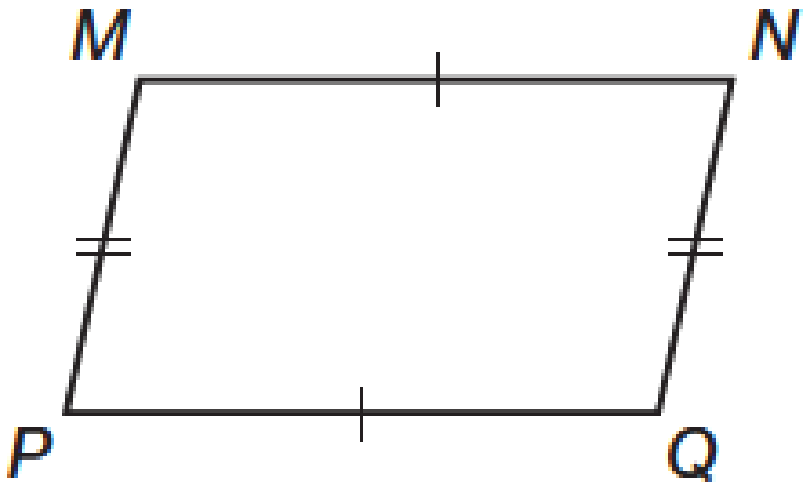
Lesson 61

Determining if a Quadrilateral is a Parallelogram

If a quadrilateral has certain characteristics, it can be identified as a parallelogram. This lesson introduces four methods of identifying parallelograms.

Identifying Parallelograms – If both pairs of opposite sides of a quadrilateral are congruent, then the quadrilateral is a parallelogram.

MNQP is a parallelogram.



Example 1 Proving a Quadrilateral is a Parallelogram Using Opposite Sides

In quadrilateral *WXYZ*, $\overline{WX} \parallel \overline{ZY}$ and $\angle Z \cong \angle X$. Is *WXYZ* a parallelogram?

SOLUTION

The diagonal $\overline{WY}$ has been added to create ΔWXY and ΔWZY.

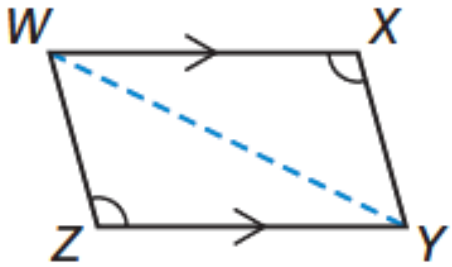
Since $\overline{WX} \parallel \overline{ZY}$, the alternate interior angles $\angle XWY$ and $\angle ZYW$ are congruent.

Segment *WY* is congruent to itself by the Reflexive Property of Congruence.

Therefore, $\Delta WXY \cong \Delta WZY$ by the AAS Triangle Congruence Theorem.

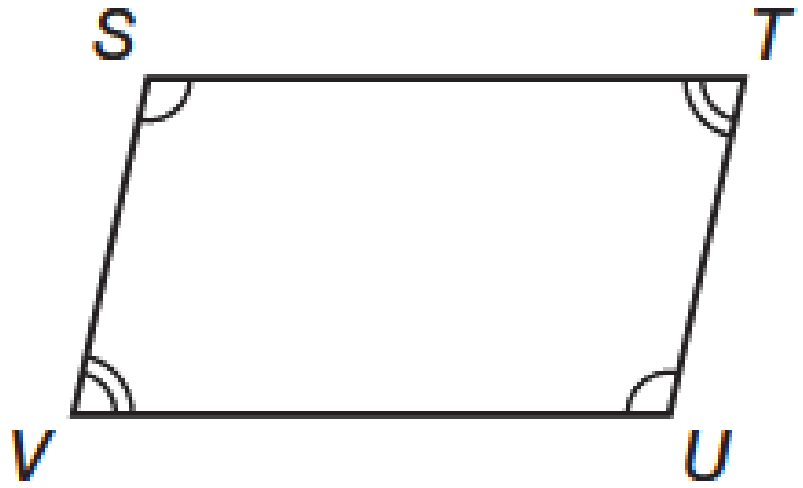
By CPCTC, $\overline{WX} \cong \overline{ZY}$ and $\overline{WZ} \cong \overline{XY}$.

