Lesson 57

Finding Perimeter and Area with Coordinates

When a figure is on a coordinate plane and no measurements are given, the distance formula can be used to determine the measurements necessary to find the area or perimeter of the figure.

Hint

The distance formula for finding the distance between points (x_1, y_1) and (x_2, y_2) is: $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Example 1 Finding Perimeter with Coordinates

Find the perimeter of the rectangle with coordinates $A(2, 4)$, $B(2, -2)$, $C(3, -2)$ and $D(3, 4)$.

SOLUTION

Plot the given points on a coordinate plane and draw the rectangle.

To find the perimeter, find the length of each side.

The distance formula can be used, but since all the segments that compose the rectangle are vertical or horizontal, the length of each segment can be calculated as if it were on a 1-dimensional number line.

$AB = |y_1 - y_2|$

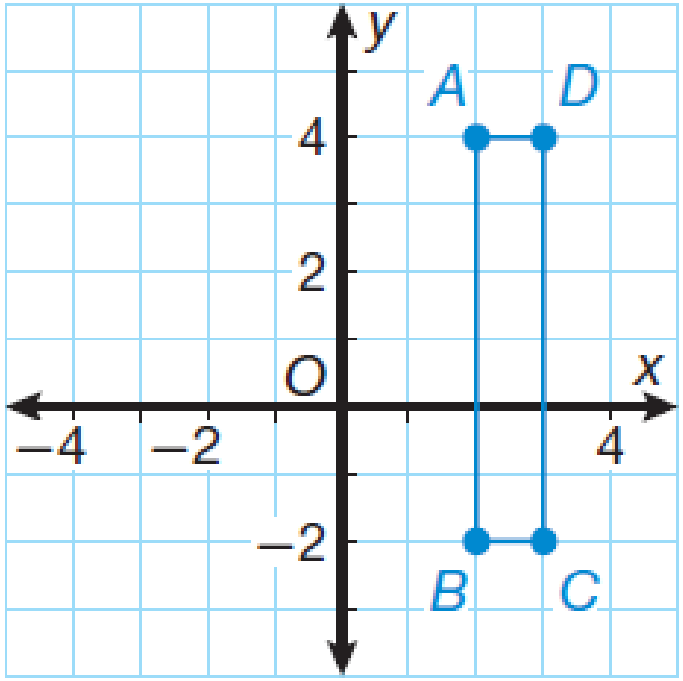
$AB = |4 - (-2)|$

$AB = 6$

Since the figure is a rectangle, $\overline{DC}$ also has a length of 6. The shorter sides, $\overline{AD}$ and $\overline{BC}$, have lengths of 1.

$P = 6 + 6 + 1 + 1$

$P = 14$



Example 2 Finding Perimeter with the Distance Formula

Find the perimeter of rectangle *EFGH* with coordinates *E*(1, 3), *F*(2, 0), *G*(−4, −2), and *H*(−5, 1). Give your answer in simplified radical form.

SOLUTION

Plot the points on a coordinate plane.

Then use the distance formula to calculate the length of each side.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$d_{EF} = \sqrt{(2-1)^2 + (0-3)^2}$$

$$d_{EF} = \sqrt{10}$$

Again, because the figure is a rectangle, $\overline{EF}$ and $\overline{HG}$ are congruent. Next, use points *F* and *G* to find the length of the rectangle's longer side.

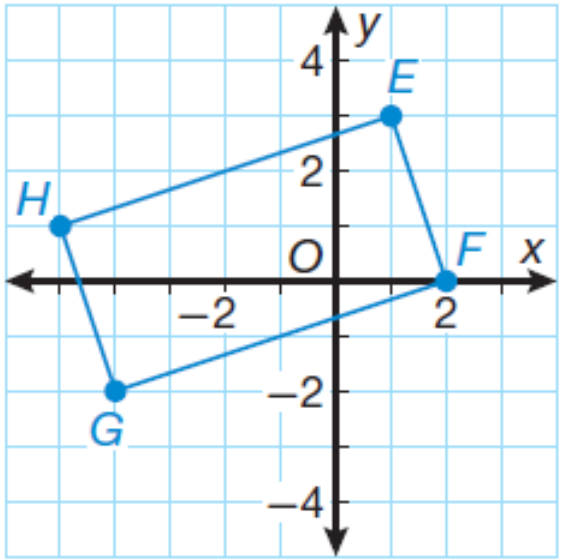
$$d_{FG} = \sqrt{(-4-2)^2 + (-2-0)^2}$$

$$d_{FG} = 2\sqrt{10}$$

Finally, find the perimeter by summing the four sides.

$$P = 2\sqrt{10} + 2\sqrt{10} + \sqrt{10} + \sqrt{10}$$

$$P = 6\sqrt{10}$$



Area can be found on a coordinate plane in the same way. It is important to know what kind of polygon a figure is before attempting to find its area.

