# Lesson 59

Finding Surface Area and Volumes of Prisms

Surface Area – The total area of all the faces and curved surfaces of a solid. The surface area of the pentagonal prism shown in the diagram, for example, is the sum of the area of the two pentagons and the five rectangles that compose the prism.

Lateral Area – The sum of the areas of the lateral faces of a prism or pyramid, or the surface area, excluding the base(s), of the lateral surface of a cylinder or cone. In the diagram, the lateral area of the pentagonal prism is the sum of the area of all the rectangular faces.
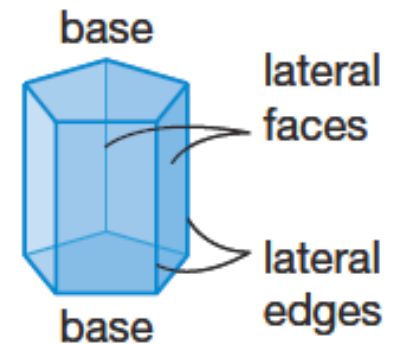
Lateral Area of a Prism – The lateral area L of a prism can be found using the following formula, where p is the perimeter of the base and h is the prism's height.

$$L = Ph$$

Example 1 Finding Lateral Area of a Prism

Find the lateral area of the regular hexagonal prism.

SOLUTION

First find the perimeter of the base, then multiply it by the height. The base is a regular hexagon with 6 side lengths of 12 feet each, so the perimeter is 72 feet. Next, multiply the perimeter by the height, or 72×18.

The lateral area of the prism is 1296 square feet.

18 ft

12 ft
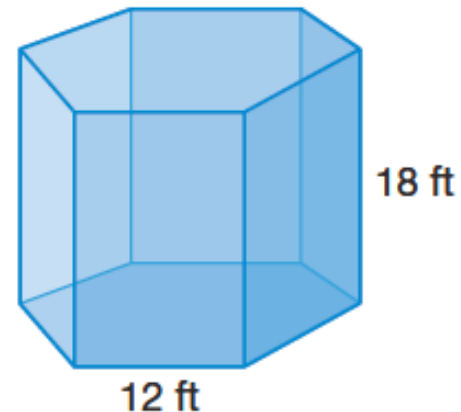
Surface area of a prism can be calculated by finding the sum of the areas of each face. Sometimes it is easiest to first determine the lateral surface area and then add the area of the two bases to the lateral surface area.

Surface Area of a Prism – The surface area of a prism is the sum of the lateral area and the area of the two bases, where B is the area of a base.

$$S = L + 2B$$

Example 2 Finding Surface Area of a Prism

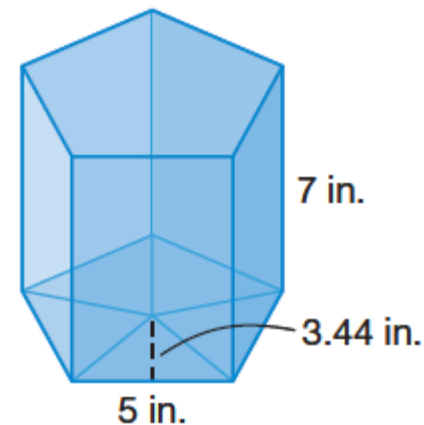Find the surface area of the regular pentagonal prism.

SOLUTION

The base is a regular pentagon, so its perimeter is given by multiplying the length of each side by the number of sides. Therefore, the perimeter is $5 \times 5 = 25$ inches. Substitute the perimeter's value and the height 7 into the lateral area formula.

$$L = Ph$$
$$L = (25)(7)$$
$$L = 175 \; in^2$$

7 in.

3.44 in.

5 in.

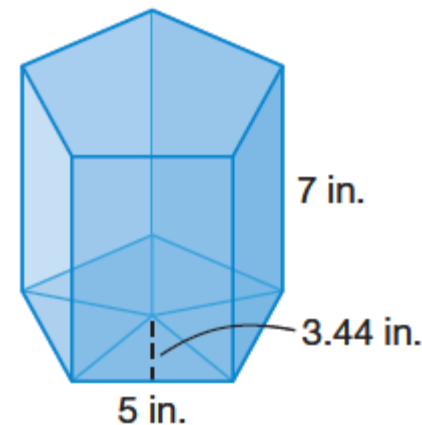# Example 2 Finding Surface Area of a Prism

Find the surface area of the regular pentagonal prism.

SOLUTION

Now find the area of the base, which is a pentagon. The pentagon can be divided into five congruent triangles as shown. For one triangle:

$$A = \frac{1}{2}bh$$

$$A = \frac{1}{2}(5)(3.44)$$

$$A \approx 8.6 \ in^2$$

7 in.

