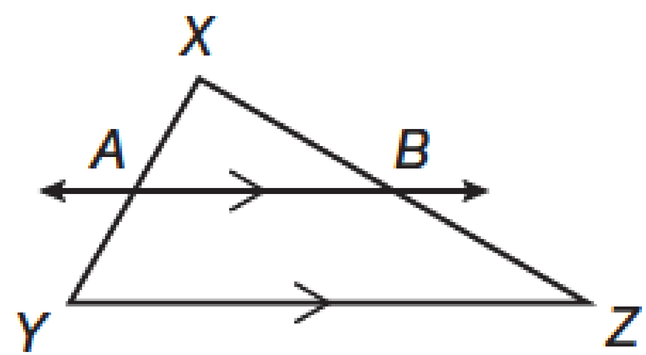
# Lesson 60

Proportionality Theorems

Previous lessons have discussed some of the proportional relationships that exist within triangles when they are divided by a midsegment. A similar relationship exists for any line that intersects two sides of a triangle and is parallel to one side.

**Theorem 60-1: Triangle Proportionality Theorem – If a line parallel to one side of a triangle intersects the other two sides, it divides those sides proportionally.**

$$\frac{XA}{AY} = \frac{XB}{BZ}$$



# Example 1 Using Triangle Proportionality to Find Unknowns

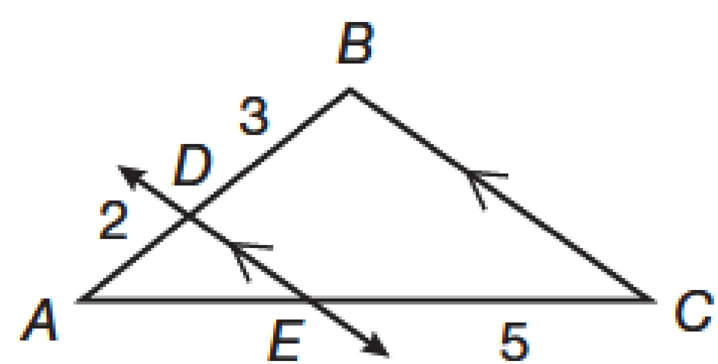
a. Find the length of $\overline{AE}$.

SOLUTION

$\frac{AE}{EC} = \frac{AD}{DB}$ Triangle Proportionality Theorem

$\frac{AE}{5} = \frac{2}{3}$ Substitute.

$AE = \frac{10}{3}$ Simplify.



# Example 1 Using Triangle Proportionality to Find Unknowns

b. Find the value of *x*.

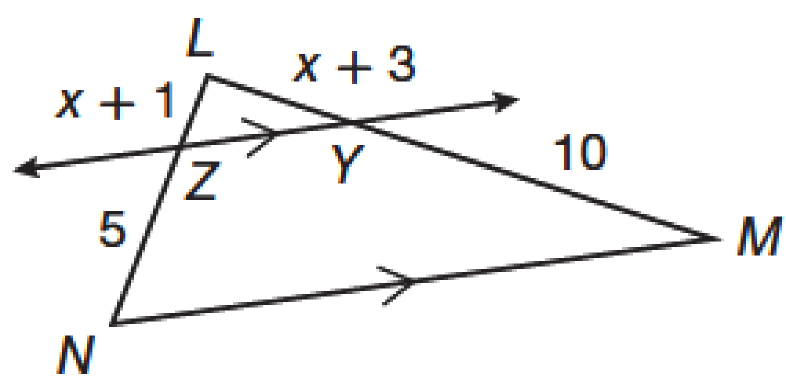
SOLUTION

Write a proportion relating the segments based on the Triangle Proportionality Theorem.

$\frac{x+1}{5} = \frac{x+3}{10}$ Triangle Proportionality Theorem

$10(x + 1) = 5(x + 3)$ Cross multiply.

