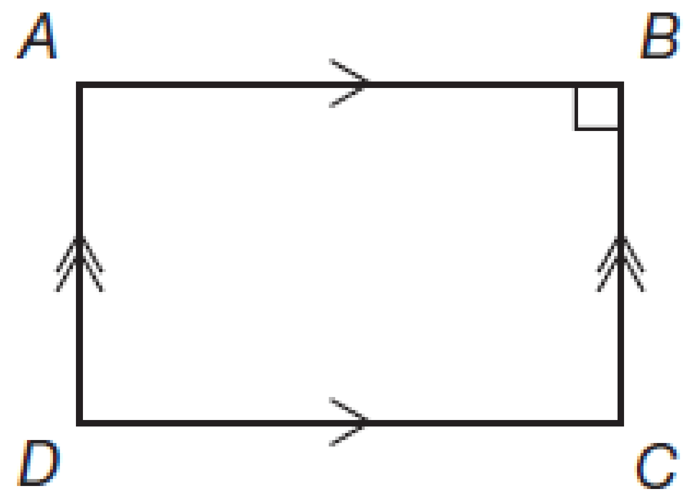
Lesson 65

Distinguishing Types of Parallelograms

Lesson 61 presented several methods for determining if a quadrilateral is a parallelogram. The properties presented in this lesson make it possible to determine if a parallelogram is a rectangle, square, or rhombus.

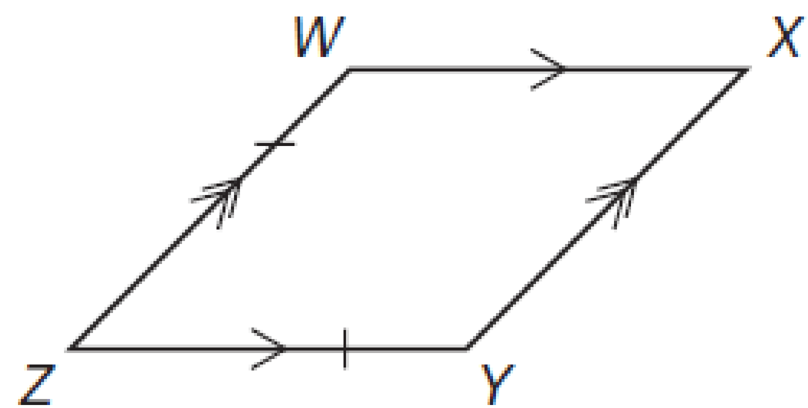
Properties of Parallelograms – If an angle in a parallelogram is a right angle then the parallelogram is a rectangle.

Since $\angle B$ is a right angle, $ABCD$ is a rectangle.



Properties of Parallelograms – If consecutive sides of a parallelogram are congruent, then the parallelogram is a rhombus.

Since $\overline{WZ} \cong \overline{ZY}$, *WXYZ* is a rhombus.



Example 1 Proving Parallelograms Are Rhombuses

Is this parallelogram a rhombus if $x = 11$?

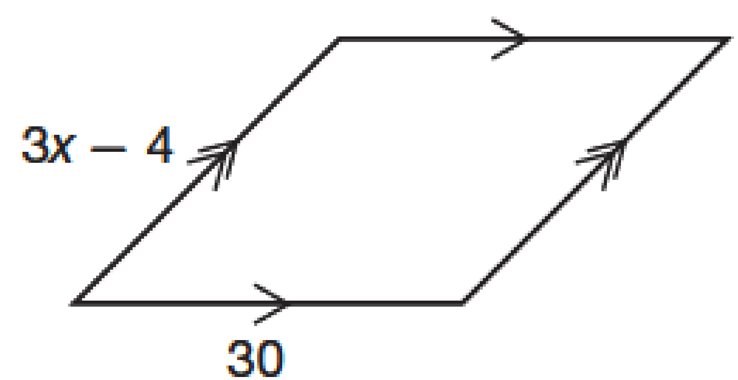
SOLUTION

To be a rhombus, two consecutive sides must be congruent. Substitute for x in the expression for the length of the side.

