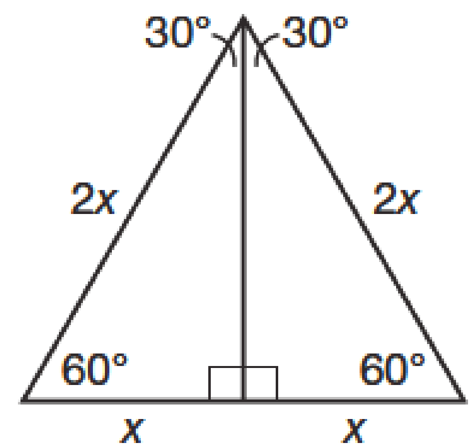
Lesson 56

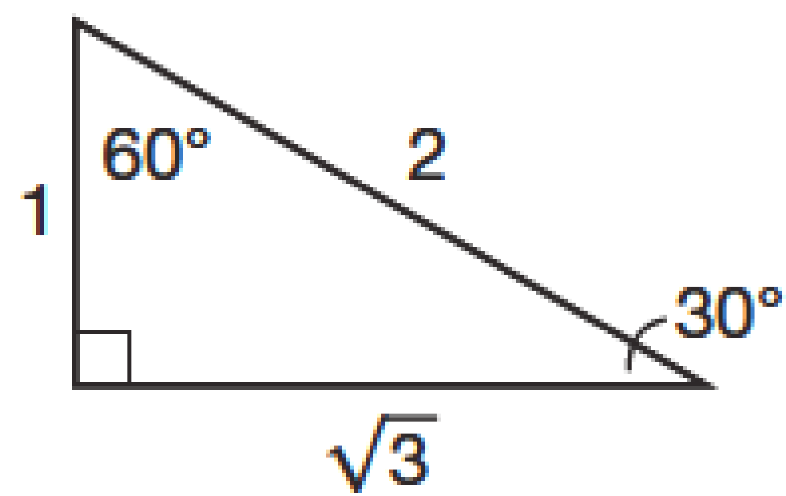
$30° - 60° - 90°$ Right Triangles

The 30°–60°–90° triangle is another special triangle. Like the 45°–45°–90° triangle, properties of the 30°–60°–90° triangle can be used to find missing measures of a triangle if the length of one side is known.

In the diagram, two 30°–60°–90° triangles are shown next to each other, with the shorter legs aligned. Placing the two triangles together so that they share a common leg makes an equilateral triangle. Since all the equilateral triangle's sides are congruent, this shows that the hypotenuse of the triangle is twice the length of the shortest leg.

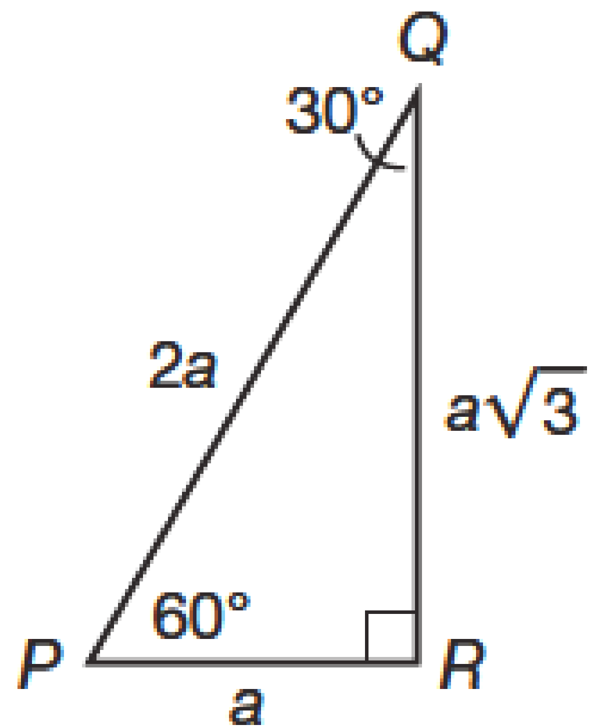


Properties of 30°–60°–90° Triangles – In a 30°–60°–90° triangle, the length of the hypotenuse is twice the length of the short leg, and the length of the longer leg is the length of the shorter leg times $\sqrt{3}$.



