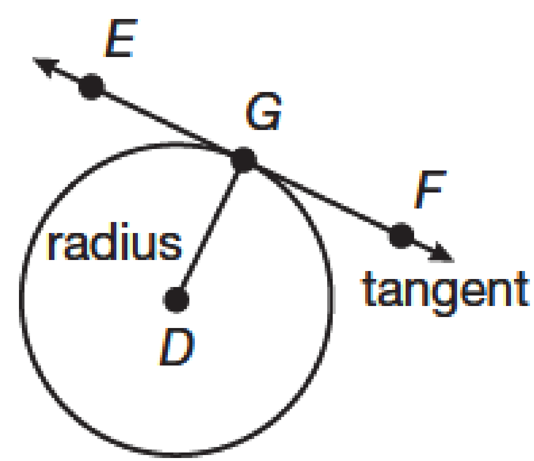
Lesson 58

Tangents and Circles, Part 1

A tangent line lies in the same plane as a circle and intersects the circle at exactly one point. A radius of a circle drawn to a point of tangency meets the tangent line at a fixed angle.

Theorem 58-1 – If a line is tangent to a circle, then the line is perpendicular to a radius drawn to the point of tangency.

$$\overleftrightarrow{EF} \perp \overleftrightarrow{DG}$$



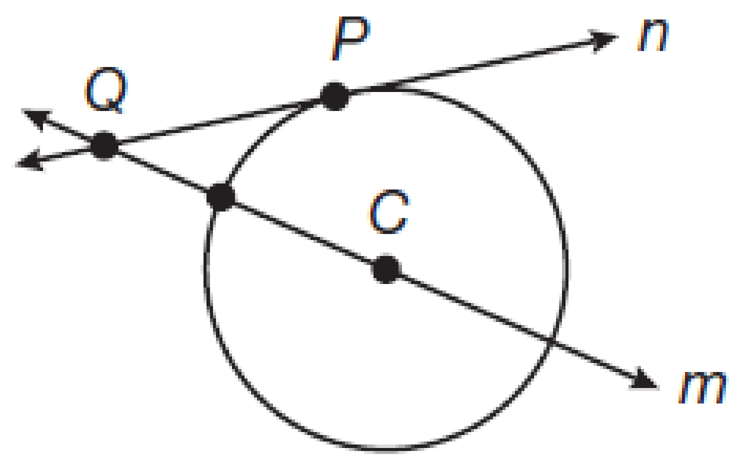
Example 1 Tangent Lines and Angle Measures

Line *n* is tangent to ⊙C at point *P*, and line *m* passes through *C*.

Lines *n* and *m* intersect at point *Q*.

a. Sketch ⊙C and lines *n* and *m*. Mark *C*, *P*, and *Q* on your sketch.

SOLUTION



Example 1 Tangent Lines and Angle Measures

b. If $m\angle CQP = 36^\circ$, determine $m\angle PCQ$.

SOLUTION

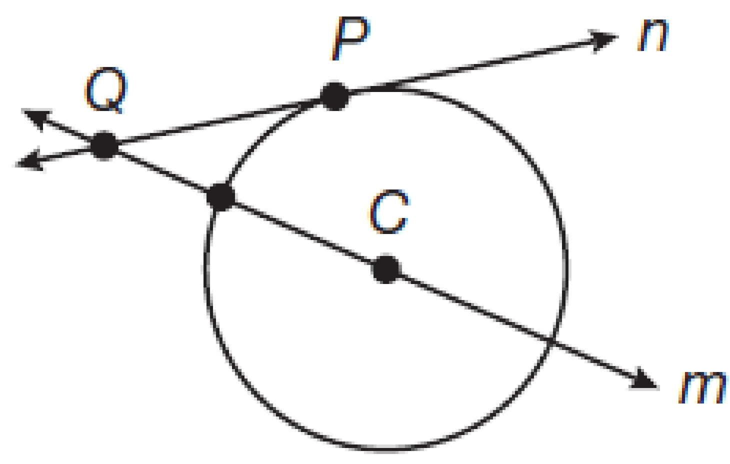
By Theorem 58-1, $\overline{CP}$ is perpendicular to n, so $\triangle CPQ$ is a right triangle.