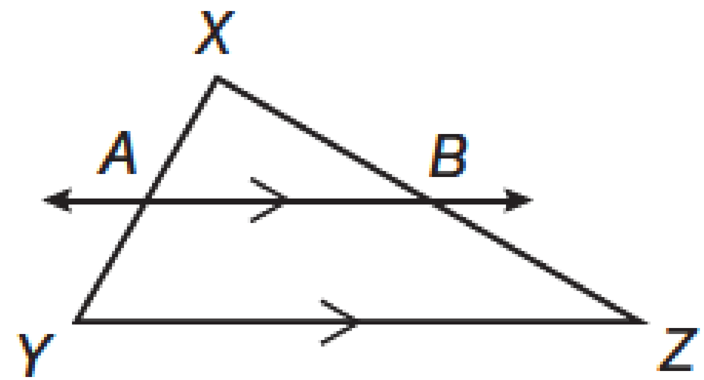
$x = 1$ Solve.



The Converse of the Triangle Proportionality Theorem is true, and can be used to check whether a line that intersects 2 sides of a triangle is parallel to the triangle's base.

**Theorem 60-2: Converse of the Triangle Proportionality Theorem – If a line divides two sides of a triangle proportionally, then it is parallel to the third side.**

**In $\Delta XYZ$, if $\frac{XA}{AY} = \frac{XB}{BZ}$, then $\overleftrightarrow{AB} \parallel \overline{YZ}$.**



# Example 2 Proving Lines Parallel

Is $\overline{ST}$ parallel to $\overline{PR}$?

SOLUTION

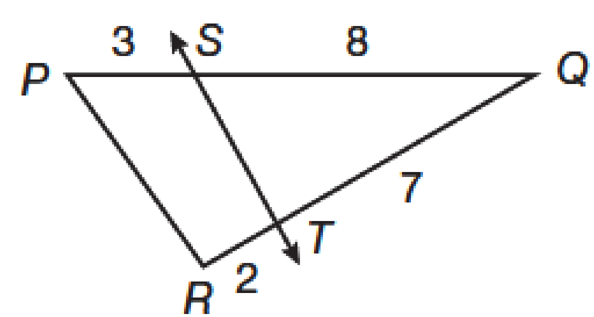
If $\overline{ST}$ divides $\overline{PQ}$ and $\overline{RQ}$ proportionally, then $\overline{ST} \parallel \overline{PR}$ by Theorem 60–2. Set up a proportion.

$\frac{PS}{SQ} = \frac{RT}{TQ}$ Triangle Proportionality Theorem

$\frac{3}{8} = \frac{2}{7}$ Substitute.

$21 = 16$ Cross multiply.

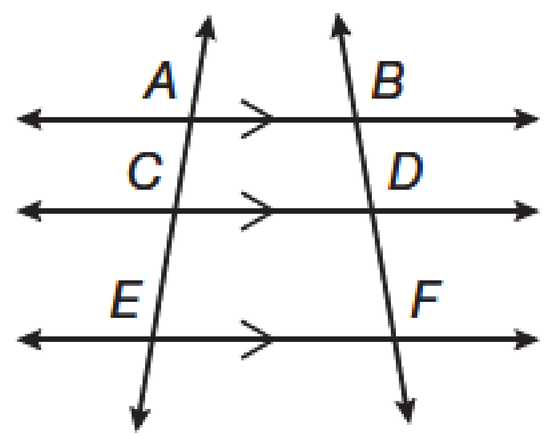
The statement is false, so $\overline{ST}$ is not parallel to $\overline{PR}$.



The Triangle Proportionality Theorem is closely related to Theorem 60-3, which uses the same proportional relationship to relate the segments of transversals that are intersected by parallel lines.

**Theorem 60-3 – If parallel lines intersect transversals, then they divide the transversals proportionally.**

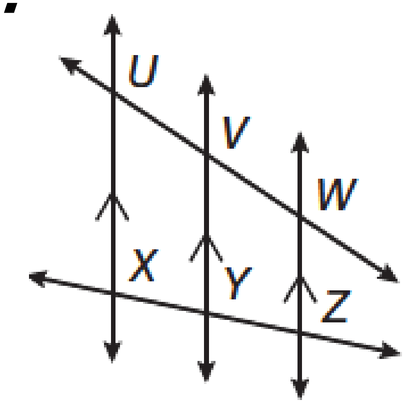
$$\frac{AC}{CE} = \frac{BD}{DF}$$



If parallel lines divide a transversal into congruent segments, then the segments are in a 1:1 ratio. By Theorem 60-3, any other transversal cut by the same parallel lines will be divided into segments that also have a 1:1 ratio, so they will also be congruent.