Since both pairs of opposite sides of *WXYZ* are congruent, it is a parallelogram.



Identifying Parallelograms – If both pairs of opposite angles of a quadrilateral are congruent, then it is a parallelogram.

STUV is a parallelogram.



Example 2 Proving a Quadrilateral is a Parallelogram Using Opposite Angles

In quadrilateral *PQRS*, $\overline{PQ} \cong \overline{SR}$. Is *PQRS* a parallelogram?

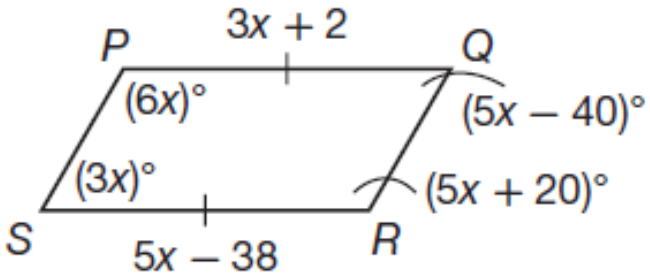
SOLUTION

Since $\overline{PQ} \cong \overline{SR}$, $PQ = SR$. Substitute the given values and solve for x.

$PQ = SR$	Given
$3x + 2 = 5x - 38$	Substitute.
$x = 20$	Solve.

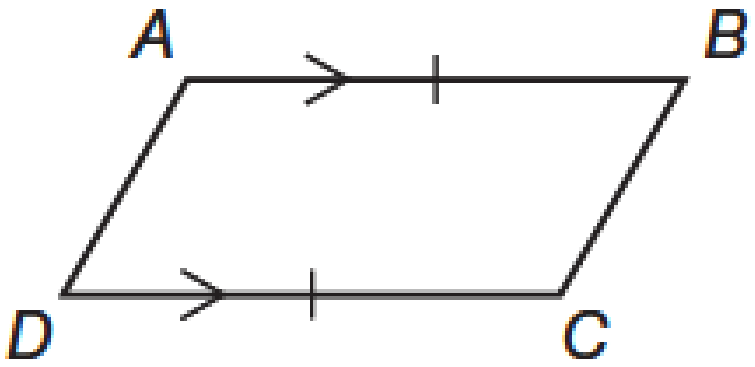
Now that x is known, substitute it into the expression for the measure of each angle. We find that $\angle P = 120°$, $\angle R = 120°$, $\angle S = 60°$, and $\angle Q = 60°$.

Since both pairs of opposite angles in *PQRS* are congruent, *PQRS* is a parallelogram.



Identifying Parallelograms – If one pair of opposite sides of a quadrilateral is both parallel and congruent, then the quadrilateral is a parallelogram.

ABCD is a parallelogram.



Example 3 Proving a Quadrilateral is a Parallelogram Using One Pair of Sides

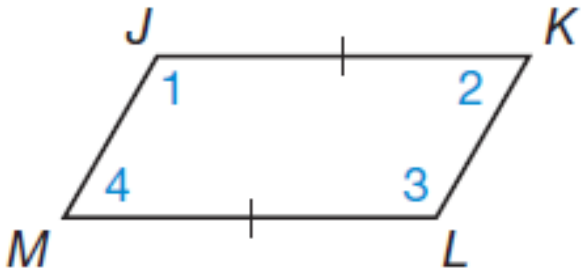
In quadrilateral *JKLM*, $\angle J$ and $\angle M$ are supplementary and $\overline{JK} \cong \overline{ML}$.

Is *JKLM* a parallelogram?

