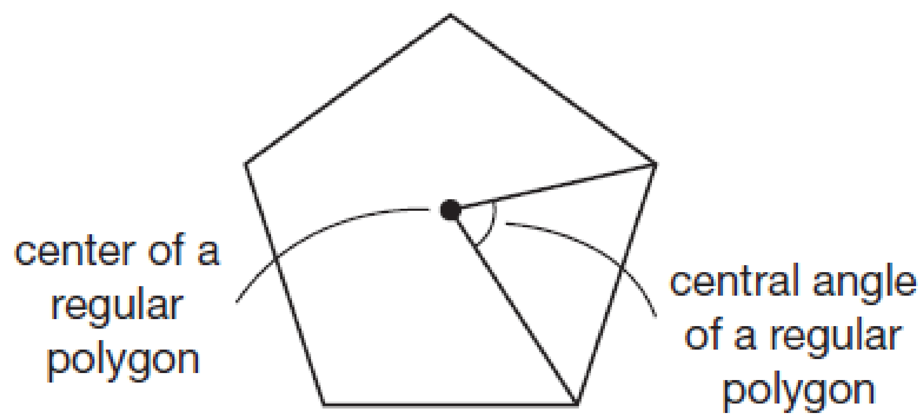
Lesson 66

Finding Perimeters and Areas of Regular Polygons

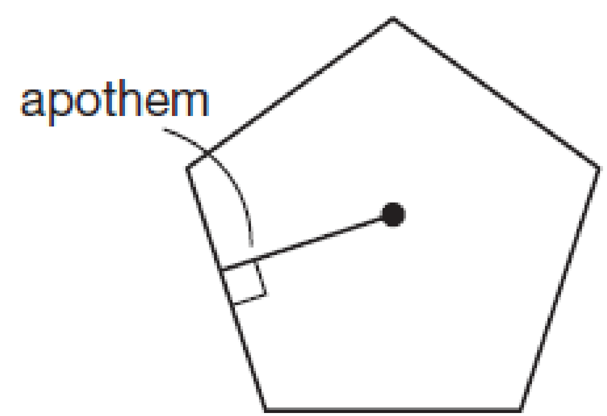
Center of a Regular Polygon – A point within the polygon that is equidistant from all vertices.

Central Angle of a Regular Polygon – The angle whose vertex is the center of a regular polygon and whose sides pass through consecutive vertices.



Apothem – The perpendicular distance from the center of a regular polygon to a side.

You can use the formula $P = ns$ to find the perimeter of a regular polygon. In the formula, P represents the perimeter, n represents the number of sides, and s represents the side length.



Example 1 Finding Perimeters of Regular Polygons

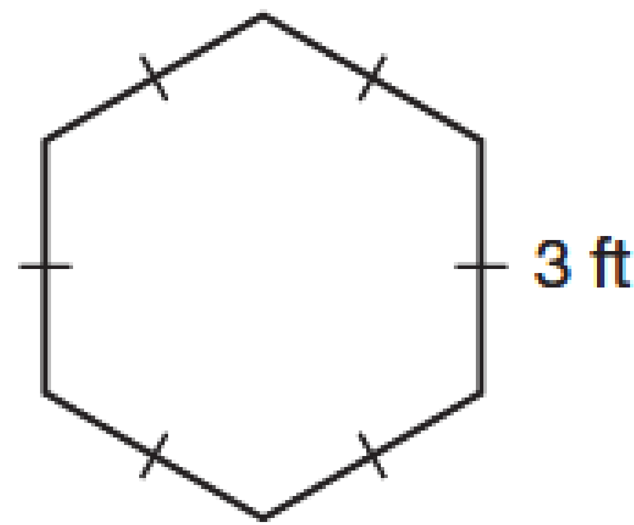
a. Find the perimeter of the polygon.

SOLUTION

$P = ns$ Formula for perimeter of a regular polygon

$P = (6)(3)$ Substitute.

