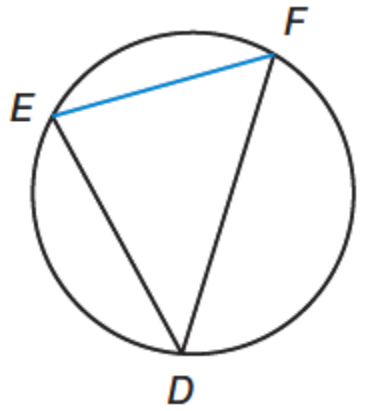
Lesson 64

Angles Interior to Circles

A segment or arc is said to subtend an angle if the endpoints of the segments or arc lie on the sides of the angle. In the diagram, $\angle EDF$ is subtended by $\widehat{EF}$ or $\overline{EF}$.

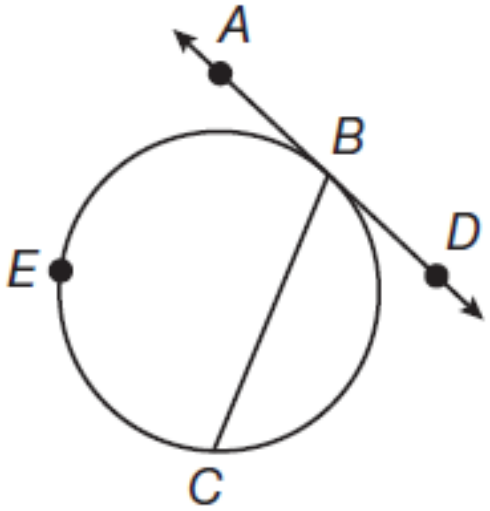
Inscribed angles are one type of subtended angle. Another type of subtended angle is one formed by a tangent to the circle and a chord of the circle.



Theorem 64-1 – The measure of an angle formed by a tangent and a chord is equal to half the measure of the arc that subtends it.

$$m\angle ABC = \frac{1}{2} m\widehat{BEC}$$

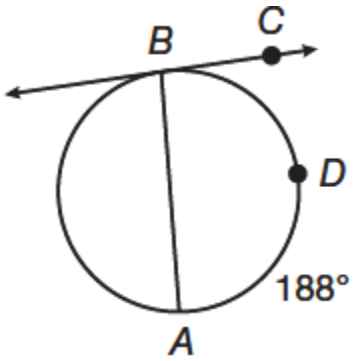
$$m\angle CBD = \frac{1}{2} m\widehat{BC}$$



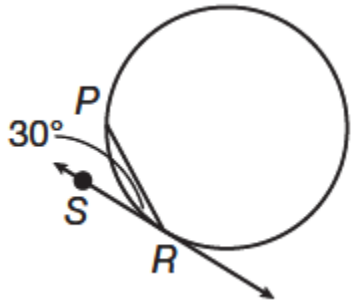
Example 1 Finding Angle Measures with Tangents and Chords

Find the indicated measure, given that $\overline{BC}$ and $\overline{SR}$ are tangents.

a. m$\angle ABC$



b. m$\overset{\frown}{PR}$



SOLUTION

In the first example, $\angle ABC$ is subtended by $\overset{\frown}{ADB}$, so its measure will be half the measure of $\overset{\frown}{ADB}$.

Since $\overset{\frown}{ADB}$ measures 188°, $\angle ABC$ measures 94°.

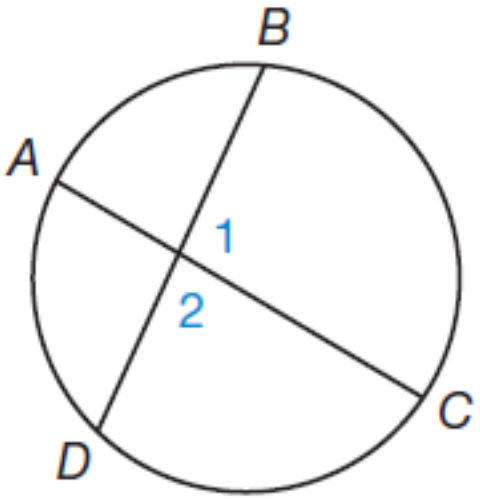
In the second example, $\overset{\frown}{PR}$ subtends $\angle PRS$, so $\angle PRS$ is half the measure of $\overset{\frown}{PR}$.

Since the measure of $\angle PRS$ is 30°, $\overset{\frown}{PR}$ measures twice that, or 60°.