Algebraically, these relationships can be written as follows.

$$PR = a \qquad PQ = 2a \qquad QR = a\sqrt{3}$$

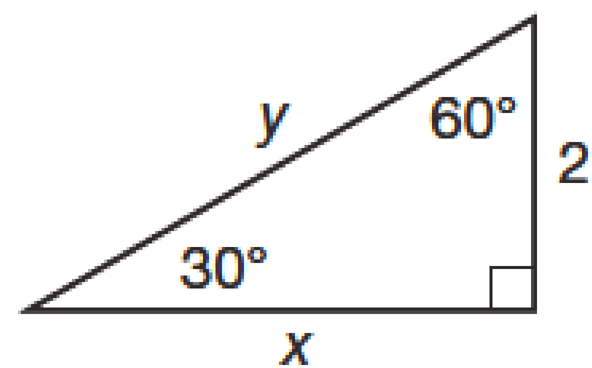


Example 1 Finding Side Lengths in a 30°–60°–90° Triangle

Find the values of x and y. Give your answer in simplified radical form.

SOLUTION

The shortest leg must be opposite the smallest angle, so the leg with a measure of 2 is the short leg. The hypotenuse is twice the short leg, so $y = 4$. The long leg is $\sqrt{3}$ times the short leg, so $x = 2\sqrt{3}$.



Example 2 Finding the Perimeter of a 30°–60°–90° Triangle with Unknown Measures

Find the perimeter of the triangle. Give your answer in simplified radical form.

SOLUTION

First, find the length of *x* and *y*. The short leg is *x*. Since the length of the short leg times the square root of 3 equals the long leg, an equation can be written to solve for *x*.

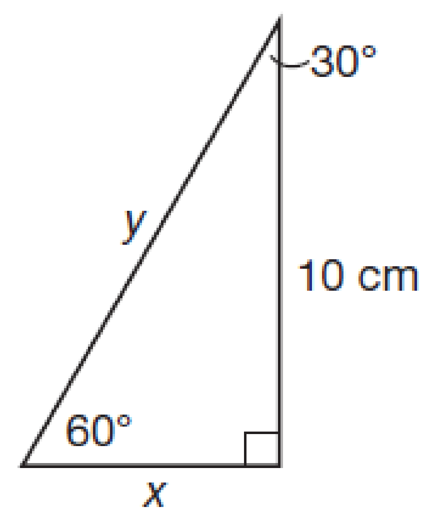
$$x\sqrt{3} = 10$$

$$\frac{x\sqrt{3}}{\sqrt{3}} = \frac{10}{\sqrt{3}}$$

$$x = \frac{10}{\sqrt{3}}$$

$$x = \frac{10}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$$

$$x = \frac{10\sqrt{3}}{3}$$



Example 2 Finding the Perimeter of a 30°–60°–90° Triangle with Unknown Measures

Find the perimeter of the triangle. Give your answer in simplified radical form.

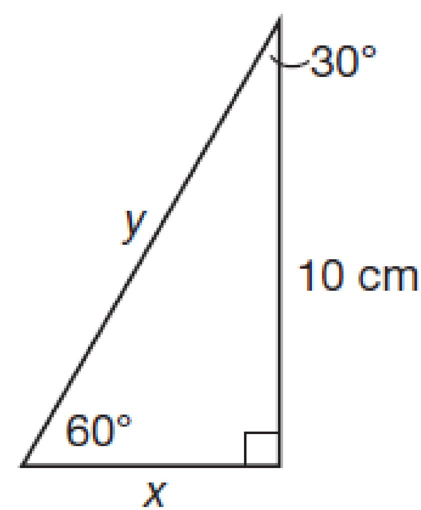
SOLUTION

Since the length of the short leg is now known, the length of the hypotenuse can be found by multiplying the short leg by 2.

$$y = 2x$$

$$y = 2\left(\frac{10\sqrt{3}}{3}\right)$$

$$y = \frac{20\sqrt{3}}{3}$$



Example 2 Finding the Perimeter of a 30°–60°–90° Triangle with Unknown Measures

Find the perimeter of the triangle. Give your answer in simplified radical form.

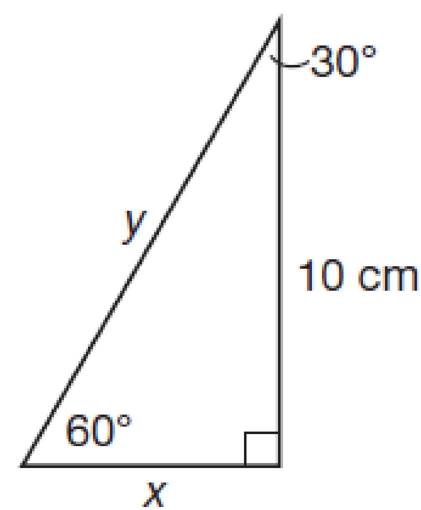
SOLUTION

Now that the lengths of all the sides are known, calculate the perimeter.

