Name: ___________________________

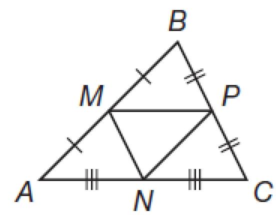
Date: ___________________________

Period: _________________________

Geometry Lesson 55

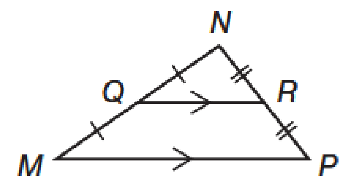
Objective: TSW use the triangle midsegment theorem.

__________________________ of a Triangle – A segment that joins the midpoints of two sides of the triangle. Every triangle has three midsegments.



The midsegment is always ___________ the length of the side that does not have a midsegment endpoint on it.

Theorem 55-1: Triangle Midsegment Theorem - The segment joining the midpoints of two sides of a triangle is parallel to, and half the length of, the third side.



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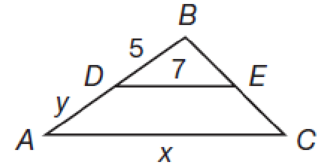
Example 1 Using the Triangle Midsegment Theorem

In the diagram, $\overline{DE}$ is a midsegment of ΔABC. Find the values of *x* and *y*.

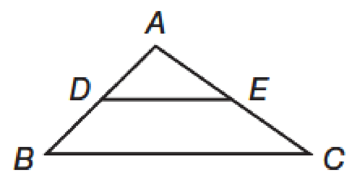
SOLUTION



Example 2 Proving the Triangle Midsegment Theorem

Given: *D* is the midpoint of $\overline{AB}$ and *E* is the midpoint of $\overline{AC}$.

Prove: $\overline{DE} \parallel \overline{BC}$ and $DE = \frac{1}{2}BC$

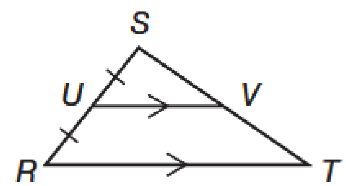


SOLUTION

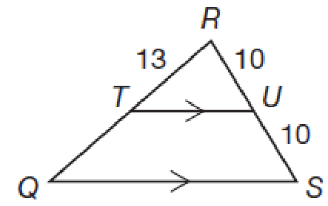
Statements	Reasons
1.	
2.	
3.	
4.	
5.	
6.	
7.	
8.	
9.	
10.	

Theorem 55-2 - If a line is parallel to one side of a triangle and it contains the midpoint of another side, then it passes through the midpoint of the third side.

Since $\overline{UV} \parallel \overline{RT}$ and $\overline{RU} \cong \overline{US}$, ______________________



The measure of $\overline{QT}$ can be determined using Theorem 55-2. Since $RU = US$ in triangle *QRS*, then *U* is the midpoint of _____. By Theorem 55-2, since $\overline{TU} \parallel \overline{QS}$ and *U* is the midpoint of $\overline{RS}$, then *T* is the midpoint of $\overline{QR}$. Since *T* is the midpoint of $\overline{QR}$, then *QT* = *TR*. The measure of $\overline{QT}$ is ______ units.



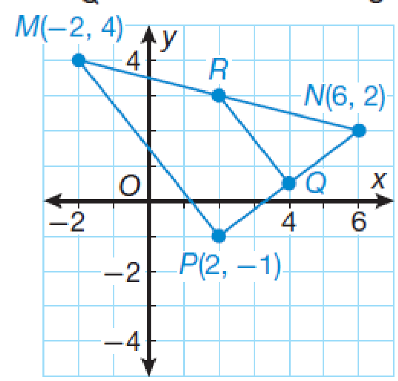
Example 3 Identifying Midpoints of Sides of a Triangle

Triangle *MNP* has vertices *M*(-2, 4), *N*(6, 2), and *P*(2, -1).

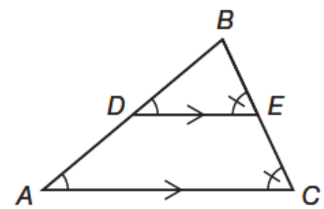
$\overline{QR}$ is a midsegment of ΔMNP.

Find the coordinates of *Q* and *R*.

SOLUTION

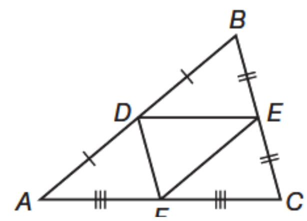


The midsegment of a triangle creates two triangles that are similar by AA-Similarity. In the diagram, since $\overline{DE} \parallel \overline{AC}$, then $\angle BAC \cong \angle BDE$ and $\angle BED \cong \angle BCA$. This shows that ___________________.



A midsegment triangle is the triangle formed by the three midsegments of a triangle. Triangle *DEF* is a midsegment triangle. Midsegment triangles are similar to the original triangle and to the triangles formed by each midsegment. In the figure,

____

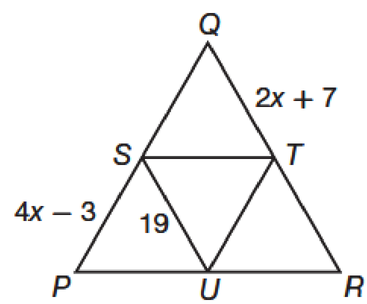


Example 4 Applying Similarity to Midsegment Triangles

Triangle *STU* is the midsegment triangle of ΔPQR.

a. Show that $\Delta STU \sim \Delta PQR$.

SOLUTION



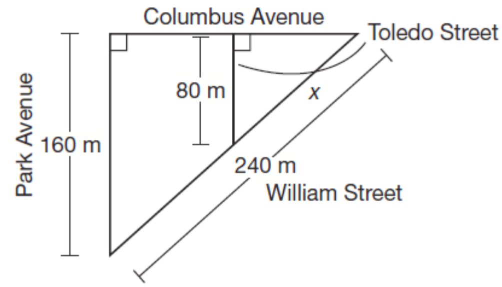
b. Find *PQ*.

SOLUTION

Hint

It may help to sketch the similar triangles separately so they can be compared more easily.

Example 5 Application: Maps

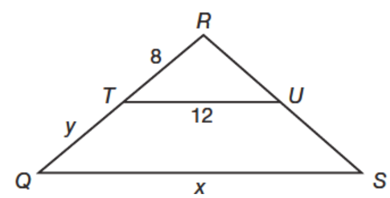


A student determined that Toledo Street is the midsegment of the triangle formed by Columbus Avenue, Park Avenue, and William Street. The distance along Park Avenue between Columbus Avenue and William Street is 160 meters, and the distance along Toledo Street in the same span is 80 meters. William Street is 240 meters long. Find the distance from the corner of Columbus and William to the corner of William and Toledo (*x* in the diagram) to the nearest meter.

SOLUTION

You Try!!!!

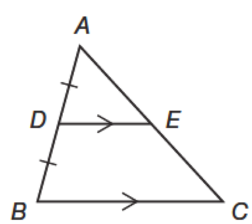
a. $\overline{TU}$ is a midsegment of ΔQRS. Find the values of *x* and *y*.



b. Prove Theorem 55-2.

Given: $\overline{DE} \parallel \overline{BC}$ and *D* is the midpoint of $\overline{AB}$.

Prove: *E* is the midpoint of $\overline{AC}$.



d.Find the perimeter of the midsegment triangle, ΔXYZ.