Though a figure may look like a rectangle, you cannot assume it is unless you are given that information in the problem or you can prove that the adjacent sides of the figure are perpendicular to each other.

Example 3 Calculating Area with Coordinates

Find the area of right triangle *ABC* with right angle $\angle BAC$.

SOLUTION

Since we know the triangle is a right triangle, finding the length of the two legs is sufficient to find the area. Since $\angle BAC$ is the right angle, the legs are $\overline{AB}$ and $\overline{AC}$. Use the distance formula to find the length of each segment.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$d_{AC} = \sqrt{(3-1)^2 + (-1-3)^2}$$

$$d_{AC} = 2\sqrt{5}$$

$$d_{AB} = \sqrt{(-1-1)^2 + (2-3)^2}$$

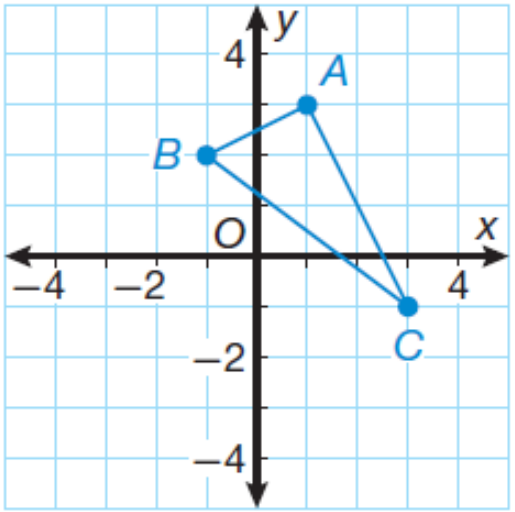
$$d_{AB} = \sqrt{5}$$

Now the formula for area of a triangle can be applied using *AC* and *AB*.

$$A = \frac{1}{2}bh$$

$$A = \frac{1}{2}(\sqrt{5})(2\sqrt{5})$$

$$A = 5$$



Sometimes it may be necessary to find the area of an irregular polygon. A coordinate plane makes this possible, because each square on the grid can be counted and added together to find the area of a figure. When a square is not entirely inside a polygon, it may be necessary to estimate and obtain an approximate area.

Example 4 Estimating Area with Coordinates

a. Estimate the area of the polygon.

SOLUTION

First, count all the squares that lie completely inside the polygon. There are 10 squares that are completely covered, as shown in the diagram.

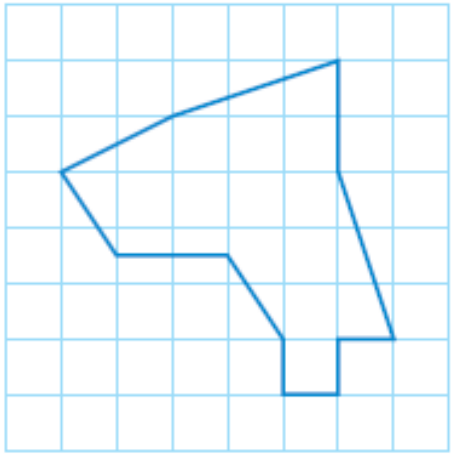
Next, estimate the area of the remaining space.

One way to do this is to look for triangles, like the right angle shown in the diagram. The triangle's legs measure 1 and 2, so the area of this triangle is 1 square unit.

Two identical right triangles together make a rectangle. Each of these right triangles has one leg that is three squares long and one leg that is one square high. Together, they make a rectangle with an area of 3 square units.

Estimate the remaining area. By looking at the remaining rectangle, which appears to have a height of approximately 0.5 and length of 3, plus the two remaining triangles, it appears that the remaining area is about 3 square units.

Add all these items together. The total area of the polygon is approximately 17 square units.