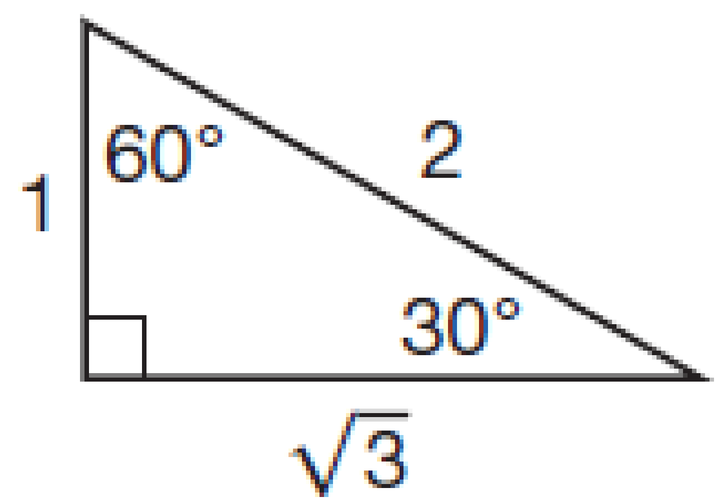
$$P = s_1 + s_2 + s_3$$

$$P = 10 + \frac{10\sqrt{3}}{3} + \frac{20\sqrt{3}}{3}$$

$$P = 10 + 10\sqrt{3}$$



Instead of memorizing the algebraic expressions for each side of a 30°–60°–90° triangle, it may be helpful to just remember the triangle in the diagram, with side lengths 1, $\sqrt{3}$, and 2.

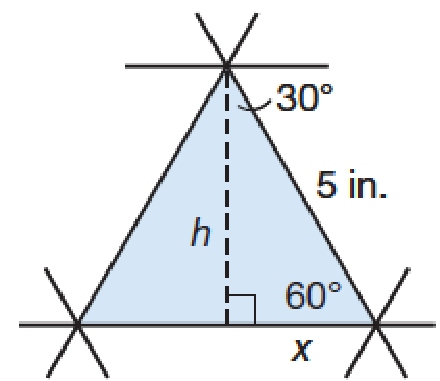


Example 3 Applying the Pythagorean Theorem with 30°–60°–90° Right Triangles

Each tile in a pattern is an equilateral triangle. Find the area of the tile. Use the Pythagorean Theorem and give your answer in simplified radical form.

SOLUTION

$a^2 + b^2 = c^2$	Pythagorean Theorem
$x^2 + x\sqrt{3}^2 = 25$	Substitute.
$4x^2 = 25$	Simplify.
$\frac{4x^2}{4} = \frac{25}{4}$	Divide both sides by 4.
$x^2 = \frac{25}{4}$	Simplify.
$x = \frac{5}{2}$	Simplify.



Example 3 Applying the Pythagorean Theorem with 30°–60°–90° Right Triangles

Each tile in a pattern is an equilateral triangle. Find the area of the tile. Use the Pythagorean Theorem and give your answer in simplified radical form.

SOLUTION

Since *h* is the longer leg of the right triangle, its length is equal to the length of the shorter leg times $\sqrt{3}$. So,

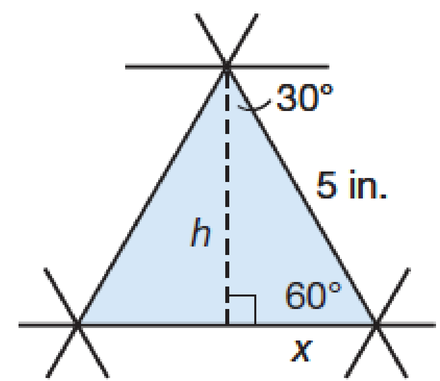
$h = \frac{5}{2}\sqrt{3}\ or\ \frac{5\sqrt{3}}{2}$.

Find the area of each tile.

$A = \frac{1}{2}bh$ Triangle Area Formula

$A = \frac{1}{2} \cdot \frac{5}{2} \cdot \frac{5\sqrt{3}}{2}$ Substitute.

$A = \frac{25\sqrt{3}}{8}$ Simplify.



Example 3 Applying the Pythagorean Theorem with 30°–60°–90° Right Triangles

Each tile in a pattern is an equilateral triangle. Find the area of the tile. Use the Pythagorean Theorem and give your answer in simplified radical form.

