).

$\overline{QR}$ is a midsegment of ΔMNP.

Find the coordinates of *Q* and *R*.

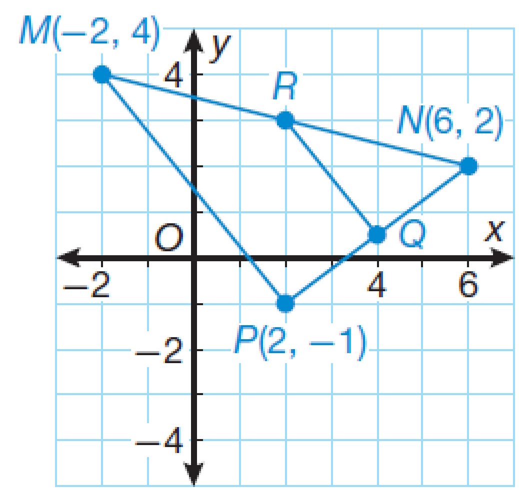
SOLUTION

R and *Q* are the midpoints of $\overline{MN}$ and $\overline{NP}$.

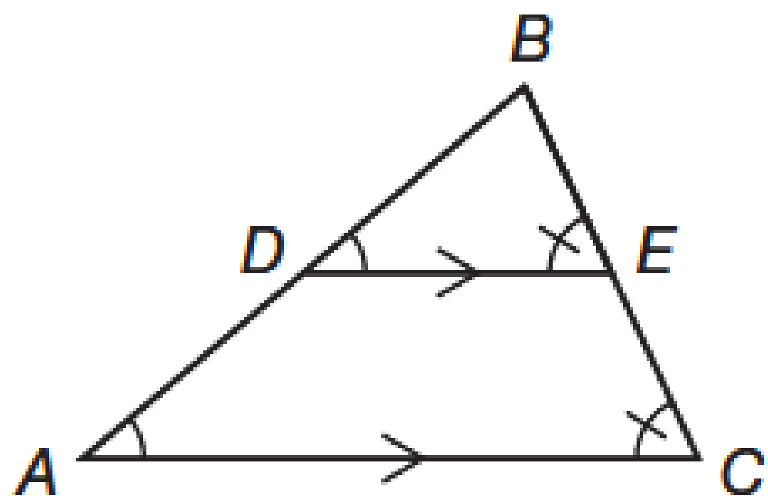
Use the Midpoint Formula to find the coordinates of *Q* and *R*.

$$Q\left(\frac{6+2}{2}, \frac{2+(-1)}{2}\right) = Q\left(4, \frac{1}{2}\right)$$

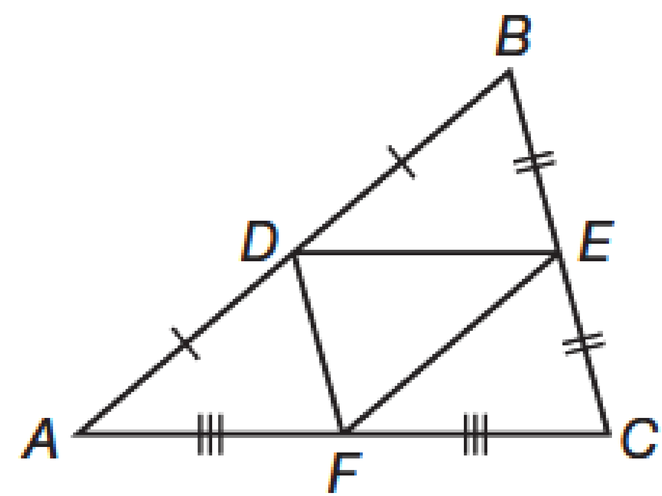
$$R\left(\frac{-2+6}{2}, \frac{4+2}{2}\right) = R(2,3)$$



The midsegment of a triangle creates two triangles that are similar by AA–Similarity. In the diagram, since $\overline{DE} \parallel \overline{AC}$, then $\angle BAC \cong \angle BDE$ and $\angle BED \cong \angle BCA$. This shows that $\Delta ABC \sim \Delta DBE$.



A midsegment triangle is the triangle formed by the three midsegments of a triangle. Triangle *DEF* is a midsegment triangle. Midsegment triangles are similar to the original triangle and to the triangles formed by each midsegment. In the figure, $\Delta ABC \sim \Delta EFD \sim \Delta ADF \sim \Delta DBE \sim \Delta FEC$.