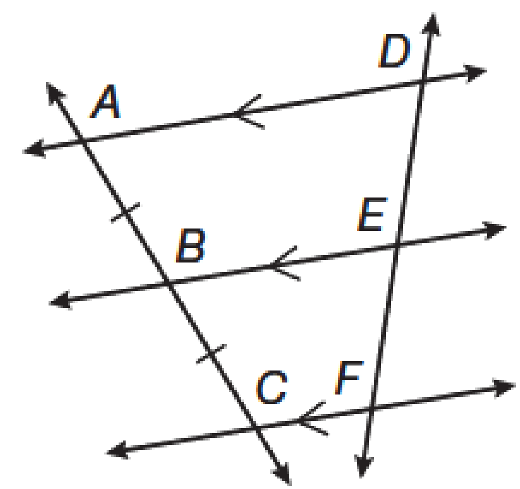
**Theorem 60-4 – If parallel lines cut congruent segments on one transversal, then they cut congruent segments on all transversals.**

**In the diagram, if $UV = VW$, then $XY = YZ$.**



# Example 3 Proving Theorem 60–4

Use a paragraph proof to prove Theorem 60–4.
Given: $\overline{AB} \cong \overline{BC}$, $\overline{AD} \parallel \overline{BE}, \overline{BE} \parallel \overline{CF}$
Prove: $\overline{DE} \cong \overline{EF}$
SOLUTION
By Theorem 60–3, we know that $\frac{AB}{BC} = \frac{DE}{EF}$.
Since $\overline{AB} \cong \overline{BC}$, the ratio of $AB$ to $BC$ by substitution is the same as $AB{:}AB$, which is equal to 1.
Substitute into the proportion given and obtain $1 = \frac{DE}{EF}$.
Taking the cross product yields $DE = EF$.
By the definition of congruent segments, $\overline{DE} \cong \overline{EF}$.



# Example 4 Finding Segment Lengths with Intersecting Transversals

a. Find the length of segment $\overline{AB}$.

SOLUTION

The parallel lines cut $\overline{DF}$ into congruent segments. Therefore, $\overline{AC}$ is also cut into congruent segments.

$3x - 1 = x + 3$

$2x = 4$

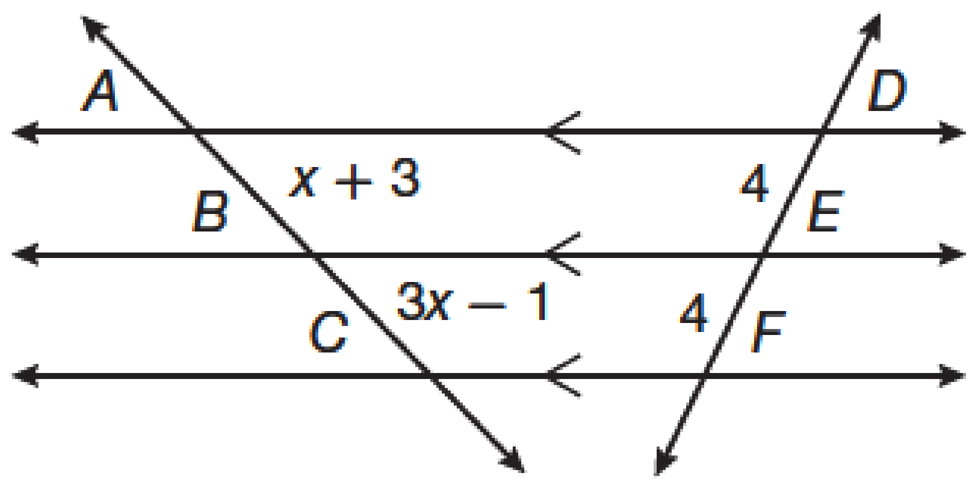
$x = 2$

Substitute this value into the equation that gives *AB*.

$AB = x + 3$

$AB = 2 + 3$

$AB = 5$



# Example 4 Finding Segment Lengths with Intersecting Transversals

b. Determine whether $\overline{UV}$, $\overline{WX}$, and $\overline{YX}$ are parallel when $x = 3$.

SOLUTION

Because $VX = XZ$, $\overline{UW}$ must equal $\overline{WY}$.