3.44 in.

5 in.

Example 2 Finding Surface Area of a Prism

Find the surface area of the regular pentagonal prism.
SOLUTION
So the area of the base is about $(5)(8.6) = 43$ square inches.
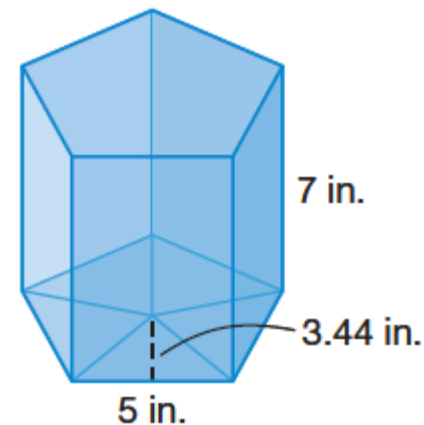
To find the total surface area of the prism, use the formula for surface area.

$$S = L + 2B$$
$$S = 175 + 2(43)$$
$$S = 261 \; in^2$$

So the surface area of the prism is 261 square inches.

7 in.

3.44 in.

5 in.

Surface area can be considered as the number of square units it takes to exactly cover the outside of a solid. Volume is the number of unit cubes of a given size that will exactly fill the interior of a solid.

Volume of a Prism – The volume of a prism can be found using the formula below, where B is the area of the base and h is the height of the prism.

$$V = Bh$$

A right prism is a prism whose lateral faces are all rectangles and whose lateral edges are perpendicular to both bases. In a right prism the height is the length of one edge that separates the bases.

Example 3 Finding the Volume of a Right Prism

Find the volume of the right prism.

SOLUTION

The base of a prism is a trapezoid, so calculate the area of the base first, then the volume of the prism:
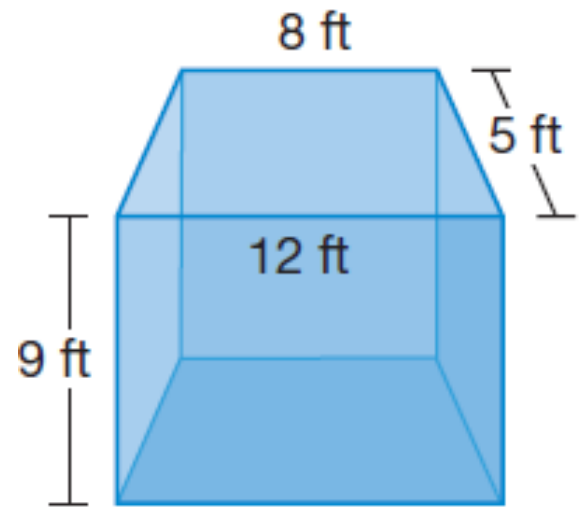
$$A = \frac{1}{2}(b_1 + b_2)h$$

$$A = \frac{1}{2}(8 + 12)5$$

$$A = 50 \; ft^2$$

$$V = Bh$$

$$V = 50 \cdot 9$$

$$V = 450 \; ft^3$$

The volume of the prism is 450 cubic feet.

Oblique Prism – A prism that has at least one nonrectangular face. An oblique prism is like a prism that has been tilted to one side.

The surface area and the volume of an oblique prism are found using the same formulas that are used with a right prism. Instead of using the height of a lateral edge of the prism, an altitude of the prism must be used. An altitude of a prism is a segment that is perpendicular to, and has its endpoints on, the planes of the bases.
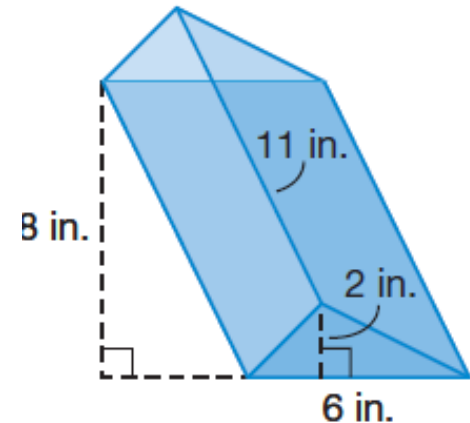
height

Oblique triangular prism
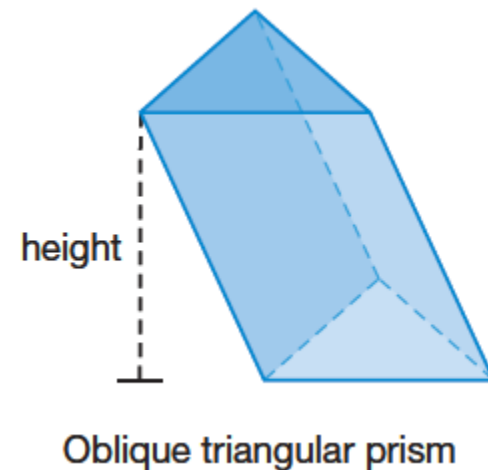
Example 4 Finding the Volume of an Oblique Prism

Find the volume of the oblique prism shown.

