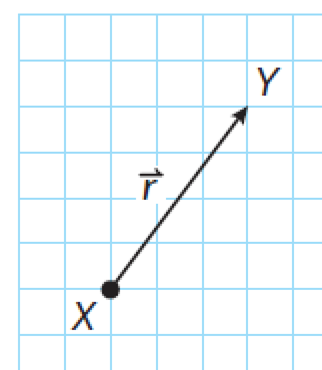
Lesson 63

Introduction to Vectors

Vector – A quantity that has both magnitude and direction.

Direction of a vector – The orientation of the vector, which is determined by the angle the vector makes with a horizontal line.

Scalar – A quantity that consists only of magnitude and has no direction.



Vectors are named by an italicized, lowercase letter with the vector symbol. For example, the vector above is named $\vec{r}$.

Initial point of a vector – The starting point of a vector.

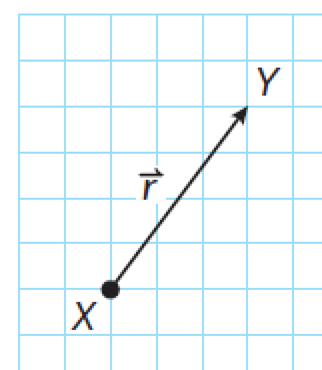
Terminal point of a vector – The endpoint of a vector.

In the diagram, X is the initial point and Y is the terminal point of $\vec{r}$.

The arrow at Y indicates the direction of the vector.

Reading Math

A vector can also be named by its initial point and terminal point. For example, the vector in the diagram could also be called $\overrightarrow{XY}$.

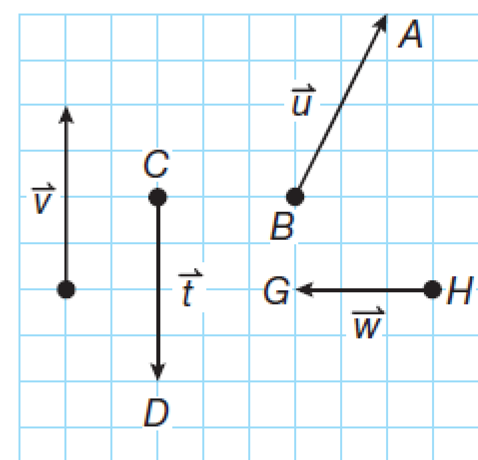


Example 1 Identifying Vectors and Scalars

Name each vector shown. Identify the terminal points of each vector, if applicable.

SOLUTION

Each vector should be named and then the terminal points given, in that order. The vectors, therefore, are $\vec{v}$, $\vec{u}$ with terminal point *A*, $\vec{t}$, with terminal point *D*, and $\vec{w}$, with terminal point *G*.



Magnitude of a vector – The length of a vector. Since magnitude is a length, absolute value bars are used to represent the magnitude of a vector. The magnitude of $\vec{v}$, for example, would be written $|\vec{v}|$.

The location of a vector on the coordinate plane is not fixed. It can be placed anywhere, so for simplicity the initial point of a vector is usually placed on the origin of the coordinate plane. To find the magnitude of a vector, place the initial point of the vector on the origin and use the distance formula.

Example 2 Finding the Magnitude of a Vector

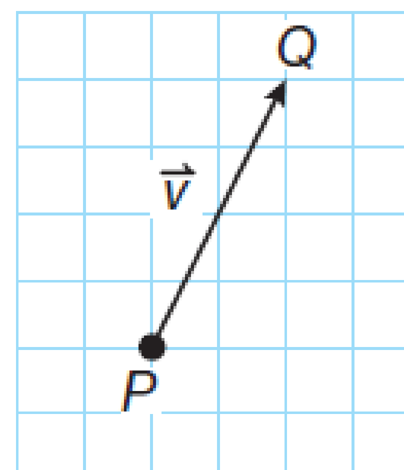
Find $|\vec{v}|$.

SOLUTION

The initial point of this vector is *P*. If *P* is placed on the origin, *Q* will be the point located two units to the right and four units up from *P*, so the coordinates of *Q* are (2, 4).

Use the distance formula to find the distance between *P*(0, 0) and *Q*(2, 4).

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$
$$d = \sqrt{(2-0)^2 + (4-0)^2}$$
$$d = \sqrt{2^2 + 4^2}$$
$$d = \sqrt{4 + 16}$$
$$d = \sqrt{20}$$
$$d = 2\sqrt{5}$$



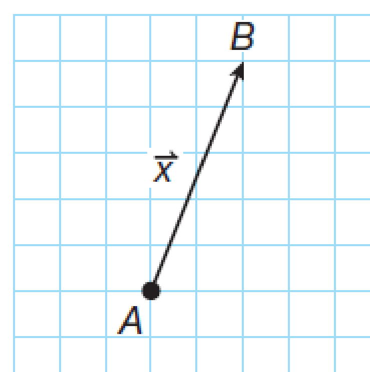
So $|\vec{v}|$ is $2\sqrt{5}$.