$3x - 4 = 30$

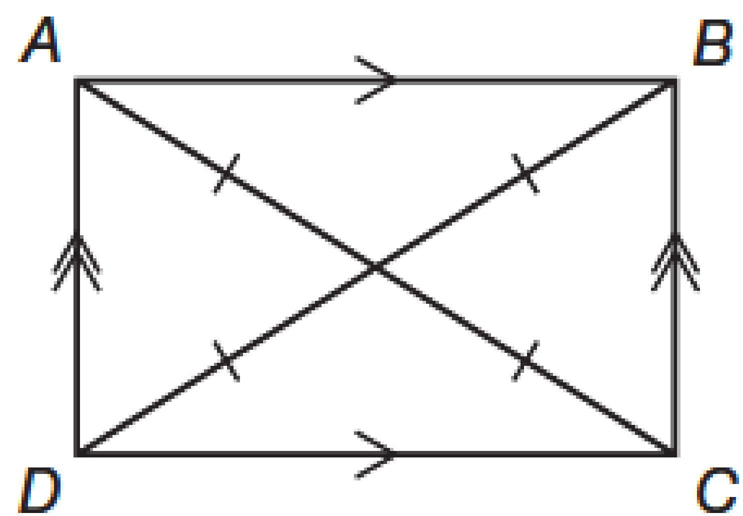
$3(11) - 4 = 29$

Since this side is not congruent to the side that measures 30 units, the quadrilateral is not a rhombus.



Properties of Parallelograms – If the diagonals of a parallelogram are congruent then it is a rectangle.

Since $\overline{AC} \cong \overline{BD}$, $ABCD$ is a rectangle.



Example 2 Proving Parallelograms are Rectangles

Is parallelogram *HIJK* a rectangle?

SOLUTION

Since $\angle HLI$ and $\angle KLJ$ are vertical angles, they are congruent.

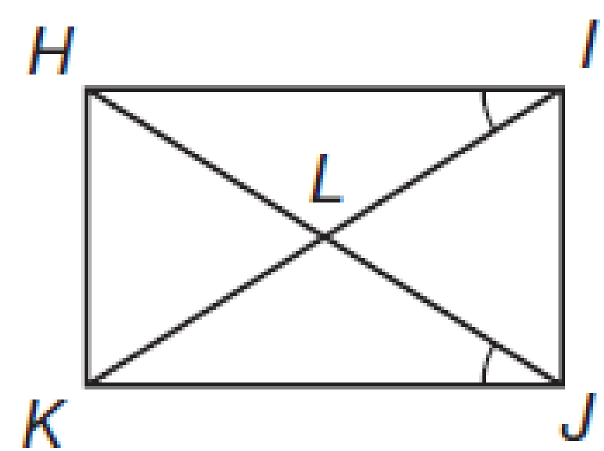
Opposite sides in a parallelogram are congruent, so $\overline{HI} \cong \overline{KJ}$.

By Angle-Angle-Side Triangle Congruence, $\Delta HLK \cong \Delta JLK$.

By CPCTC and the definition of congruent segments, $LI = LJ$ and $LH = LK$.

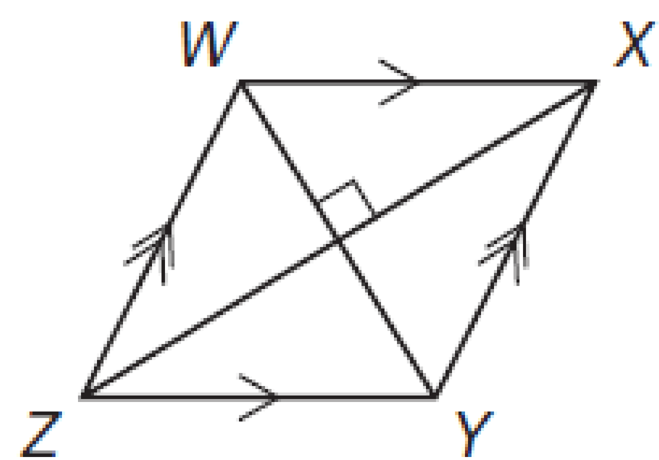
By the Addition Property of Equality $LI + LK = LJ + LK$, and by substitution, $LI + LK = LJ + LH$.

Therefore, the two diagonals are congruent and the parallelogram is a rectangle.



Properties of Parallelograms – If the diagonals of a parallelogram are perpendicular then it is a rhombus.

Since $\overline{WY}$ is a perpendicular to $\overline{ZX}$, *WXYZ* is a rhombus.



Example 3 Proving Parallelograms are Rhombuses

Is parallelogram *KLMN* a rhombus?

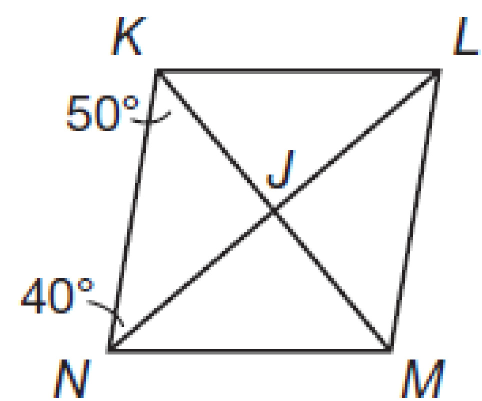
SOLUTION

Use the Triangle Angle Sum Theorem in ΔKJN to determine the angle measure of $\angle KJN$.