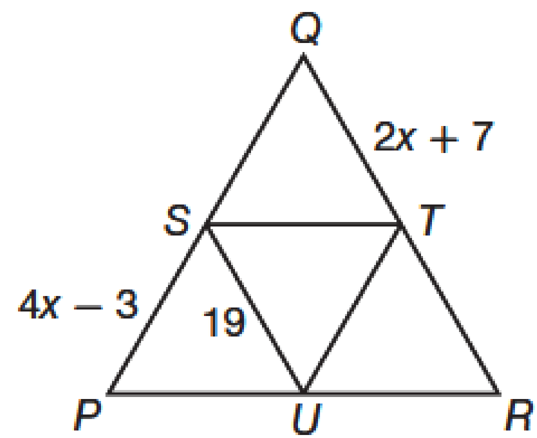
Example 4 Applying Similarity to Midsegment Triangles

Triangle STU is the midsegment triangle of ΔPQR.

a. Show that $\Delta STU \sim \Delta PQR$.

SOLUTION

Since ΔSTU is the midsegment triangle of ΔPQR, by the definition of midsegment:



$ST = \frac{1}{2}PR$

$SU = \frac{1}{2}PR$

$TU = \frac{1}{2}QP$

Therefore, $\Delta STU \sim \Delta PQR$ by SSS similarity.

Example 4 Applying Similarity to Midsegment Triangles

Triangle *STU* is the midsegment triangle of ΔPQR.

b. Find *PQ*.

SOLUTION

QR is twice *SU*, and *T* is the midpoint of $\overline{QR}$, so *QT* = *SU*.

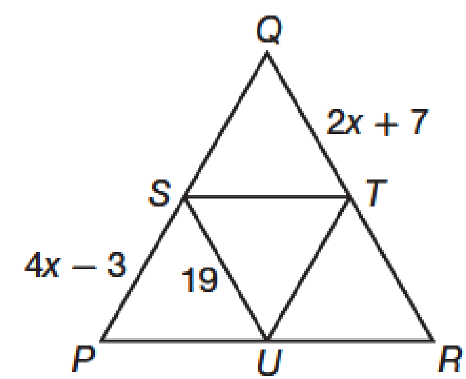
$2x + 7 = 19$

$x = 6$

Since *S* is the midpoint of $\overline{PQ}$, $PQ = 2PS$.

$PQ = 2(4x - 3) = 2[4(6) - 3] = 42$

The length of $\overline{PQ}$ *PQ* is 42 units.



Example 5 Application: Maps

A student determined that Toledo Street is the midsegment of the triangle formed by Columbus Avenue, Park Avenue, and William Street. The distance along Park Avenue between Columbus Avenue and William Street is 160 meters, and the distance along Toledo Street in the same span is 80 meters. William Street is 240 meters long. Find the distance from the corner of Columbus and William to the corner of William and Toledo (*x* in the diagram) to the nearest meter.

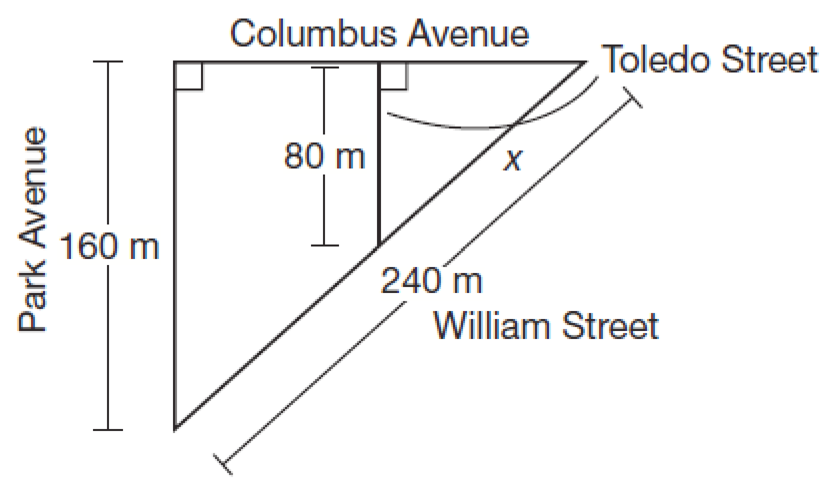
SOLUTION

Since Toledo Street is the midsegment of the triangle, it creates two similar triangles. Use the lengths of Toledo Street and Park Avenue to find the similarity ratio of 160:80. Name the shorter segment *x* and write a proportion.