$P = 18$ ft

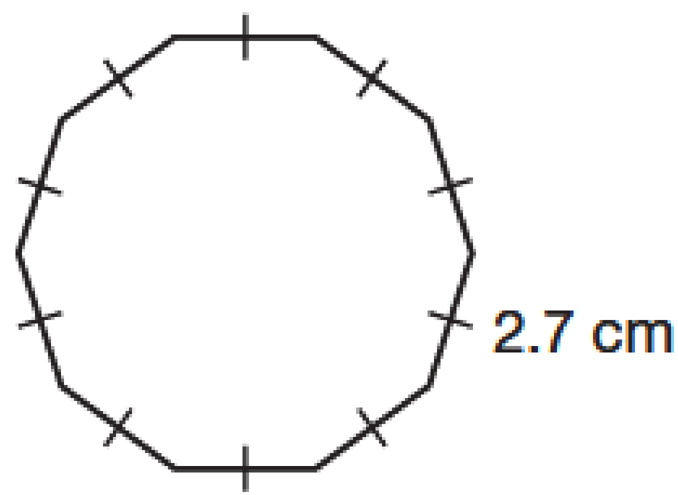


Example 1 Finding Perimeters of Regular Polygons

b. Find the perimeter of the polygon.

SOLUTION

$P = ns$	Formula for perimeter of a regular polygon
$P = 10 \cdot 2.7$	Substitute.
$P = 27$ cm	



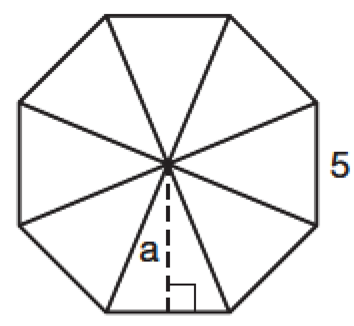
You can find the area, A, of a regular polygon using only the apothem and perimeter. Consider an n–sided regular polygon with a side length of s. Divide the polygon into n triangles so the vertices of each triangle are the center of the polygon and the endpoints of a side as shown. By definition, the height of each triangle is the apothem, a.

The base of each triangle has a length of s. So, the area of each triangle is $\frac{1}{2}as$. The total area of the polygon is n times the area of one triangle, or

$$A = \frac{1}{2}nas$$

The formula for the perimeter of a regular polygon is $P = ns$. By substitution, the area of a regular polygon is

$$A = \frac{1}{2}aP$$



Area Formula for Regular Polygons – The area, A, of a regular polygon is half the apothem length a and the perimeter P of the regular polygon.

$$A = \frac{1}{2}aP$$

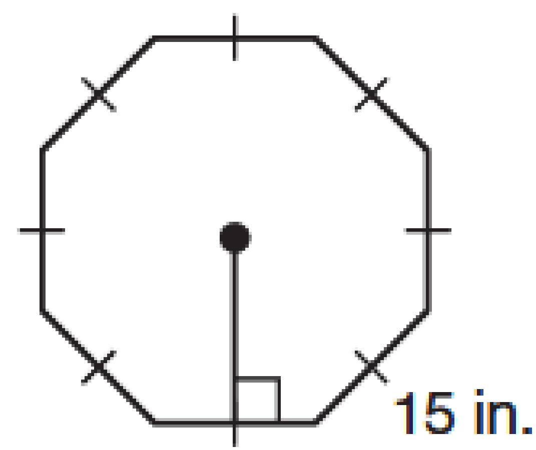
Example 2 Using the Area Formula

Find the area of a regular octagon with an apothem about 18 inches.

SOLUTION

$P = ns$	Formula for perimeter of a regular polygon
$P = (8)(15)$	Substitute.
$P = 120$	Simplify.
$A = \frac{1}{2}aP$	Area formula for regular polygons
$A = \frac{1}{2}(18)(120)$	Substitute.
$A = 1080$	Simplify.

The area is 1080 square inches.