$50° + 40° + m\angle KJN = 180°$

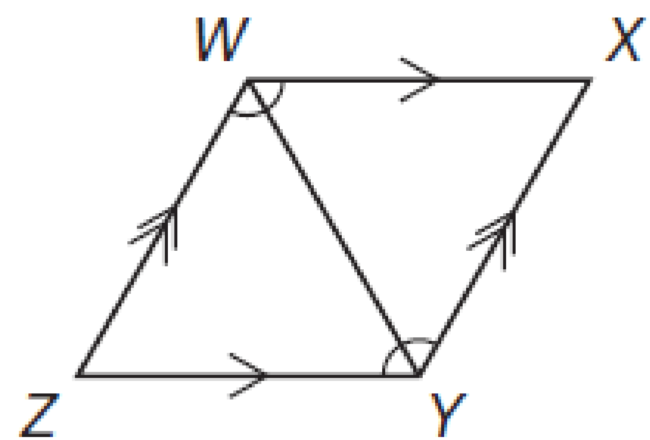
$m\angle KJN = 90°$

Since they form a right angle, $\overline{KM}$ and $\overline{NL}$ are perpendicular, which means *KLMN* is a rhombus.



Properties of Parallelograms – If a diagonal in a parallelogram bisects opposite angles, then it is a rhombus.

Since $\angle XWY \cong \angle ZWY$ and $\angle XYW \cong \angle ZYW$, *WXYZ* is a rhombus.



Example 4 Proving Parallelograms are Rhombuses

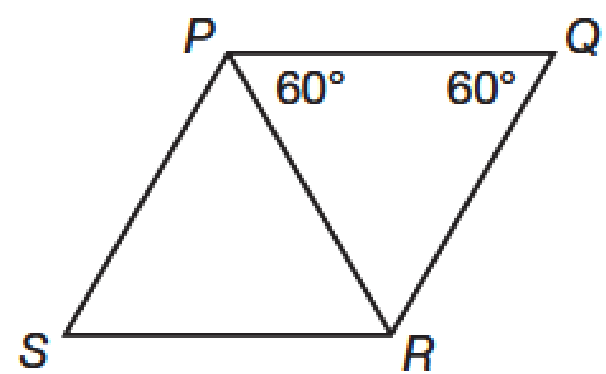
Is parallelogram *PQRS* a rhombus?

SOLUTION

From the diagram, ΔPQR is an equilateral triangle, with $m\angle PRQ = 60^\circ$.

Since *PQRS* is a parallelogram, the Alternate Interior Angles Theorem can be used to show that $\angle QPR \cong \angle PRS$ and $\angle PRQ \cong \angle RPS$.

Therefore, $\overline{PR}$ bisects both $\angle P$ and $\angle R$, and *PQRS* is a rhombus.



Example 5 Application: Signs

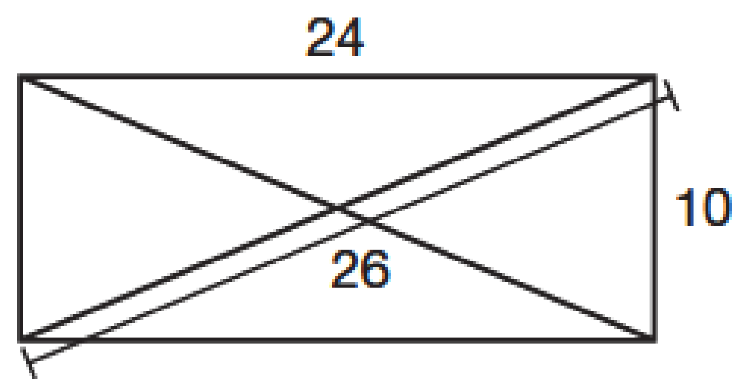
A sign maker is commissioned to make a rectangular sign. The sign needs to be a perfect rectangle. Given the measurements shown in the diagram, is the sign a rectangle? How do you know?

SOLUTION

The length of one diagonal is given. The length of the other one can be determined using the Pythagorean Theorem.

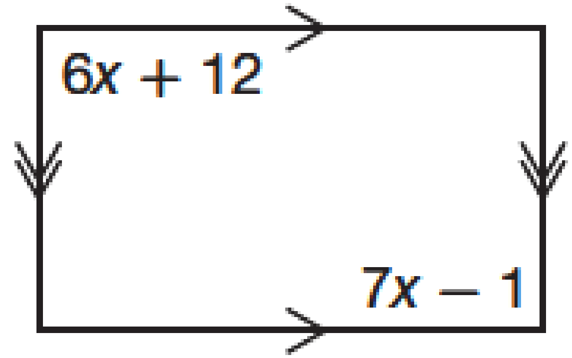
$a^2 + b^2 = c^2$	Pythagorean Theorem
$10^2 + 24^2 = c^2$	Substitute.
$c = 26$	Solve.