SOLUTION

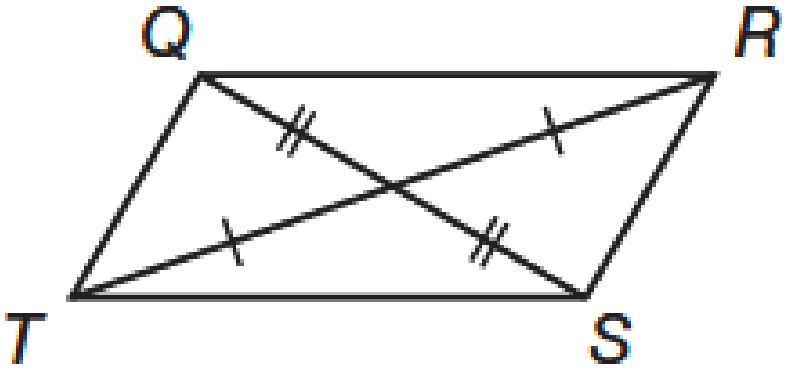
Since $\angle J$ and $\angle M$ are supplementary, then by the Converse of the Same-Side Interior Angles Theorem, we know that $\overline{JK} \parallel \overline{ML}$.

Since the opposite sides $\overline{JK}$ and $\overline{ML}$ are both parallel and congruent, *JKLM* is a parallelogram.



Identifying Parallelograms – If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram.

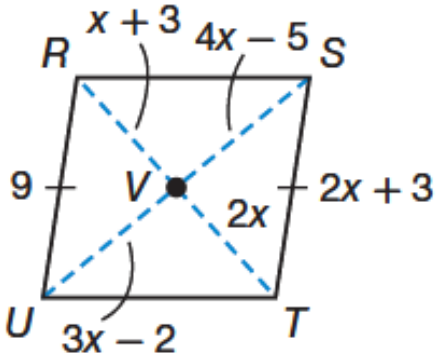
QRST is a parallelogram.



Example 4 Proving a Quadrilateral is a Parallelogram Using Diagonals

In quadrilateral *RSTU*, $\overline{RU} \cong \overline{ST}$. Is *RSTU* a parallelogram?

SOLUTION



Since $\overline{RU} \cong \overline{ST}$, $RU = ST$.

Substitute the given values and solve for x.

$RU = ST$		Given
$9 = 2x + 3$		Substitute.
$x = 3$		Solve.

Now that x is known, it can be used to find the lengths of each diagonal segment in the quadrilateral. We find that $RV = 6$, $VT = 6$, $UV = 7$, and $VS = 7$.

The segments of each diagonal are equal, so V is the midpoint of each one.

Therefore, the diagonals bisect each other, which proves that *RSTU* is a parallelogram.

Example 5 Application: Gardening

A gardener wants to know how much fencing to buy for the perimeter of her garden, shown below. The garden has two paths that bisect each other to form an "X." How much fencing does the gardener need?

SOLUTION

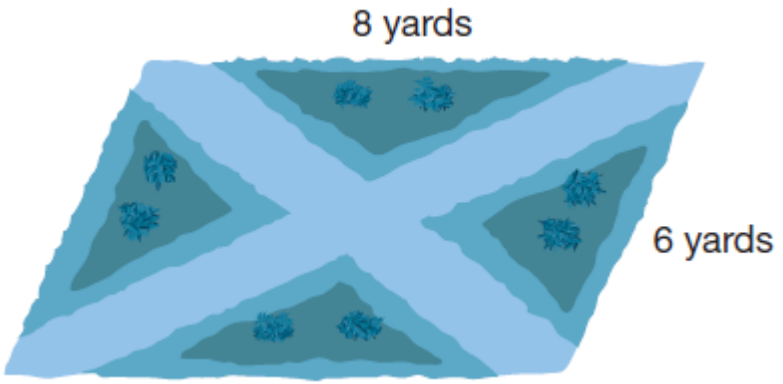
The diagonals bisect each other, so the quadrilateral is a parallelogram and opposite sides are equal.