Example 3 Finding the Area of a Regular Hexagon

Use the apothem and perimeter to find the area of this regular hexagon.

SOLUTION

First, find the apothem length of the regular hexagon.

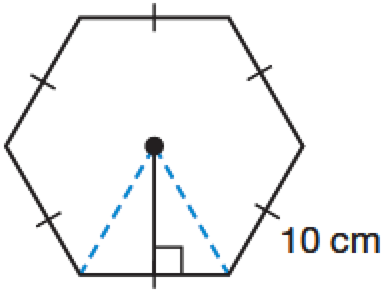
Draw an isosceles triangle whose vertices are the center of the hexagon and the endpoints of a side.

The triangle contains a central angle whose measure is 60°.

The apothem bisects the central angle and the side, forming a 30°–60°–90° triangle.

The shorter leg of the triangle is 5 centimeters long.

Therefore, the apothem measures $5\sqrt{3}$ centimeters.



Example 3 Finding the Area of a Regular Hexagon

Use the apothem and perimeter to find the area of this regular hexagon.

SOLUTION

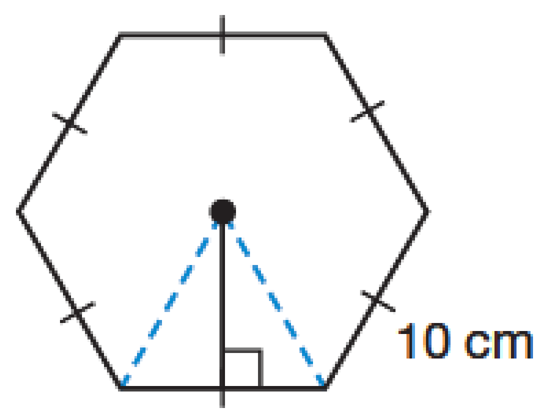
Next, find the perimeter of the hexagon.

$P = ns$	Formula for perimeter of a regular polygon
$P = (6)(10)$	Substitute.
$P = 60$	Simplify.

Finally, find the area of the hexagon.

$A = \frac{1}{2}aP$	Area formula for regular polygons
$A = \frac{1}{2}(5\sqrt{3})(60)$	Substitute.
$A = 150\sqrt{3}$	Simplify.

The area is $150\sqrt{3}$ centimeters squared.



Example 4 Finding the Area of an Equilateral Triangle

Find the area of an equilateral triangle with 18-inch sides.

SOLUTION

Use your knowledge of 30°-60°-90° triangles to find the apothem, *a*, and then use your result to find the area.