The component form of a vector lists its horizontal and vertical change from the initial point to the terminal point.

For example, $\vec{x}$ written in component form would be $\langle 2, 5\rangle$. The horizontal change is listed first, followed by the vertical change.

Opposite vectors – Two vectors with opposite components. Vectors that have the same magnitude but opposite directions. The opposite vector of $\langle 2, 5\rangle$ is $\langle -2, -5\rangle$.

Any two vectors can be added together by summing their components. The vector that represents the sum or difference of two given vectors is a resultant vector.



Reading Math

The brackets $\langle\ \rangle$ used in component form show that the pair indicates a vector, instead of coordinates on a grid.

Example 3 Adding Vectors

a. Add vectors $\vec{r}$ and $\vec{t}$.

SOLUTION

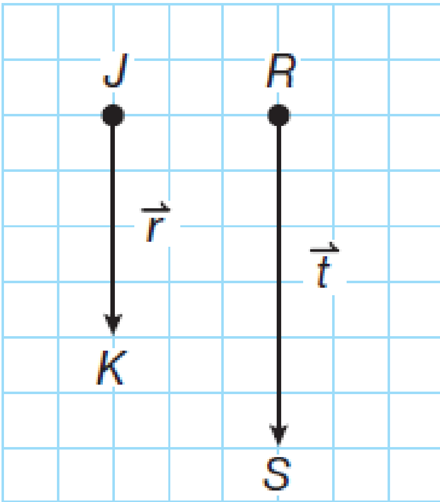
First, write each vector in component form.

The component form of $\vec{r}$ is $\langle 0, -4 \rangle$, because there is no horizontal distance between *J* and *K*, but there is a negative vertical change of 4 units.

The component form of $\vec{t}$ is $\langle 0, -6 \rangle$.

Add the components: $\langle 0 + 0, -4 + -6 \rangle = \langle 0, -10 \rangle$

The resultant vector from adding these two vectors is $\langle 0, -10 \rangle$.



Example 3 Adding Vectors

b. Add vectors $\vec{u}$ and $\vec{v}$.

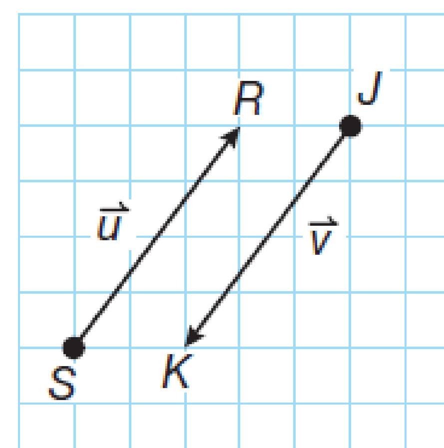
SOLUTION

First, write each vector in component form.

The component form of $\vec{u}$ is $\langle 3, 4\rangle$.

The component form of $\vec{v}$ is $\langle -3, -4\rangle$

Since $\vec{u}$ and $\vec{v}$ are opposite vectors, their components sum to 0. The resultant vector is $\langle 0, 0\rangle$.



Equal vectors – Vectors that have the same magnitude and direction. An easy way to add equal vectors is to multiply the vector by a constant. This is known as scalar multiplication of a vector. For example, to add $\langle 1, 2\rangle$ and

$\langle 1, 2\rangle$, simply multiply $\langle 1, 2\rangle$ by the scalar 2. The resultant vector is $\langle 2, 4\rangle$, which has a magnitude that is twice that of $\langle 1, 2\rangle$.

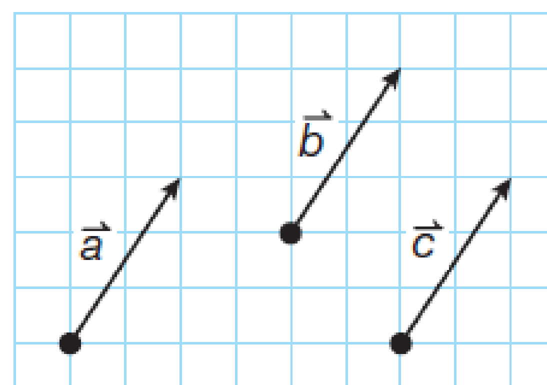
Example 4 Adding Equal Vectors

Add the equal vectors $\vec{a}$, $\vec{b}$, and $\vec{c}$.