Therefore, $\angle CQP$ and $\angle PCQ$ are complementary.

$m\angle CQP + m\angle PCQ = 90^\circ$

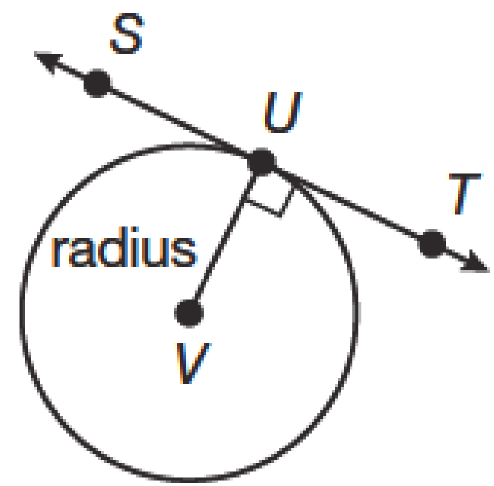
$36^\circ + m\angle PCQ = 90^\circ$

$m\angle PCQ = 54^\circ$



Theorem 58-2 – If a line in the plane of a circle is perpendicular to a radius at its endpoint on the circle, then the line is tangent to the circle.

$$\overleftrightarrow{ST} \text{ is tangent to } \odot V$$



Theorem 58-2 is the converse of Theorem 58-1. Together, they can be used to show that this biconditional statement is true:

A line in the plane of a circle is tangent to the circle if and only if it is perpendicular to a radius drawn to the point of tangency.

Example 2 Identifying Tangent Lines

If $m\angle BEA = 45°$, show that $\overleftrightarrow{BD}$ is tangent to ⊙C.

SOLUTION

To show that $\overleftrightarrow{BD}$ is tangent to ⊙C, it has to be shown that $\angle BEC$ is a right angle.

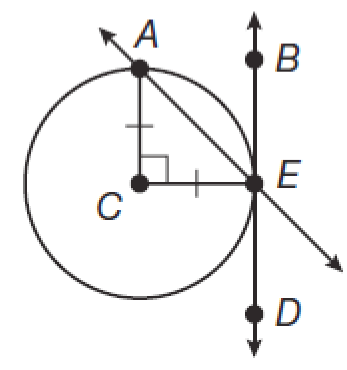
From the diagram, ΔAEC is an isosceles right triangle, so $\angle CAE \cong \angle CEA$.

The acute angles of a right triangle are complimentary, so both $\angle CAE$ and $\angle CEA$ are 45° angles.

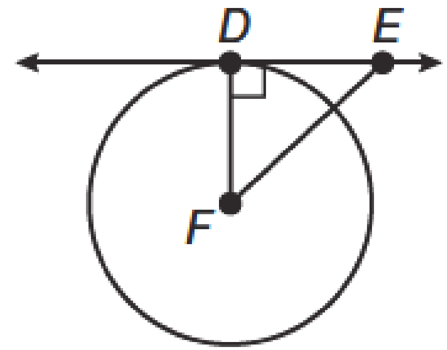
By the Angle Addition Postulate, $\angle CEA + \angle BEA = \angle BEC$.

Substituting shows that $\angle BEC = 45° + 45°$, so $\angle BEC$ is a right angle.

Therefore, by Theorem 58-2, $\overleftrightarrow{BD}$ is tangent to ⊙C.



Example 3 Proving Theorem 58-2



Prove Theorem 58-2.

Given: $\overleftrightarrow{DE} \perp \overline{FD}$ and D is on ⊙.

Prove: $\overleftrightarrow{DE}$ is tangent to ⊙.