SOLUTION

The volume of an oblique prism is the area of the base times the height. Do not use the slanted height of the lateral face. Instead, use the altitude, which is 8 inches.

$$V = Bh$$

$$V = \frac{1}{2}(6)(2) \cdot 8$$

$$V = 48 \ in^3$$

8 in.

11 in.

2 in.

6 in.

The volume of the oblique triangular prism is 48 cubic inches.

Example 5 Application: Packaging

Rick has a gift that he needs to wrap. He has 15 square feet of wrapping paper. Does Rick have enough wrapping paper to cover the gift shown in the diagram?

SOLUTION

First find the lateral surface area by multiplying the perimeter of the 2-by-3-foot base by the height.
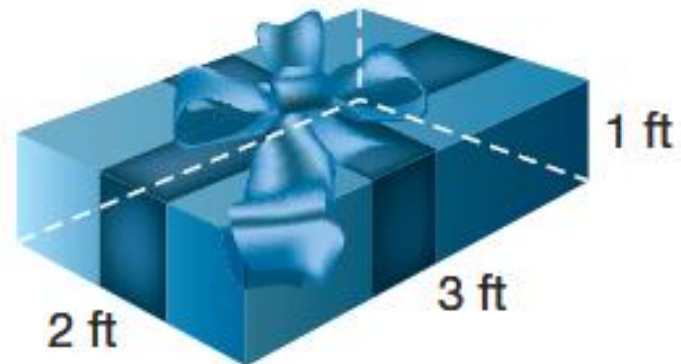
$$L = Ph$$
$$L = (2 + 2 + 3 + 3)(1)$$
$$L = 10 \ ft^2$$

Now add the area of the two bases.

$$S = L + 2B$$
$$S = 10 + 2(2 \cdot 3)$$
$$S = 22 \ ft^2$$

1 ft
3 ft
2 ft

Since the surface area of the gift is 22 square feet, Rick will need to buy more wrapping paper to wrap the entire gift.
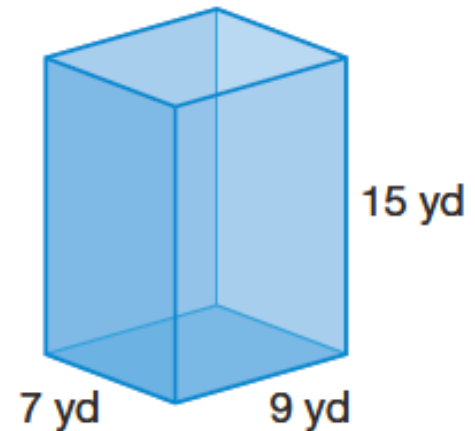
You Try!!!!!

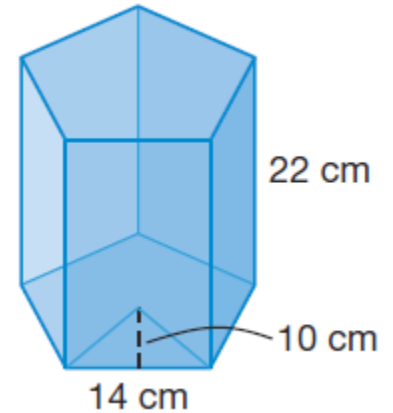Use the figure at right to answer problems a and b.

a. Find the lateral area of the prism.

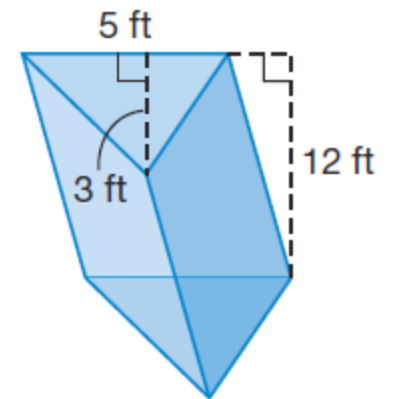b. Find the surface area of the prism.

15 yd

7 yd   9 yd

You Try!!!!!

c. Find the volume of the right prism.

22 cm

10 cm

14 cm

d. Find the volume of the oblique prism.

5 ft

3 ft

12 ft

Assignment

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    Lesson Practice (Ask Mr. Heintz)

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    Practice 1-30 (Do the starred ones first)