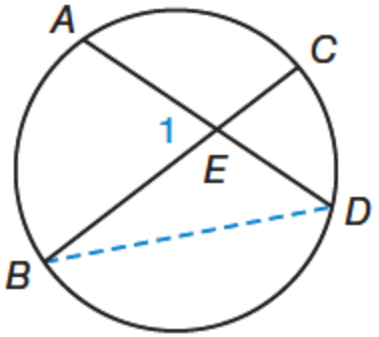
Theorem 64-2 – The measure of an angle formed by two chords B intersecting in a circle is equal to half the sum of the intersected arcs.

$$m\angle 1 = \frac{1}{2}\left(m\widehat{AD} + m\widehat{BC}\right)$$

$$m\angle 2 = \frac{1}{2}\left(m\widehat{AB} + m\widehat{DC}\right)$$



Example 2 Proving Theorem 64-2



Given: $\overline{AD}$ and $\overline{BC}$ intersect at *E*.

Prove: $m\angle 1 = \frac{1}{2}\left(m\widehat{AB} + m\widehat{CD}\right)$

SOLUTION

Statements	Reasons
1. $\overline{AD}$ and $\overline{BC}$ intersect at *E*.	1. Given
2. Draw $\overline{BD}$	2. Two points determine a line
3. $m\angle 1 = m\angle EDB + m\angle EBD$	3. Exterior Angle Theorem
4. $m\angle EDB = \frac{1}{2}m\widehat{AB}$ $m\angle EBD = \frac{1}{2}m\widehat{CD}$	4. Inscribed Angle Theorem
5. $m\angle 1 = \frac{1}{2}m\widehat{AB} + \frac{1}{2}m\widehat{CD}$	5. Substitution Property of Equality
6. $m\angle 1 = \frac{1}{2}\left(m\widehat{AB} + m\widehat{CD}\right)$	6. Distributive Property

Example 3 Finding Angle Measures of the Intersection of Two Chords

Find x.

SOLUTION

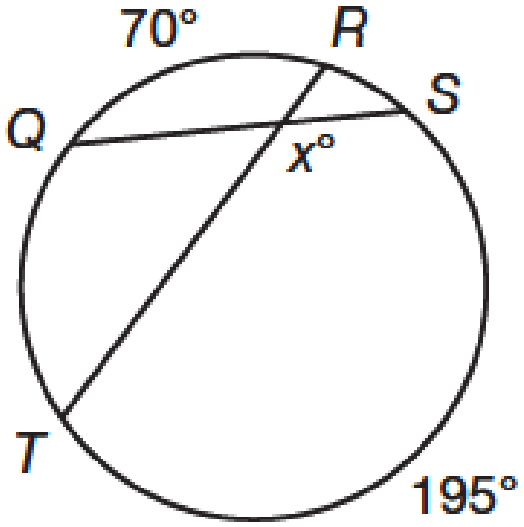
Theorem 64–2 says that the value of x will be equal to half the sum of the two arcs that subtend it.

Apply the formula from 64–2.

$$x = \frac{1}{2}\left(m\widehat{QR} + m\widehat{ST}\right)$$

$$x = \frac{1}{2}(70° + 195°)$$

$$x = 132.5°$$



Example 4 Application: Tiling

Albert is laying tile in his kitchen in a circular pattern as shown. He knows the $m\widehat{AB} = 50°$ and $m\widehat{CD} = 86°$. He wants to know the measure of angle 1 so he can cut the tile accordingly.