$$\frac{a}{9} = \frac{1}{\sqrt{3}}$$

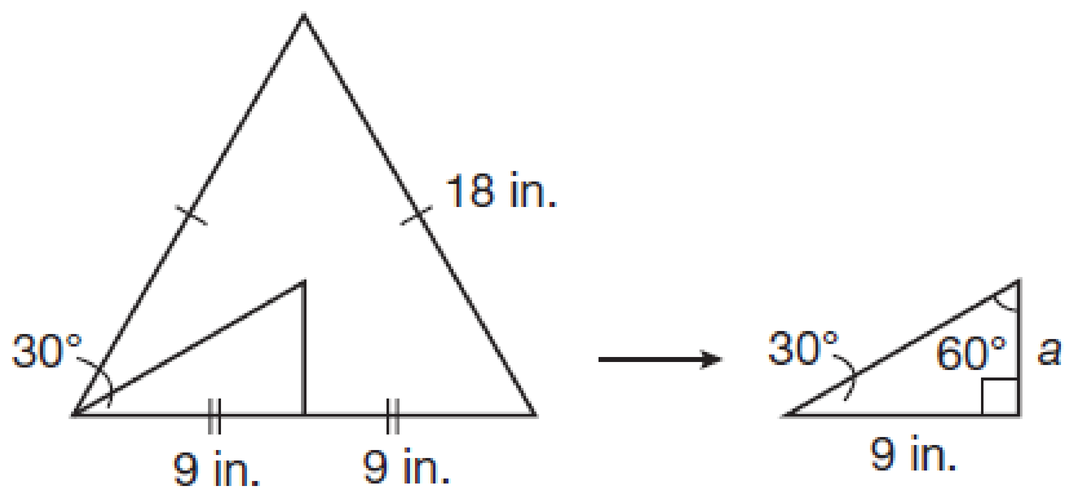
$$a = \frac{9}{\sqrt{3}}$$

$$a = 3\sqrt{3}$$

$$A = \frac{1}{2}aP$$

$$A = \frac{1}{2}(3\sqrt{3})(54)$$

$$A = 81\sqrt{3}$$



Example 5 Application: Land Survey

A plot of land is in the shape of a regular octagon with 10-mile side lengths and apothem of about 12 miles. The plot needs be divided into eight equal parcels of land. What will the area of land be in each parcel?

SOLUTION

First, find the area of the plot of land. Because the plot is in the shape of an octagon where each side is 10 miles long, its perimeter is 80 miles.

$A = \frac{1}{2}aP$	Area formula for regular polygons
$A = \frac{1}{2}(12)(80)$	Substitute.
$A = 480$	Simplify.

The area of the octagonal plot is $480\ mi^2$.

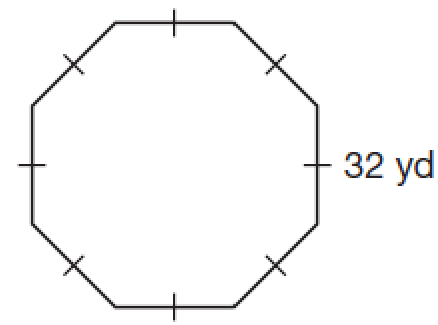
Next, divide the total area by 8 to find the area in each equal parcel.

$$\frac{480}{8} = 60$$

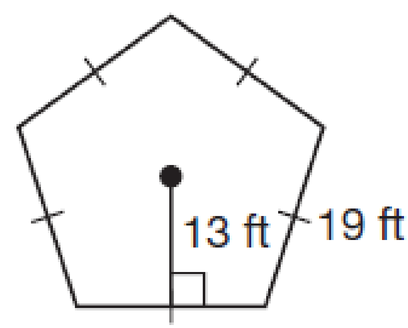
Each of the 8 parcels has an area of about $60\ mi^2$.

You Try!!!!!

a. Find the perimeter of this octagon.

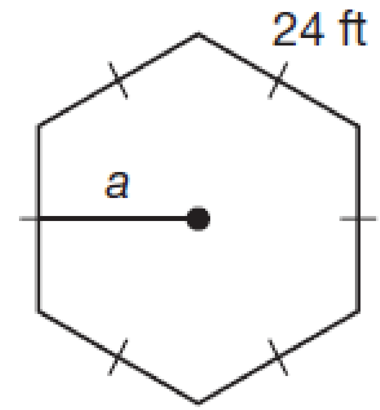


b. Use the area formula for regular polygons to find the area of this pentagon.

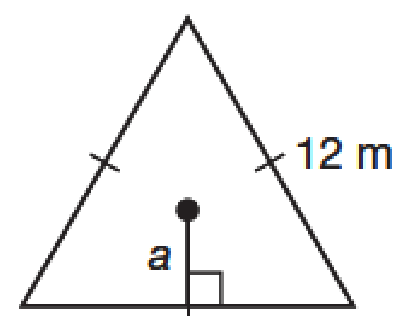


You Try!!!!!

c. Find the area of this hexagon.



d. Find the area of this equilateral triangle.



You Try!!!!!

e. The shape of a playground is a regular hexagon where each side length is 78 feet long. The playground is to be resurfaced with a nonslip rubber material. What is the total area that must be surfaced?

Assignment

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Lesson Practice (Ask Mr. Heintz)

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Practice 1–30 (Do the starred ones first)