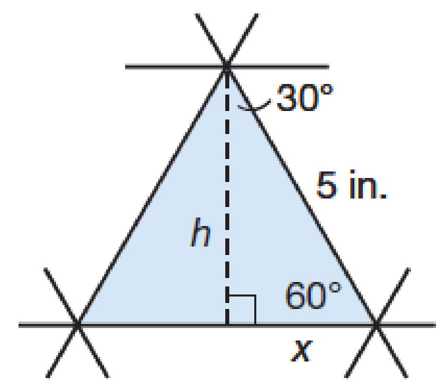
SOLUTION

Double the answer because only half the area of the tile has been found.

$$A = 2 \cdot \frac{25\sqrt{3}}{8}$$

$$A = \frac{25\sqrt{3}}{4} in^2$$

Notice that the sides match the 30°–60°–90° ratios.



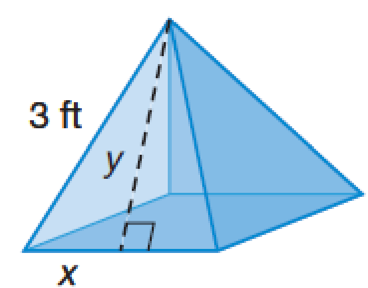
Example 4 Application: Engineering

A museum exhibit contains a model of a pyramid. It has equilateral triangles for faces, and each side of a face is 3 feet long. For a restoration project, the exhibit designers need to know the area of one face of the pyramid. Find this area.

SOLUTION

First, find the dimensions of the 30°–60°–90° triangle. Set up an equation to solve for *x*.

$2x = 3$	Hypotenuse of a 30°–60°–90° is twice the shorter leg
$x = \frac{3}{2}$	Simplify.



Example 4 Application: Engineering

Use the value for x to find y, the longer leg of the triangle.

$y = x\sqrt{3}$ — Longer leg of a 30°–60°–90° is $\sqrt{3}$ times the shorter leg

$y = \frac{3\sqrt{3}}{2}$ — Substitute.

Next, find the area of the triangle.

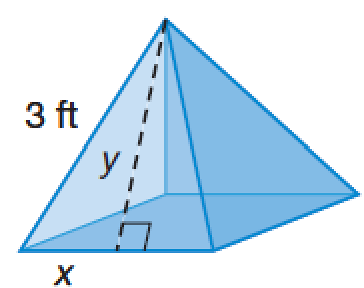
$A = \frac{1}{2}bh$ — Triangle Area Formula

$A = \frac{1}{2} \cdot \frac{3}{2} \cdot \frac{3\sqrt{3}}{2}$ — Substitute.

$A = \frac{9\sqrt{3}}{8}$ — Simplify.

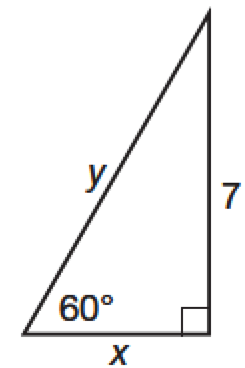
$A = 2 \cdot \frac{9\sqrt{3}}{8}$ — Multiply by 2 to find the area of one face.

$A = \frac{9\sqrt{3}}{4} ft^2$

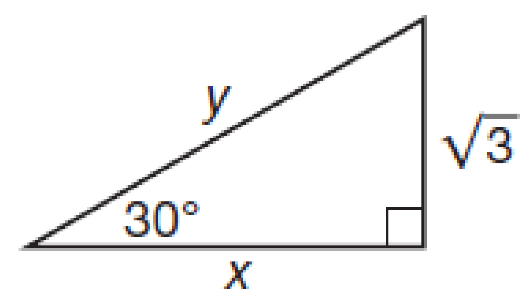


You Try!!!!!

a.Find the length of each side of the triangle.



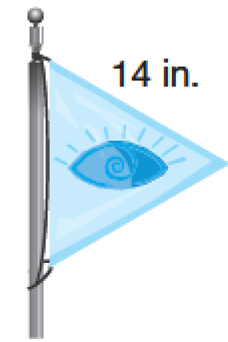
b.Find the perimeter of the triangle in simplified radical form.



You Try!!!!!

c.A school's banner is an equilateral triangle shown here.

Use the Pythagorean Theorem to find the area of the equilateral triangle.



d. Find the area of one triangular face of a pyramid. The faces of the pyramid are equilateral triangles with sides that are 12 centimeter each.

Assignment

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Lesson Practice (Ask Mr. Heintz)

Page 371

Practice 1–30 (Do the starred ones first)