Example 4 Estimating Area with Coordinates

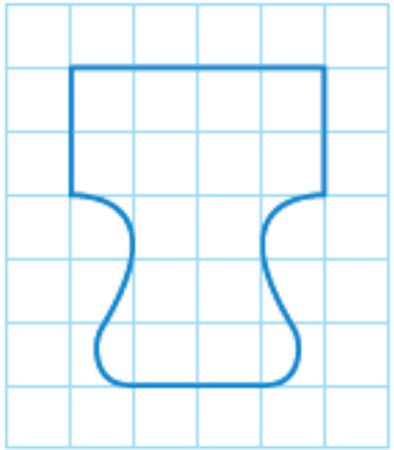
b. Estimate the area of the figure.

SOLUTION

First, count complete square units. There are 14.

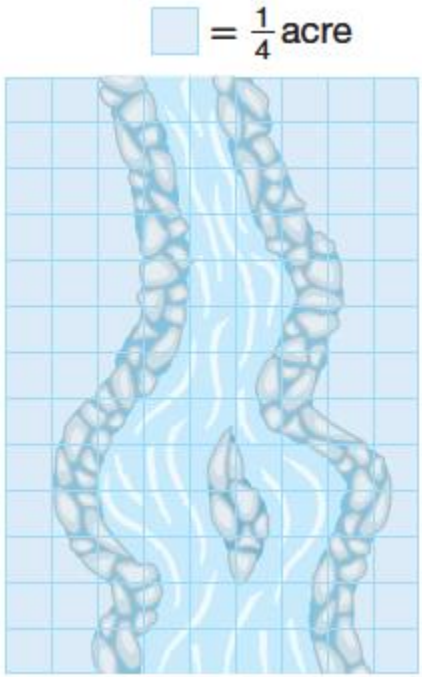
Then, estimate the area of the remaining area.

The curved area covers around one square unit on either side. Therefore, the total area is approximately 16 square units.



Example 5 Application: Farming

A farmer wants to estimate how much seed she needs to buy for her land. She cannot farm in the river or on the riverbank, which is shaded in the diagram. For every acre, she needs 2 bags of seed. Estimate how many bags of seed she will need. Each square unit on the grid represents one fourth of an acre.



SOLUTION

The entire plot of land is 9 by 13, or 117 units. Every four units equals one acre. Including the river, the farmer has $\frac{117}{4} = 29.25$ acres.

There are 49 full units that are unusable because of the river and riverbank, as shown in the diagram. Next, identify triangles and their approximate measurements.

The remaining parts of the river and riverbank add to approximately 14 square units. Your answers may vary, but anything between 13 and 15 units is a reasonable estimation.

Therefore, the total amount of land that is not available for farming is approximately 63 square units.

Every four units equals one acre, so there are $\frac{63}{4} = 15.75$ acres that are unusable.

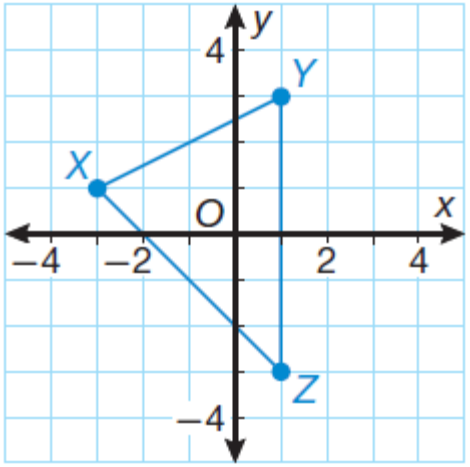
In total, she can use 29.25 – 15.75 = 13.5 acres.

The farmer will need 13.5(2) = 27 bags of seed.

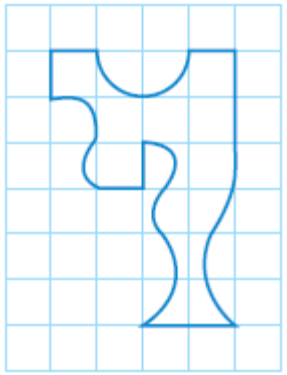
You Try!!!!

a. Find the perimeter of rectangle *LMNO* with coordinates *L*(−5, 3), *M*(−5, −1), *N*(4, −1), and *O*(4, 3).

c.Calculate the area of ΔXYZ.



d.Estimate the area of the figure.



Assignment

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Lesson Practice (Ask Mr. Heintz)

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Practice 1-30 (Do the starred ones first)