SOLUTION

In component form, all three of these vectors are $\langle 2, 3\rangle$. Since there are three equal vectors, multiply the component form of the vectors by the scalar 3.

The resultant vector is $\langle 3 \times 2, 3 \times 3\rangle = \langle 6, 9\rangle$.



Example 5 Application: Currents

A rower on a lake is rowing a boat at a rate of 5 miles per hour. A current is moving at 2 miles per hour in the opposite direction as the boat. How fast is the rower traveling over the ground below?

SOLUTION

Use the four-step problem-solving plan.

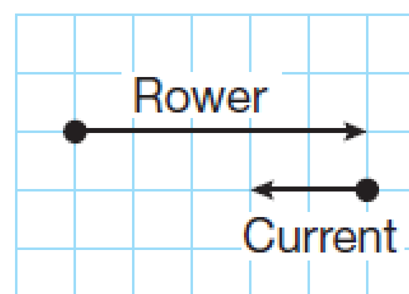
Understand: Sketch the vectors for the rower and the current. The direction of the rower's vector does not matter, as long as the current's vector is pointing in the opposite direction. The magnitude of the rower's vector is 5, and the magnitude of the current's vector is 2.

Plan: As in Examples 3 and 4, the vectors need to be added. First, find the component form of the vectors. Then, add them together.

Solve: The component form of the rower's vector is $\langle 5, 0\rangle$, and the current's vector is $\langle -2, 0\rangle$. Add the vectors. $\langle -2 + 5, 0 + 0\rangle = \langle 3, 0\rangle$

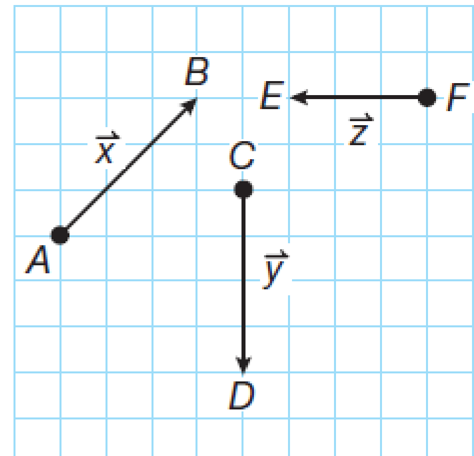
So the rower is traveling at 3 miles per hour.

Check: Does it make sense that the current would be slowing the boat's progress? It does, because the current is flowing in the opposite direction.



You Try!!!!

a. Name the vectors and identify the initial point of each one.

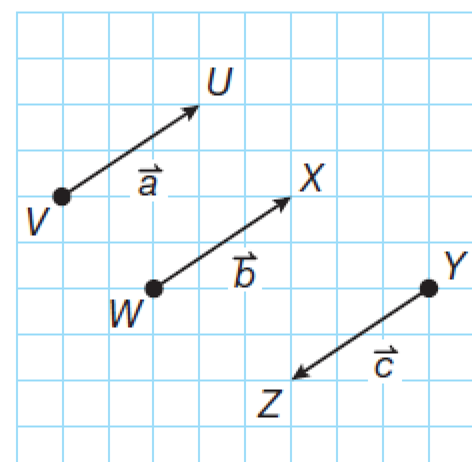


b. Find the magnitude of the vector $\langle 5, 3\rangle$ in simplified radical form.

You Try!!!!

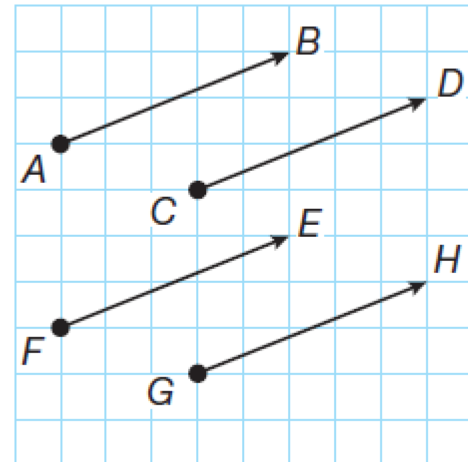
c. Add vectors $\vec{a}$ and $\vec{b}$.

d. Add vectors $\vec{b}$ and $\vec{c}$.



You Try!!!!

e.Add the four vectors.



f.A canoe is traveling down a river. In still water, the canoe would be traveling at 2 miles per hour. The river is flowing 1.5 miles per hour in the same direction as the canoe.

How fast is the canoe actually traveling?

Assignment

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Lesson Practice (Ask Mr. Heintz)

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Practice 1–30 (Do the starred ones first)