$UW = 4x - 1$

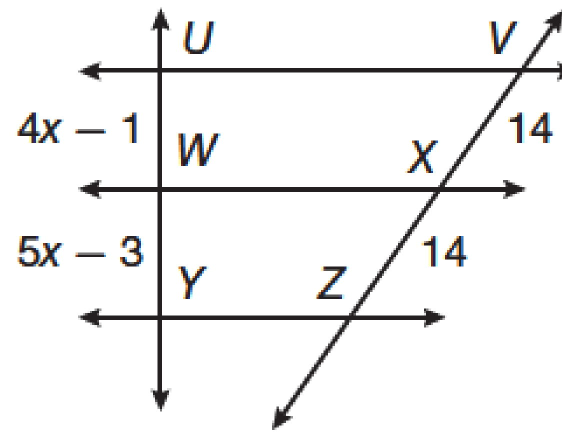
$= 4(3) - 1$

$= 11$

$WY = 5x - 3$

$= 5(3) - 3$

$= 12$



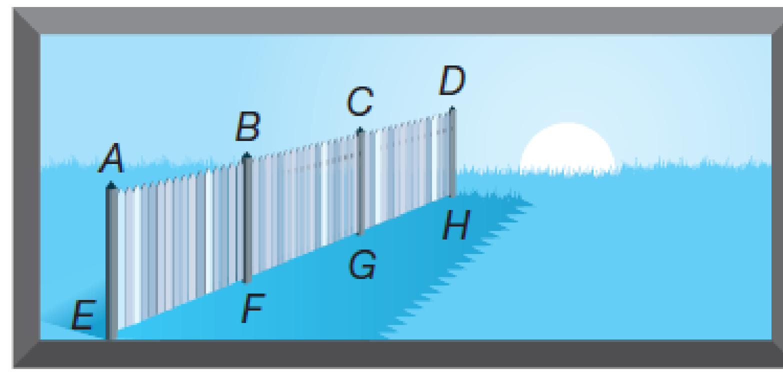

$UX \neq WY$. Therefore the lines $\overline{UV}$, $\overline{WX}$, and $\overline{YZ}$ are not parallel.

# Example 5 Application: Art

Perspective is a method artists use to make an object appear as if it is receding into the distance. If the fence posts are parallel, then what is the length of $\overline{AB}$ if $EH = 22$, $BC = 4$, $CD = 6$, and $FH = 18$?

SOLUTION

Use Theorem 60-3 to write a proportion relating the segments given.




$$\frac{EH}{FH} = \frac{AD}{BD}$$

$$\frac{22}{18} = \frac{AD}{(4+6)}$$

$$AD = 12.\overline{2}$$

Use the Segment Addition Postulate:

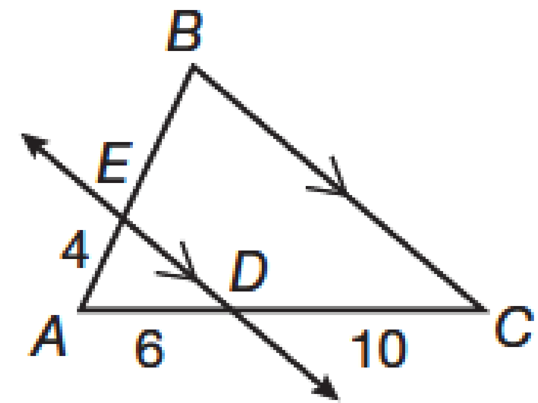
$$AB = AD - BC - CD$$

$$AB = 12.\overline{2} - 4 - 6$$

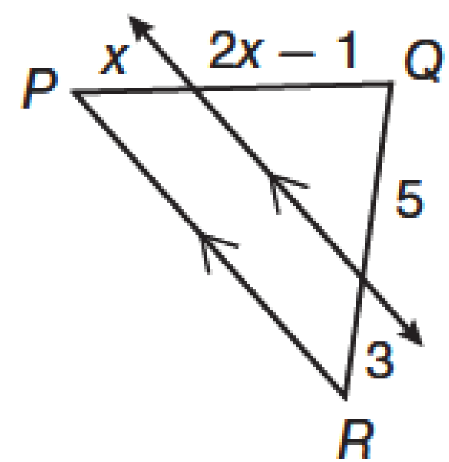
$$AB = 2.\overline{2}$$

# You Try!!!!!

a. Find the length of $\overline{EB}$.




b. Find the length of $\overline{PQ}$.