SOLUTION

We will write an indirect proof. Assume that $\overleftrightarrow{DE}$ is not tangent ⊙F.

Then $\overleftrightarrow{DE}$ ⊙F at two points, D and point P. Then $FD = FP$.

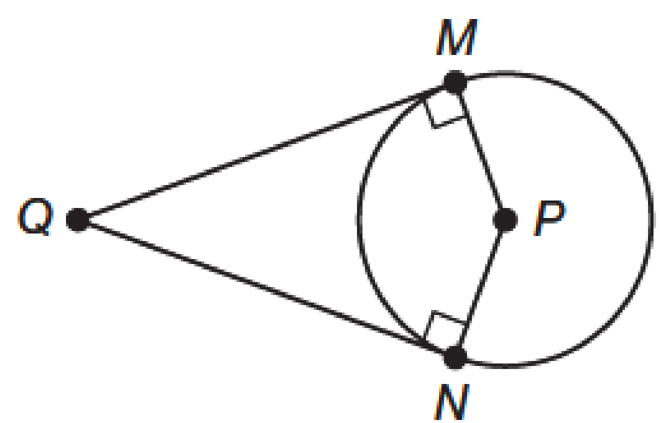
But if $\triangle DFP$ is isosceles, and the base angles of an isosceles triangle are congruent, then $\angle DPF$ is a right angle.

That means there are two right angles from F to $\overleftrightarrow{DE}$, which is a contradiction of the theorem which states that through a line and a point not on a line, there is only one perpendicular line, so the assumption was incorrect and $\overleftrightarrow{DE}$ is tangent to ⊙F.

If two tangents to the same circle intersect, the tangent segments exhibit a special property, stated in Theorem 58-3.

Theorem 58-3 - If two tangent segments are drawn to a circle from the same exterior point, then they are congruent.

$$\overline{MQ} \cong \overline{NQ}$$



Example 4 Applying Relationships of Tangents from an Exterior Point

In this figure, $\overline{JK}$ and $\overline{JM}$ are tangent to ⊙L.

Determine the perimeter of quadrilateral *JKLM*. What type of quadrilateral is *JKLM*?

SOLUTION

Since $\overline{ML}$ and $\overline{LK}$ are radii of the same circle, $KL = 8$ in.

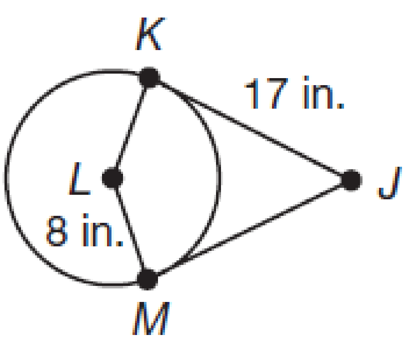
By Theorem 58-3, tangents to the same circle are congruent, so $JM = 17$ in.

To get the perimeter, add the lengths of the four sides.

$P = JK + KL + LM + JM$

$P = 17 + 8 + 8 + 17 = 50$ in.

Since *JKLM* has exactly two pairs of congruent consecutive sides, it is a kite.



Example 5 Application: Glass Cutting

An ornamental window has several glass panes oriented to look like an eye. The radius of the eye's iris is 3 feet, and *DC* is 5 feet. What are *AI* and *AH*?

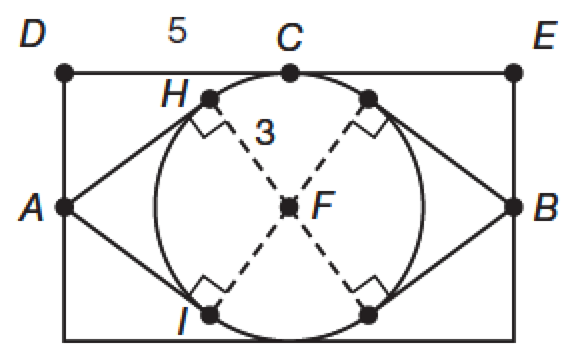
SOLUTION

The right angles on the diagram indicate that the four segments that form the corners of the eye are tangent to the circle.