Since the lengths of the two diagonals are the same, they are congruent and the sign is a perfect rectangle.

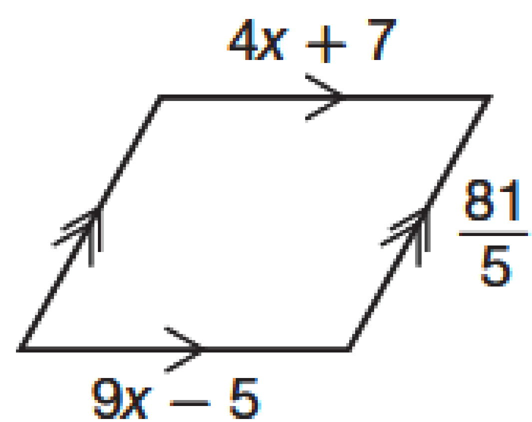


You Try!!!!!

a. Is this parallelogram a rectangle?

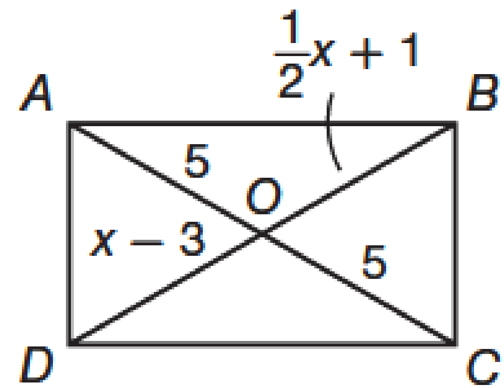


b. Is this parallelogram a rhombus?

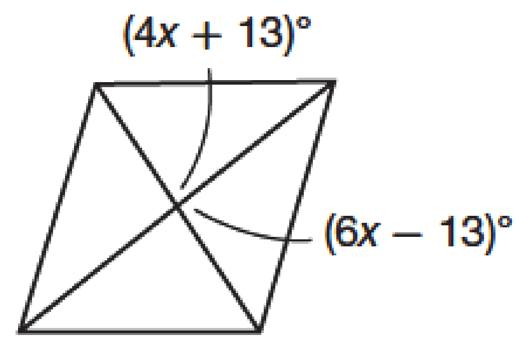


You Try!!!!!

c. Is this parallelogram a rectangle?

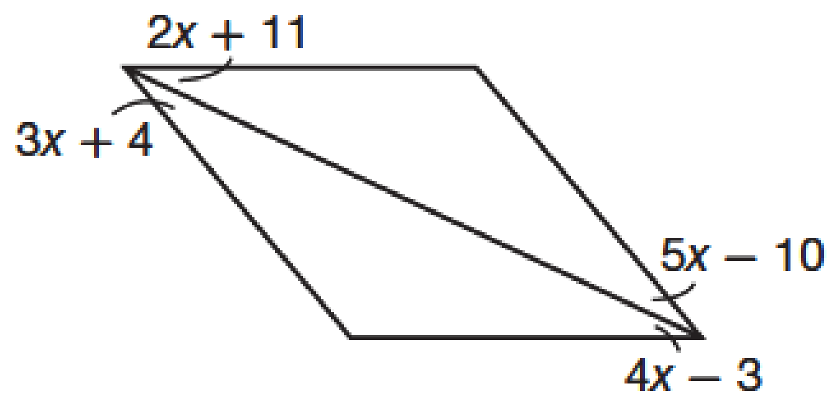


d. Is this parallelogram a rhombus?

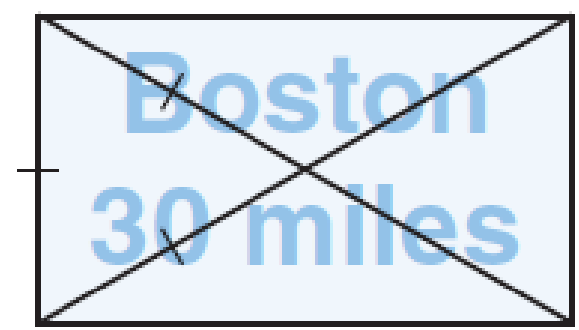


You Try!!!!!

e.Is this parallelogram a rhombus?



f.A sign in the shape of a parallelogram has diagonals that create an equilateral triangle as shown. Is the sign a perfect rectangle? Explain how you know.



Assignment

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Lesson Practice (Ask Mr. Heintz)

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Practice 1–30 (Do the starred ones first)