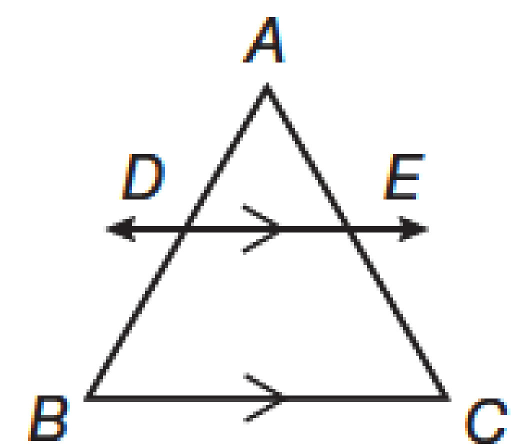


# You Try!!!!!

c. Use a paragraph proof to prove the Triangle Proportionality Theorem.

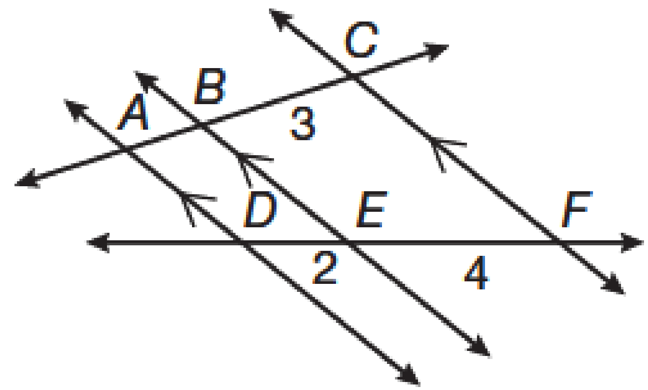
Given: $\overline{DE} \parallel \overline{BC}$

Prove: $\frac{AD}{DB} = \frac{AE}{EC}$

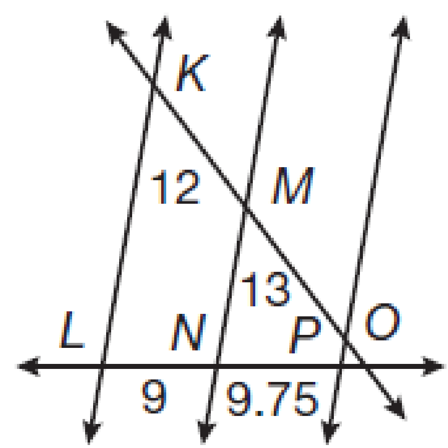


# You Try!!!!!

d. Find the length of $\overline{AC}$.

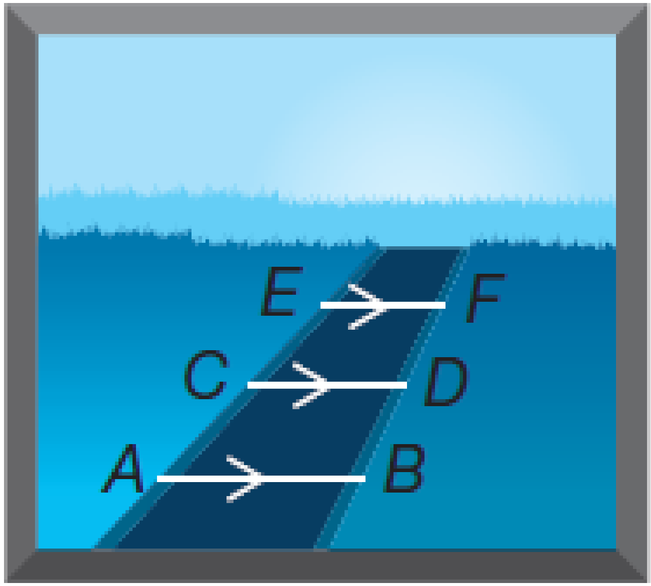



e. Determine whether $\overline{KL}, \overline{MN}$, and $\overline{OP}$ are parallel.



# You Try!!!!!

f.Art: A road is drawn with perspective. Find the length of $\overline{AE}$ if $AC = 10$, $BF = 20$, and $BD = 8$.



# Assignment

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Lesson Practice (Ask Mr. Heintz)

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Practice 1–30 (Do the starred ones first)