Calculate the perimeter.

$P = 2(8) + 2(6)$

$P = 28$

The gardener needs 28 yards of fencing.



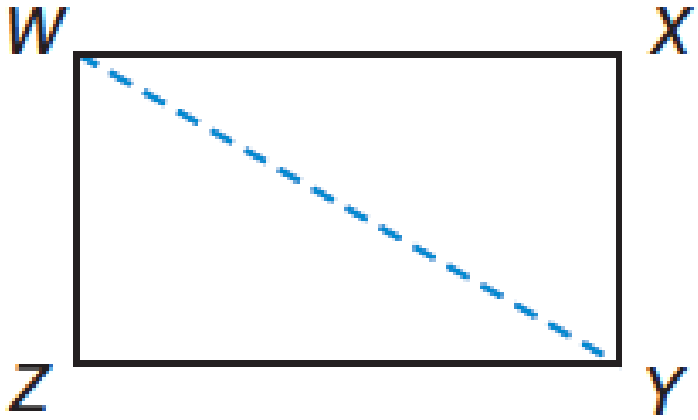
You Try!!!!!

a.In quadrilateral *ABCD*, $\overline{AD} \cong \overline{BC}$ and $\overline{AB} \cong \overline{DC}$. Prove that the diagonals of *ABCD* bisect each other.

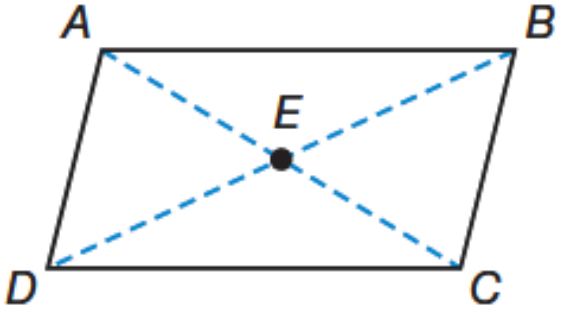
b. In quadrilateral *EFGH*, $\angle E \cong \angle G$ and $\angle F \cong \angle H$. Prove that the opposite sides are congruent.

You Try!!!!!

c.In quadrilateral *WXYZ*, $\Delta WXY \cong \Delta YZW$. Prove that *WXYZ* is a parallelogram by showing that $\overline{WX} \parallel \overline{ZY}$ and $\overline{WX} \cong \overline{ZY}$.

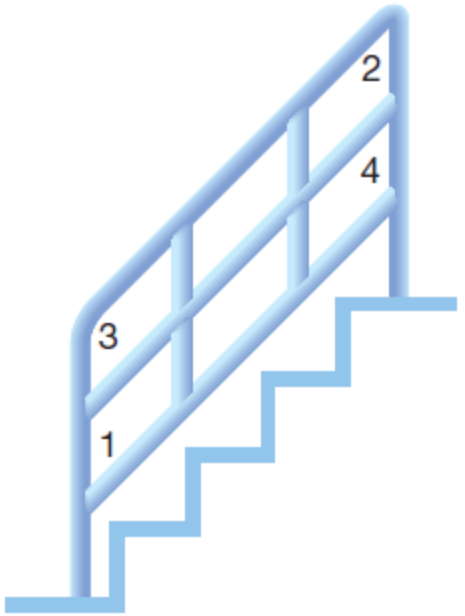


d.In the diagram, $\Delta AED \cong \Delta CEB$. Prove that quadrilateral *ABCD* is a parallelogram.



You Try!!!!!

e.A school has a railing on the front staircase. If $\angle 1 \cong \angle 2$ and $\angle 3 \cong \angle 4$, prove that the top railing and the bottom railing are parallel.



Assignment

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Lesson Practice (Ask Mr. Heintz)

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Practice 1-30 (Do the starred ones first)