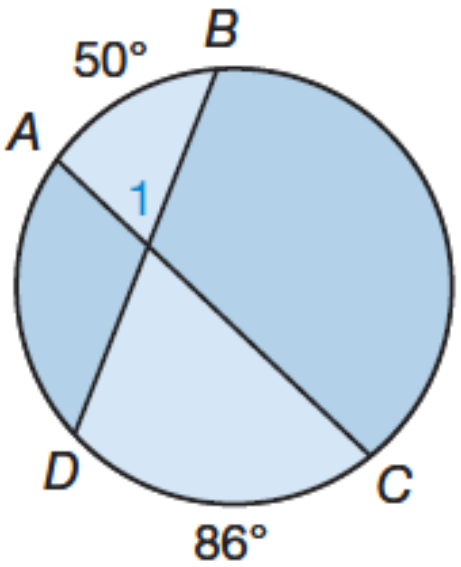
SOLUTION

$$m\angle 1 = \frac{1}{2}\left(m\widehat{AB} + m\widehat{CD}\right)$$

$$m\angle 1 = \frac{1}{2}(50° + 86°)$$

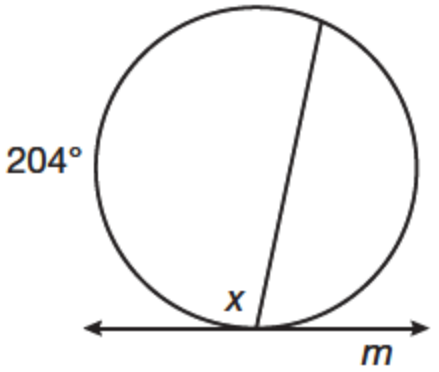
$$m\angle 1 = 68°$$

So, m$\angle$1 = 68°.

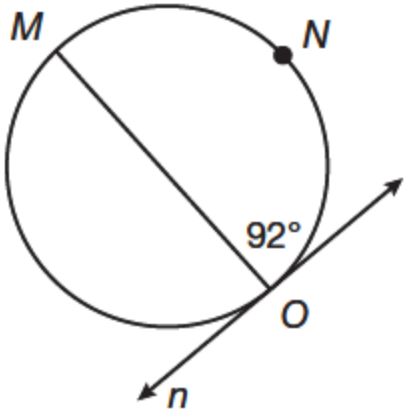


You Try!!!!!

a. Find the measure of angle x in the figure. Line m is tangent to the circle.



b. Find the measure of $\overset{\frown}{MNO}$ in the figure. Line n is tangent to the circle.



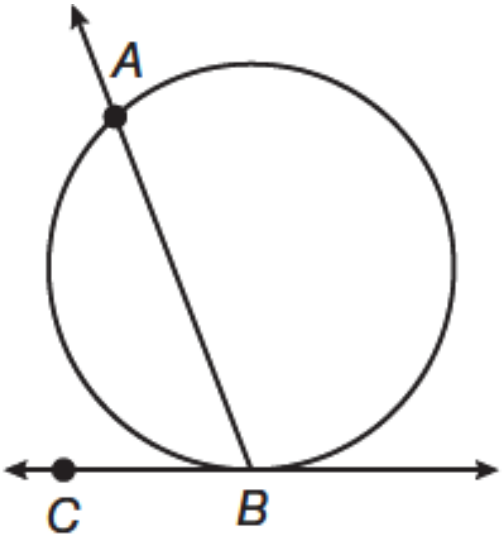
You Try!!!!!

c. Prove Theorem 64-1.

Given: Tangent $\overleftrightarrow{BC}$ and secant $\overrightarrow{BA}$.

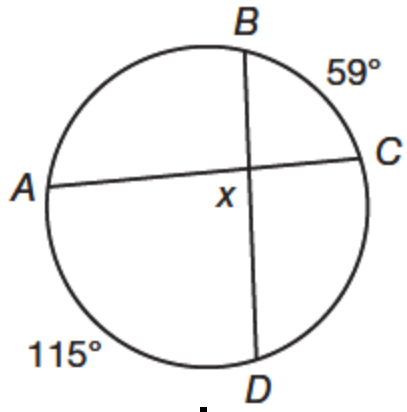
Prove: $m\angle ABC = \frac{1}{2}m\widehat{AB}$

Hint: There are two cases you must prove: one where $\overline{AB}$ is a diameter and one where $\overline{AB}$ is not a diameter.

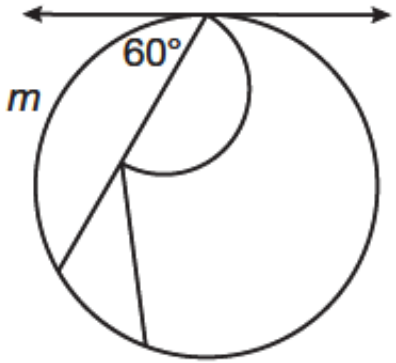


You Try!!!!!

d. Find the measure of angle *x*.



e. An artist is drawing a design for a company logo that has a capital "R" inside a large circle as shown. She first draws a baseline at the top of the R. The R is supposed to be at a 60° angle in relation to the baseline. What is the measure of the arc *m*, which extends leftward from the top of the R?



Assignment

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Lesson Practice (Ask Mr. Heintz)

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Practice 1-30 (Do the starred ones first)