$\frac{160}{80} = \frac{240}{X}$

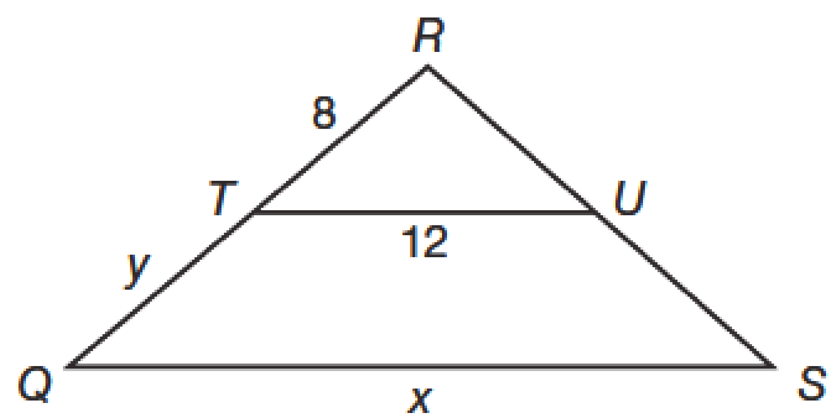
$160x = (80)(240)$

$x = 120$ m



You Try!!!!

a. $\overline{TU}$ is a midsegment of ΔQRS. Find the values of x and y.

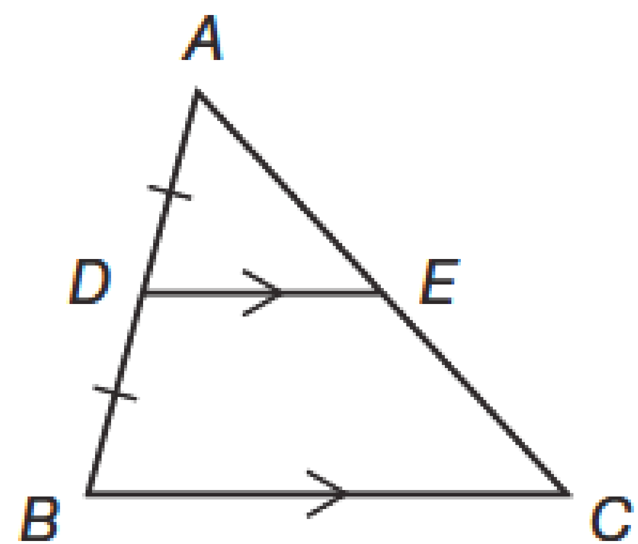


You Try!!!!

b. Prove Theorem 55–2.

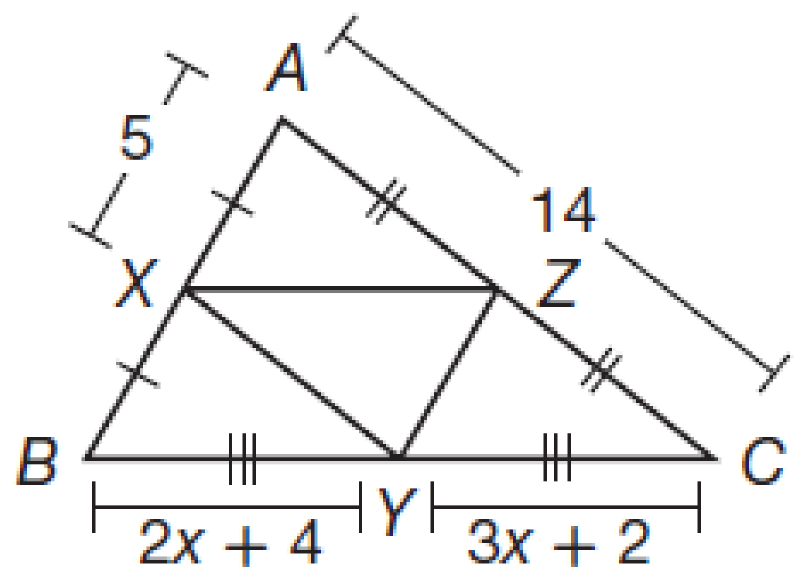
Given: $\overline{DE} \parallel \overline{BC}$ and D is the midpoint of $\overline{AB}$.

Prove: E is the midpoint of $\overline{AC}$.



You Try!!!!

d. Find the perimeter of the midsegment triangle, ΔXYZ.



Assignment

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Lesson Practice (Ask Mr. Heintz)

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Practice 1–30 (Do the starred ones first)