Draw in the segment $\overline{FA}$. This forms right triangles, *FAI* and *FAH*.

The hypotenuse of the triangles are 5 feet long, and their shorter legs are each 3 feet long.

By using the Pythagorean Theorem, *AI* and *AH* are each 4 feet long.



You Try!!!!!

a. Line a is tangent to $\odot R$ at D, and line b passes through R. Lines a and b intersect at E. If $m\angle RED = 42°$, determine $m\angle DRE$.

b. Let $\overline{CA}$ be a radius of $\odot C$. Let line m be tangent to $\odot C$ at A. Let B be an exterior point of $\odot C$, with $m\angle BAC < 90°$.

Is $\overleftrightarrow{AB}$ a tangent to $\odot C$? Why or why not?

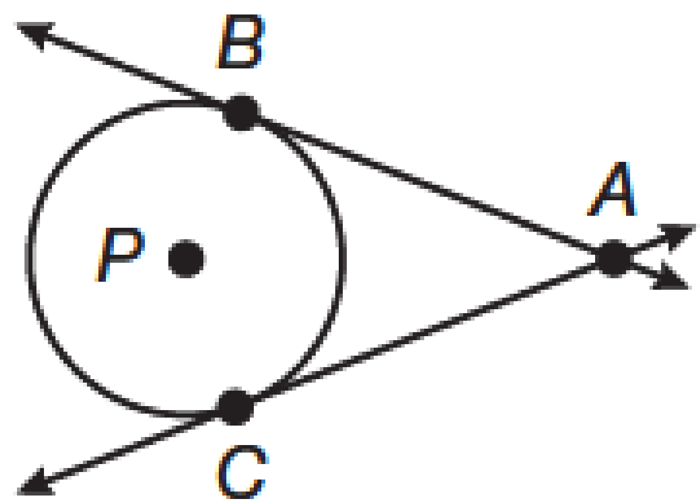
You Try!!!!!

c. Give a paragraph proof of Theorem 58-3.

Hint: Draw $\overline{PA}$, $\overline{PB}$, *and* $\overline{PC}$.

Given: $\overline{AB}$ and $\overline{AC}$ are tangent to ⊙P

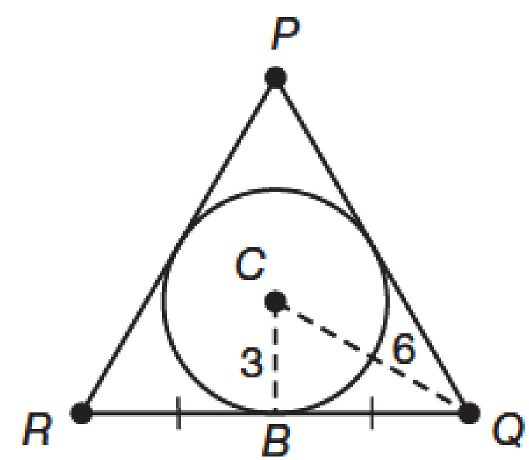
Prove: $\overline{AB} \cong \overline{AC}$



You Try!!!!!

d. Circle C has a 5-inch radius. $\overline{XZ}$ and $\overline{YZ}$ are tangents to ⊙C and Z is exterior to ⊙C.
If $\angle XCY$ is a right angle, what is the area of quadrilateral $CXZY$?

e. A decorative window is shaped like a triangle with an inscribed circle. If the triangle is an equilateral triangle, the circle has a radius of 3 feet, and CQ is 6 feet, what is the perimeter of the triangle in simplified radical form?



Assignment

Page 384

Lesson Practice (Ask Mr. Heintz)

Page 384

Practice 1–30 (Do the starred ones first)